Computer Algebra
Independent Integration Tests

Summer 2023 edition

1-Algebraic-functions/1.1-Binomial-products/1.1.2-Quadratic/22-1.1.2.5-a+b-x^2-^p-c+d-x^2-^q-e+f-x^2-^r

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September 5, 2023 Compiled on September 5, 2023 at 10:53pm

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [115]. This is test number [22].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (115)	0.00 (0)
Mathematica	99.13 (114)	0.87 (1)
Maple	93.04 (107)	6.96 (8)
Fricas	59.13 (68)	40.87 (47)
Giac	26.96 (31)	73.04 (84)
Mupad	23.48 (27)	76.52 (88)
Sympy	22.61 (26)	77.39 (89)
Maxima	13.04 (15)	86.96 (100)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

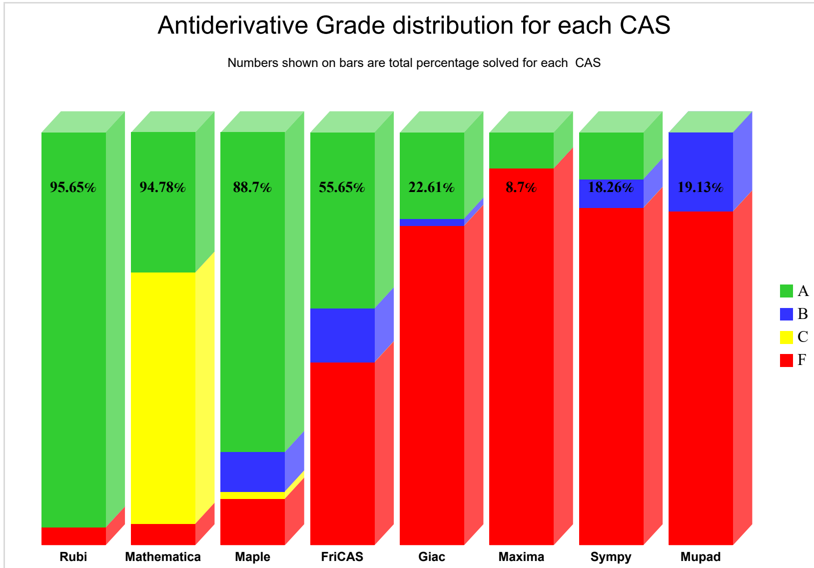
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

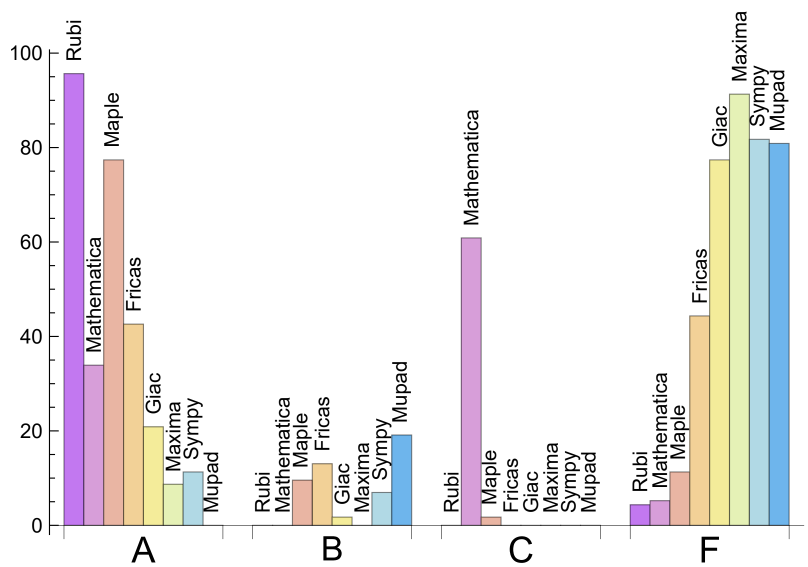
System	% A grade	% B grade	% C grade	% F grade
Rubi	95.652	0.000	0.000	4.348
Maple	77.391	9.565	1.739	11.304
Fricas	42.609	13.043	0.000	44.348
Mathematica	33.913	0.000	60.870	5.217
Giac	20.870	1.739	0.000	77.391
Sympy	11.304	6.957	0.000	81.739
Maxima	8.696	0.000	0.000	91.304
Mupad	0.000	19.130	0.000	80.870

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Maple	8	100.00	0.00	0.00
Fricas	47	25.53	74.47	0.00
Giac	84	95.24	0.00	4.76
Mupad	88	0.00	100.00	0.00
Sympy	89	92.13	7.87	0.00
Maxima	100	87.00	0.00	13.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.18
Maxima	0.23
Giac	0.47
Fricas	2.86
Mupad	3.54
Mathematica	3.83
Maple	4.25
Sympy	5.22

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	121.27	0.99	100.00	1.02
Mupad	184.81	1.13	158.00	1.05
Giac	199.97	1.28	182.00	1.27
Sympy	251.08	1.60	211.00	1.35
Mathematica	258.54	0.95	210.00	0.99
Rubi	285.26	1.00	242.00	1.00
Fricas	488.59	2.31	330.00	1.27
Maple	498.62	1.66	334.00	1.19

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

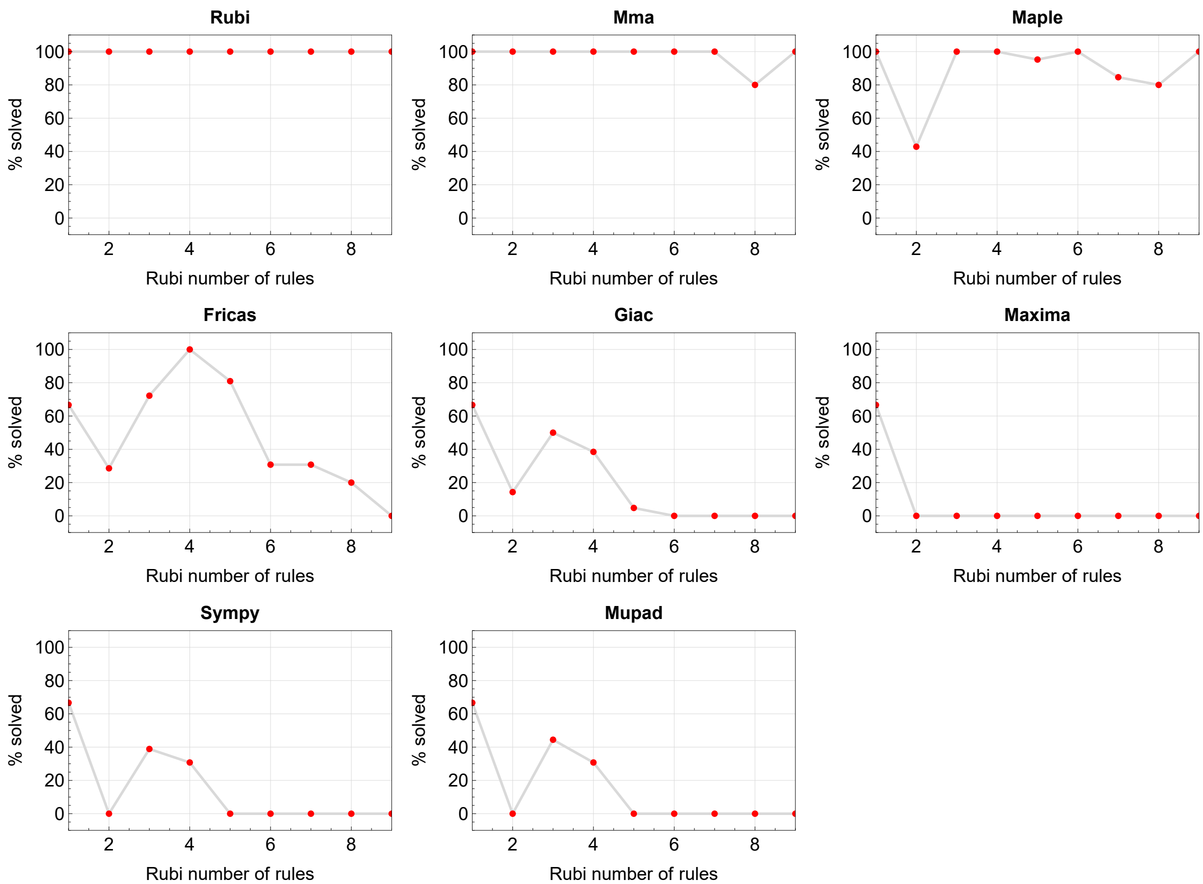


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

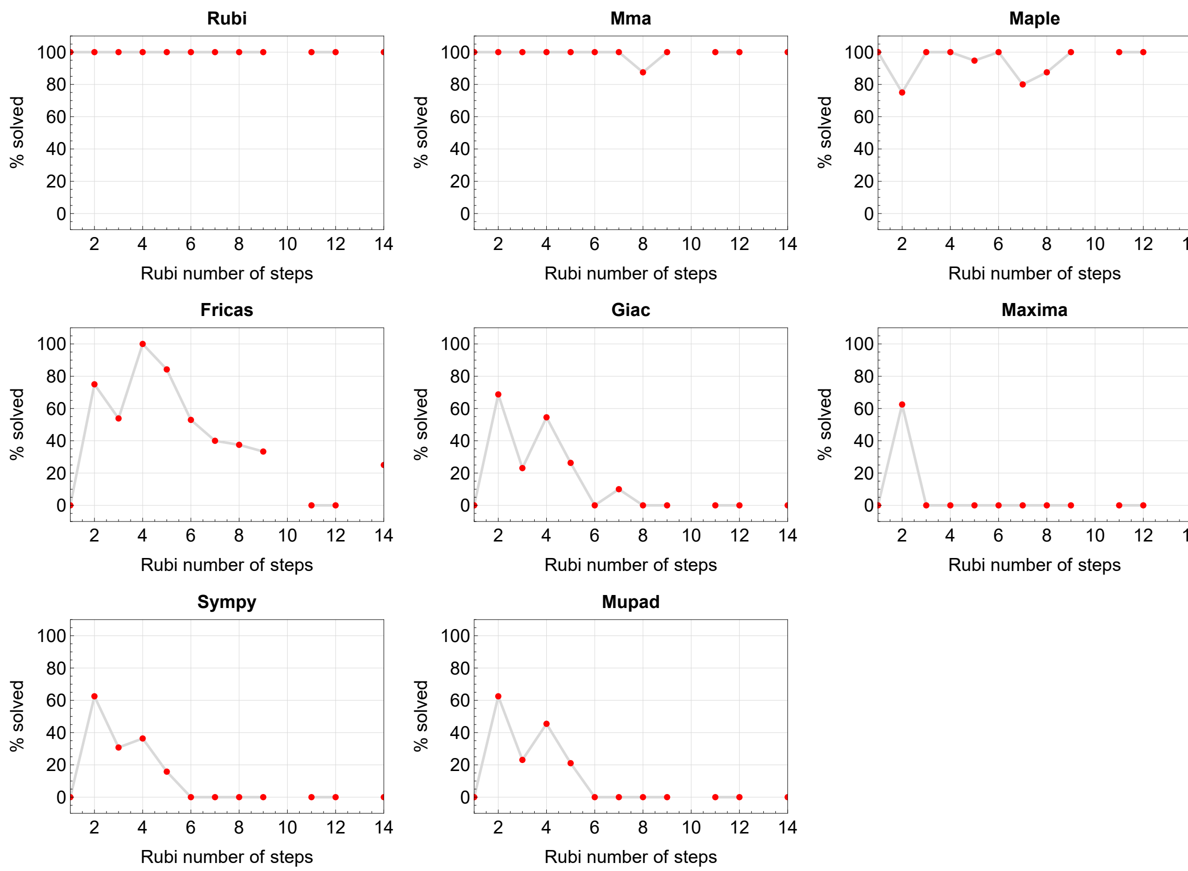


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

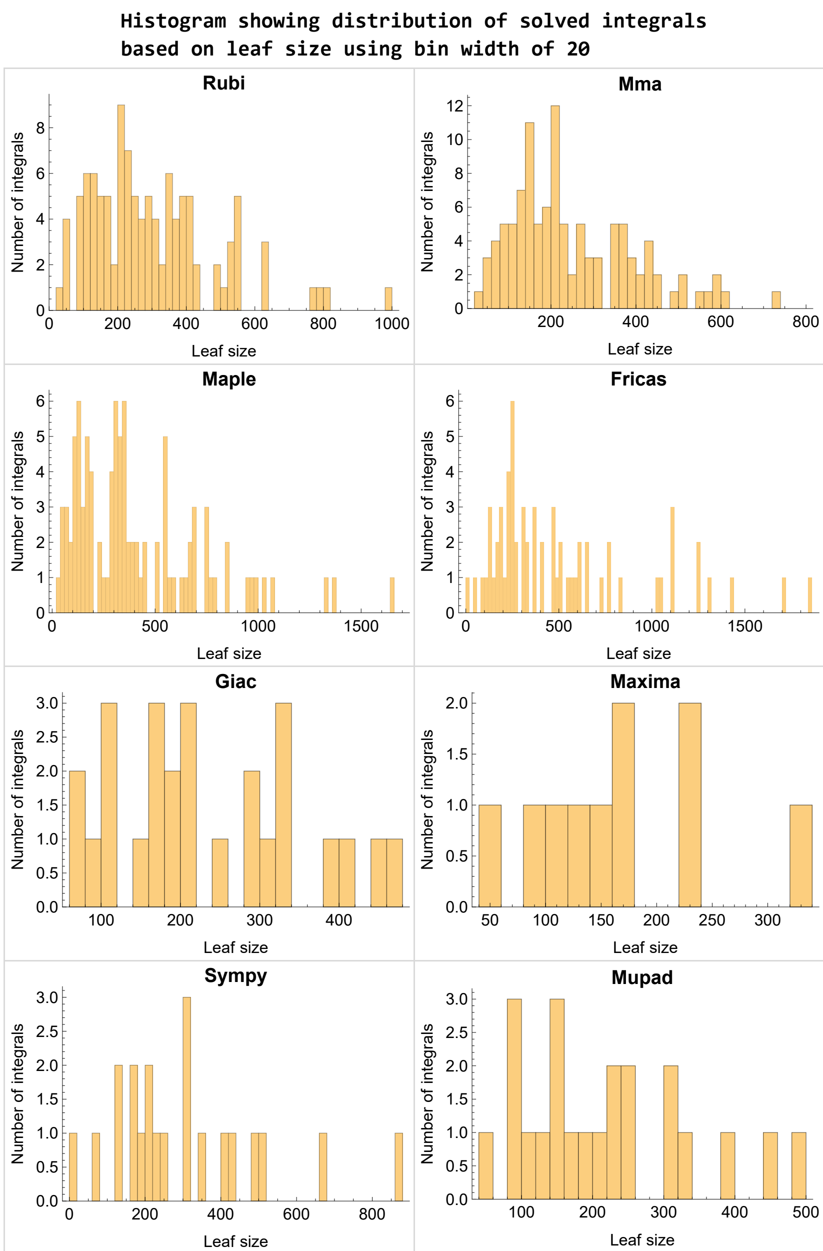


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

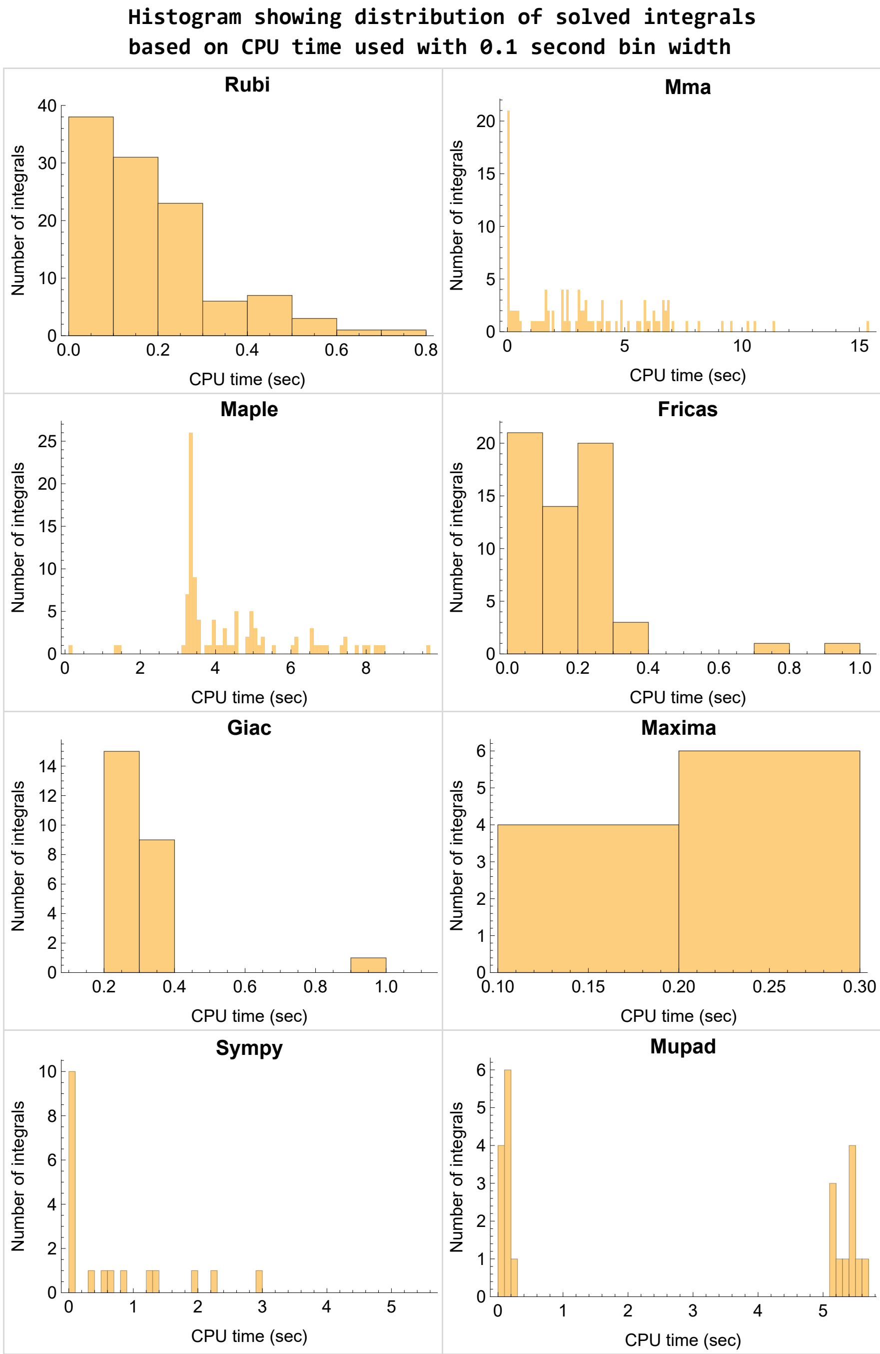


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

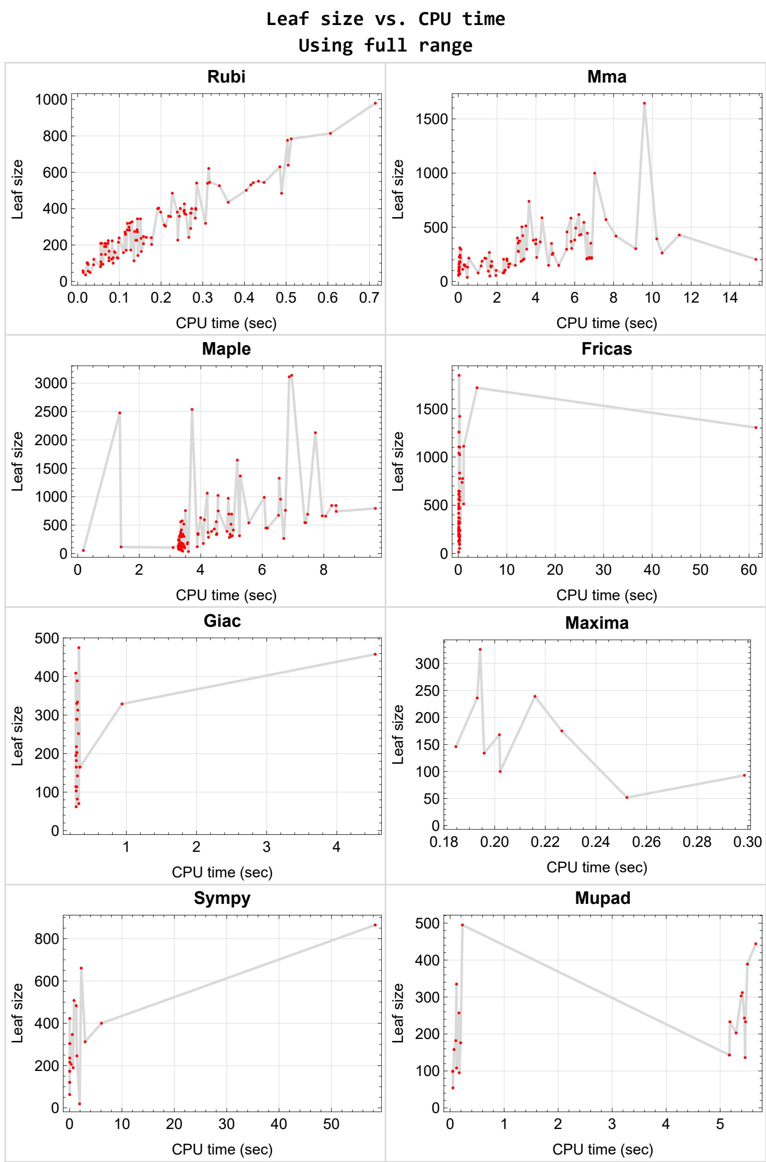


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{103, 107, 110, 112, 115\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple $\{55, 56\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

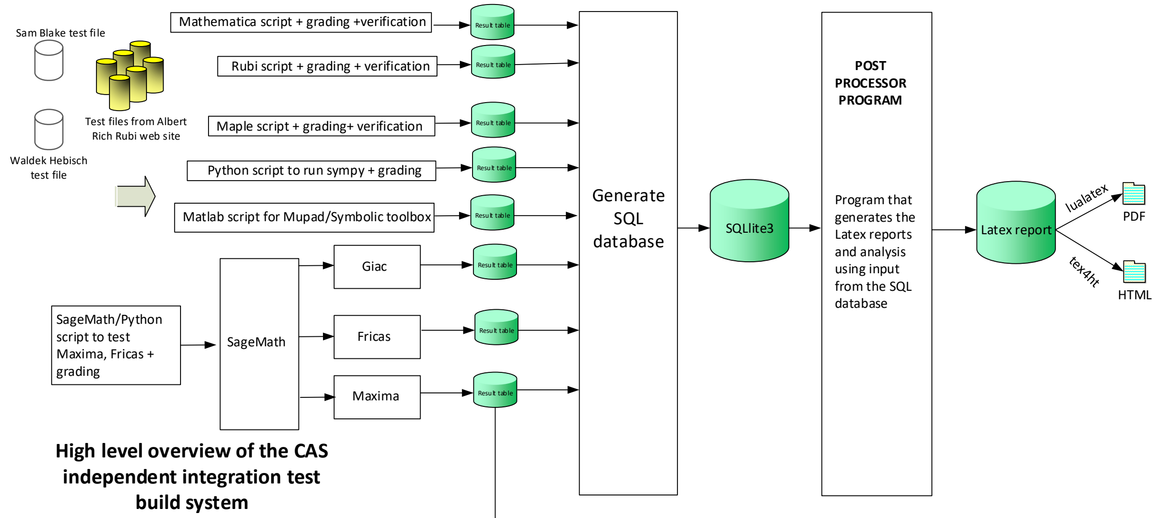
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
June 27, 2023
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 57, 58, 59, 60, 61, 62, 63, 96, 97, 98, 104, 105, 106, 109, 111, 113, 114 }

B grade { }

C grade { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102 }

F normal fail { 108 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 77, 78, 79, 80, 81, 83, 85, 86, 87, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102 }

B grade { 55, 56, 69, 70, 74, 75, 76, 82, 84, 88, 101 }

C grade { 89, 90 }

F normal fail { 104, 105, 106, 108, 109, 111, 113, 114 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 29, 30, 31, 32, 35, 36, 37, 38, 39, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 56, 57, 58, 59, 97 }

B grade { 8, 15, 22, 28, 33, 34, 40, 41, 46, 47, 55, 60, 61, 62, 98 }

C grade { }

F normal fail { 89, 90, 91, 92, 93, 94, 95, 104, 106, 108, 109, 114 }

F(-1) timedout fail { 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 96, 99, 100, 101, 102, 103, 105, 111, 113 }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 9, 10, 11, 16, 17, 18 }

B grade { }

C grade { }

F normal fail { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

F(-1) timedout fail { }

F(-2) exception fail { 5, 6, 7, 8, 12, 13, 14, 15, 19, 20, 21, 22, 57 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 61, 62 }

B grade { 63, 98 }

C grade { }

F normal fail { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

F(-1) timedout fail { }

F(-2) exception fail { 57, 58, 59, 60 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22 }

C grade { }

F normal fail { }

F(-1) timedout fail { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 6, 8, 9, 10, 11, 16, 17, 18, 97 }

B grade { 5, 7, 12, 13, 14, 19, 20, 21 }

C grade { }

F normal fail { 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 98, 99, 100, 101, 102, 104, 105, 106, 108, 109, 111, 113, 114 }

F(-1) timedout fail { 15, 22, 34, 47, 75, 76, 93 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **`Mathematica`** was abbreviated to **`MMA`**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	172	176	175	175	236	218	182
N.S.	1	1.00	1.00	1.02	1.02	1.02	1.37	1.27	1.06
time (sec)	N/A	0.127	0.058	4.084	0.227	0.288	0.033	0.287	0.105

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	130	135	134	134	173	165	143
N.S.	1	1.00	1.00	1.04	1.03	1.03	1.33	1.27	1.10
time (sec)	N/A	0.085	0.043	3.295	0.196	0.270	0.028	0.285	5.171

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	96	94	93	93	121	114	99
N.S.	1	1.00	1.02	1.00	0.99	0.99	1.29	1.21	1.05
time (sec)	N/A	0.060	0.025	3.378	0.299	0.273	0.024	0.280	0.048

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	56	53	52	52	63	62	54
N.S.	1	1.00	1.00	0.95	0.93	0.93	1.12	1.11	0.96
time (sec)	N/A	0.025	0.009	0.183	0.252	0.271	0.020	0.285	0.051

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	72	74	0	191	206	82	108
N.S.	1	1.00	0.89	0.91	0.00	2.36	2.54	1.01	1.33
time (sec)	N/A	0.055	0.036	3.279	0.000	0.291	0.315	0.300	0.118

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	95	97	0	318	190	103	95
N.S.	1	1.00	0.88	0.90	0.00	2.94	1.76	0.95	0.88
time (sec)	N/A	0.056	0.044	3.272	0.000	0.292	0.685	0.285	0.169

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	130	132	0	471	246	142	136
N.S.	1	1.00	1.00	1.02	0.00	3.62	1.89	1.09	1.05
time (sec)	N/A	0.075	0.053	3.270	0.000	0.314	1.389	0.303	5.463

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	171	163	0	642	313	195	176
N.S.	1	1.00	1.00	0.95	0.00	3.75	1.83	1.14	1.03
time (sec)	N/A	0.116	0.067	3.295	0.000	0.290	2.979	0.280	0.194

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	226	237	236	236	304	289	233
N.S.	1	1.00	1.00	1.05	1.04	1.04	1.35	1.28	1.03
time (sec)	N/A	0.142	0.056	3.304	0.193	0.257	0.035	0.290	5.470

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	158	169	168	168	216	202	158
N.S.	1	1.00	1.00	1.07	1.06	1.06	1.37	1.28	1.00
time (sec)	N/A	0.110	0.038	3.426	0.202	0.249	0.031	0.295	0.073

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	96	100	100	100	121	114	99
N.S.	1	1.00	1.02	1.06	1.06	1.06	1.29	1.21	1.05
time (sec)	N/A	0.056	0.021	3.306	0.202	0.250	0.024	0.292	0.047

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	115	170	0	366	347	182	203
N.S.	1	1.00	0.81	1.20	0.00	2.58	2.44	1.28	1.43
time (sec)	N/A	0.144	0.041	3.303	0.000	0.273	0.523	0.287	5.295

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	134	182	0	552	483	203	257
N.S.	1	1.00	0.82	1.11	0.00	3.37	2.95	1.24	1.57
time (sec)	N/A	0.153	0.063	3.314	0.000	0.296	1.278	0.293	0.164

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	183	230	0	777	400	252	243
N.S.	1	1.00	0.88	1.11	0.00	3.75	1.93	1.22	1.17
time (sec)	N/A	0.158	0.081	3.282	0.000	0.280	6.092	0.317	5.450

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	240	240	242	289	0	1024	0	330	303
N.S.	1	1.00	1.01	1.20	0.00	4.27	0.00	1.38	1.26
time (sec)	N/A	0.177	0.097	3.313	0.000	0.274	0.000	0.293	5.390

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	310	310	339	326	326	423	409	335
N.S.	1	1.00	1.00	1.09	1.05	1.05	1.36	1.32	1.08
time (sec)	N/A	0.207	0.076	3.387	0.194	0.244	0.041	0.279	0.118

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	226	226	226	244	239	239	304	289	233
N.S.	1	1.00	1.00	1.08	1.06	1.06	1.35	1.28	1.03
time (sec)	N/A	0.139	0.057	3.275	0.216	0.327	0.045	0.298	5.181

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	130	144	146	146	173	165	143
N.S.	1	1.00	1.00	1.11	1.12	1.12	1.33	1.27	1.10
time (sec)	N/A	0.095	0.034	3.332	0.185	0.254	0.028	0.338	5.172

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	179	300	0	586	508	313	312
N.S.	1	1.00	0.79	1.32	0.00	2.58	2.24	1.38	1.37
time (sec)	N/A	0.240	0.059	3.307	0.000	0.264	0.850	0.307	5.411

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	176	308	0	834	661	334	389
N.S.	1	1.00	0.73	1.27	0.00	3.45	2.73	1.38	1.61
time (sec)	N/A	0.267	0.088	3.428	0.000	0.270	2.225	0.304	5.506

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	219	344	0	1102	865	389	495
N.S.	1	1.00	0.75	1.18	0.00	3.79	2.97	1.34	1.70
time (sec)	N/A	0.271	0.106	3.354	0.000	0.287	58.357	0.299	0.227

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	295	417	0	1422	0	475	444
N.S.	1	1.00	0.85	1.20	0.00	4.09	0.00	1.36	1.28
time (sec)	N/A	0.282	0.132	3.372	0.000	0.293	0.000	0.323	5.660

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	544	544	373	695	0	463	0	0	0
N.S.	1	1.00	0.69	1.28	0.00	0.85	0.00	0.00	0.00
time (sec)	N/A	0.447	3.132	5.006	0.000	0.104	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	267	431	0	305	0	0	0
N.S.	1	1.00	0.70	1.13	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.239	1.602	4.441	0.000	0.107	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	283	283	212	312	0	191	0	0	0
N.S.	1	1.00	0.75	1.10	0.00	0.67	0.00	0.00	0.00
time (sec)	N/A	0.119	1.422	5.026	0.000	0.099	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	271	271	192	328	0	223	0	0	0
N.S.	1	1.00	0.71	1.21	0.00	0.82	0.00	0.00	0.00
time (sec)	N/A	0.113	2.513	3.328	0.000	0.099	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	297	518	0	515	0	0	0
N.S.	1	1.00	1.08	1.89	0.00	1.88	0.00	0.00	0.00
time (sec)	N/A	0.137	3.527	3.450	0.000	0.101	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	379	756	0	1105	0	0	0
N.S.	1	1.00	0.98	1.96	0.00	2.87	0.00	0.00	0.00
time (sec)	N/A	0.255	3.864	3.502	0.000	0.133	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	372	695	0	465	0	0	0
N.S.	1	1.00	0.69	1.28	0.00	0.86	0.00	0.00	0.00
time (sec)	N/A	0.421	3.165	4.915	0.000	0.098	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	275	448	0	318	0	0	0
N.S.	1	1.00	0.69	1.12	0.00	0.80	0.00	0.00	0.00
time (sec)	N/A	0.273	3.099	6.167	0.000	0.095	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	248	544	0	416	0	0	0
N.S.	1	1.00	0.67	1.47	0.00	1.13	0.00	0.00	0.00
time (sec)	N/A	0.260	4.841	7.384	0.000	0.099	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	373	373	296	559	0	561	0	0	0
N.S.	1	1.00	0.79	1.50	0.00	1.50	0.00	0.00	0.00
time (sec)	N/A	0.256	5.581	4.504	0.000	0.107	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	382	749	0	1040	0	0	0
N.S.	1	1.00	1.02	1.99	0.00	2.77	0.00	0.00	0.00
time (sec)	N/A	0.271	5.997	4.566	0.000	0.138	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	531	531	545	1023	0	1847	0	0	0
N.S.	1	1.00	1.03	1.93	0.00	3.48	0.00	0.00	0.00
time (sec)	N/A	0.416	6.470	4.561	0.000	0.174	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	551	551	386	691	0	489	0	0	0
N.S.	1	1.00	0.70	1.25	0.00	0.89	0.00	0.00	0.00
time (sec)	N/A	0.434	4.004	7.475	0.000	0.102	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	396	396	279	448	0	334	0	0	0
N.S.	1	1.00	0.70	1.13	0.00	0.84	0.00	0.00	0.00
time (sec)	N/A	0.284	3.086	6.117	0.000	0.094	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	215	312	0	201	0	0	0
N.S.	1	1.00	0.76	1.11	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.124	1.372	5.260	0.000	0.095	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	131	158	0	130	0	0	0
N.S.	1	1.00	0.64	0.77	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	0.068	2.589	3.357	0.000	0.085	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	206	334	0	254	0	0	0
N.S.	1	1.00	0.99	1.60	0.00	1.22	0.00	0.00	0.00
time (sec)	N/A	0.055	6.623	3.911	0.000	0.088	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	284	284	302	540	0	611	0	0	0
N.S.	1	1.00	1.06	1.90	0.00	2.15	0.00	0.00	0.00
time (sec)	N/A	0.142	9.140	5.563	0.000	0.110	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	393	760	0	1257	0	0	0
N.S.	1	1.00	0.98	1.90	0.00	3.13	0.00	0.00	0.00
time (sec)	N/A	0.283	10.226	6.752	0.000	0.142	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	501	369	794	0	648	0	0	0
N.S.	1	1.00	0.74	1.58	0.00	1.29	0.00	0.00	0.00
time (sec)	N/A	0.404	5.810	9.676	0.000	0.101	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	260	543	0	418	0	0	0
N.S.	1	1.00	0.73	1.52	0.00	1.17	0.00	0.00	0.00
time (sec)	N/A	0.244	4.875	7.410	0.000	0.093	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	208	378	0	239	0	0	0
N.S.	1	1.00	0.81	1.47	0.00	0.93	0.00	0.00	0.00
time (sec)	N/A	0.114	2.507	3.346	0.000	0.088	0.000	0.000	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	212	349	0	258	0	0	0
N.S.	1	1.00	1.01	1.67	0.00	1.23	0.00	0.00	0.00
time (sec)	N/A	0.058	6.870	3.906	0.000	0.099	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	272	272	262	515	0	618	0	0	0
N.S.	1	1.00	0.96	1.89	0.00	2.27	0.00	0.00	0.00
time (sec)	N/A	0.143	10.501	4.985	0.000	0.107	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	375	375	428	673	0	1259	0	0	0
N.S.	1	1.00	1.14	1.79	0.00	3.36	0.00	0.00	0.00
time (sec)	N/A	0.272	11.387	6.528	0.000	0.152	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	212	349	0	260	0	0	0
N.S.	1	1.00	1.01	1.67	0.00	1.24	0.00	0.00	0.00
time (sec)	N/A	0.065	6.628	3.912	0.000	0.094	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	247	247	220	349	0	257	0	0	0
N.S.	1	1.00	0.89	1.41	0.00	1.04	0.00	0.00	0.00
time (sec)	N/A	0.158	6.872	4.525	0.000	0.092	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	213	333	0	254	0	0	0
N.S.	1	1.00	0.90	1.41	0.00	1.07	0.00	0.00	0.00
time (sec)	N/A	0.152	6.753	4.510	0.000	0.092	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	221	338	0	257	0	0	0
N.S.	1	1.00	0.91	1.40	0.00	1.06	0.00	0.00	0.00
time (sec)	N/A	0.164	6.742	4.953	0.000	0.094	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	81	105	0	124	0	0	0
N.S.	1	1.00	0.42	0.55	0.00	0.65	0.00	0.00	0.00
time (sec)	N/A	0.068	2.358	3.100	0.000	0.087	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	142	282	0	181	0	0	0
N.S.	1	1.00	0.54	1.08	0.00	0.69	0.00	0.00	0.00
time (sec)	N/A	0.115	1.172	4.930	0.000	0.088	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	356	356	186	390	0	274	0	0	0
N.S.	1	1.00	0.52	1.10	0.00	0.77	0.00	0.00	0.00
time (sec)	N/A	0.223	1.223	4.852	0.000	0.094	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	113	113	104	2538	0	369	0	0	0
N.S.	1	1.00	0.92	22.46	0.00	3.27	0.00	0.00	0.00
time (sec)	N/A	0.135	2.417	3.717	0.000	0.110	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	526	526	100	2477	0	372	0	0	0
N.S.	1	1.00	0.19	4.71	0.00	0.71	0.00	0.00	0.00
time (sec)	N/A	0.340	2.387	1.368	0.000	0.099	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	141	115	0	777	0	0	0
N.S.	1	1.00	1.10	0.90	0.00	6.07	0.00	0.00	0.00
time (sec)	N/A	0.097	0.356	1.410	0.000	0.904	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	214	191	0	1718	0	0	0
N.S.	1	1.00	0.70	0.63	0.00	5.65	0.00	0.00	0.00
time (sec)	N/A	0.211	0.544	3.571	0.000	3.868	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	151	132	0	1111	0	0	0
N.S.	1	1.00	0.91	0.80	0.00	6.69	0.00	0.00	0.00
time (sec)	N/A	0.075	0.297	3.457	0.000	1.115	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	111	76	0	737	0	0	0
N.S.	1	1.00	1.22	0.84	0.00	8.10	0.00	0.00	0.00
time (sec)	N/A	0.038	0.231	3.349	0.000	0.788	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	70	41	0	241	0	70	0
N.S.	1	1.00	1.43	0.84	0.00	4.92	0.00	1.43	0.00
time (sec)	N/A	0.013	0.081	3.412	0.000	0.302	0.000	0.323	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	153	120	0	1305	0	165	0
N.S.	1	1.00	1.25	0.98	0.00	10.70	0.00	1.35	0.00
time (sec)	N/A	0.082	0.330	3.433	0.000	61.426	0.000	0.337	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	203	169	0	0	0	458	0
N.S.	1	1.00	1.00	0.83	0.00	0.00	0.00	2.26	0.00
time (sec)	N/A	0.177	15.335	3.564	0.000	0.000	0.000	4.547	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	608	776	456	657	0	0	0	0	0
N.S.	1	1.28	0.75	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.503	5.603	8.061	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	346	845	0	0	0	0	0
N.S.	1	1.00	0.86	2.11	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.192	3.985	8.397	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	321	321	184	340	0	0	0	0	0
N.S.	1	1.00	0.57	1.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.125	1.720	3.360	0.000	0.000	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	143	191	0	0	0	0	0
N.S.	1	1.00	1.40	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.022	1.714	3.365	0.000	0.000	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	347	390	0	0	0	0	0
N.S.	1	1.00	1.66	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.072	4.002	4.353	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	401	401	427	1366	0	0	0	0	0
N.S.	1	1.00	1.06	3.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.246	6.245	5.285	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	630	630	584	3138	0	0	0	0	0
N.S.	1	1.00	0.93	4.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.485	5.802	6.956	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	659	784	445	663	0	0	0	0	0
N.S.	1	1.19	0.68	1.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.512	6.650	7.950	0.000	0.000	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	403	403	739	845	0	0	0	0	0
N.S.	1	1.00	1.83	2.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.194	3.644	8.258	0.000	0.000	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	184	300	0	0	0	0	0
N.S.	1	1.00	0.56	0.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.130	3.232	3.457	0.000	0.000	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	492	630	0	0	0	0	0
N.S.	1	1.00	2.20	2.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	6.050	3.997	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	999	1645	0	0	0	0	0
N.S.	1	1.00	2.55	4.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	7.026	5.190	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	639	639	570	3112	0	0	0	0	0
N.S.	1	1.00	0.89	4.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.505	7.609	6.876	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	621	621	350	741	0	0	0	0	0
N.S.	1	1.00	0.56	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.314	4.802	8.404	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	197	341	0	0	0	0	0
N.S.	1	1.00	0.62	1.07	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.119	3.328	3.417	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	143	191	0	0	0	0	0
N.S.	1	1.00	1.40	1.87	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.023	1.669	3.339	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	101	118	0	0	0	0	0
N.S.	1	1.00	1.01	1.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.085	1.955	3.887	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	365	413	0	0	0	0	0
N.S.	1	1.00	1.06	1.20	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.143	4.222	5.058	0.000	0.000	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	433	1325	0	0	0	0	0
N.S.	1	1.00	1.00	3.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.361	6.330	6.551	0.000	0.000	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	980	980	352	1063	0	0	0	0	0
N.S.	1	1.00	0.36	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.714	6.826	4.210	0.000	0.000	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	304	594	0	0	0	0	0
N.S.	1	1.00	1.36	2.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.083	5.843	4.118	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	207	285	0	0	0	0	0
N.S.	1	1.00	0.99	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.072	3.384	4.250	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	221	303	0	0	0	0	0
N.S.	1	1.00	0.64	0.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.151	4.036	4.983	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	539	539	418	956	0	0	0	0	0
N.S.	1	1.00	0.78	1.77	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.312	8.126	6.595	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	814	814	1645	2127	0	0	0	0	0
N.S.	1	1.00	2.02	2.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.606	9.591	7.726	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	239	204	265	0	0	0	0	0
N.S.	1	0.99	0.84	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.099	2.334	6.699	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	78	121	0	0	0	0	0
N.S.	1	1.00	0.41	0.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	1.020	3.326	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	54	64	0	0	0	0	0
N.S.	1	1.00	0.93	1.10	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.013	1.938	3.344	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	78	147	0	0	0	0	0
N.S.	1	1.00	0.64	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.039	2.311	3.347	0.000	0.000	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	353	372	0	0	0	0	0
N.S.	1	1.00	1.64	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.098	3.084	4.234	0.000	0.000	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	134	293	0	0	0	0	0
N.S.	1	1.00	0.45	0.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.123	1.690	3.382	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	94	133	0	0	0	0	0
N.S.	1	1.00	1.01	1.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.025	1.549	3.355	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	49	53	0	0	0	0	0
N.S.	1	1.00	1.00	1.08	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.030	1.625	3.389	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	37	35	0	14	19	0	0
N.S.	1	1.00	1.03	0.97	0.00	0.39	0.53	0.00	0.00
time (sec)	N/A	0.019	0.464	3.599	0.000	0.080	1.919	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	131	100	0	513	0	329	0
N.S.	1	1.00	1.16	0.88	0.00	4.54	0.00	2.91	0.00
time (sec)	N/A	0.075	0.478	3.483	0.000	1.109	0.000	0.938	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	422	574	0	0	0	0	0
N.S.	1	1.00	1.18	1.60	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.218	3.318	3.397	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	401	559	0	0	0	0	0
N.S.	1	1.00	1.05	1.47	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.199	3.047	3.349	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	617	989	0	0	0	0	0
N.S.	1	1.00	1.45	2.32	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.256	6.210	6.064	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	485	485	587	973	0	0	0	0	0
N.S.	1	1.00	1.21	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.227	4.310	4.894	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	28	30	0	31	30	30
N.S.	1	1.00	1.06	0.82	0.88	0.00	0.91	0.88	0.88
time (sec)	N/A	0.037	18.211	0.020	0.239	0.000	12.864	0.351	5.900

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	545	545	503	0	0	0	0	0	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.316	3.278	0.000	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	162	0	0	0	0	0	0
N.S.	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.088	2.654	0.000	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	148	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	4.650	0.000	0.000	0.000	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	28	30	50	31	30	30
N.S.	1	1.00	1.06	0.82	0.88	1.47	0.91	0.88	0.88
time (sec)	N/A	0.040	20.482	0.084	0.273	54.452	17.592	0.404	6.217

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	484	484	0	0	0	0	0	0	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.489	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	319	319	148	0	0	0	0	0	0
N.S.	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.307	5.174	0.000	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	28	30	111	31	30	30
N.S.	1	1.00	1.06	0.82	0.88	3.26	0.91	0.88	0.88
time (sec)	N/A	0.045	21.111	0.087	0.265	0.261	11.111	0.359	6.260

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	541	541	512	0	0	0	0	0	0
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.285	3.484	0.000	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	28	30	53	31	30	30
N.S.	1	1.00	1.06	0.82	0.88	1.56	0.91	0.88	0.88
time (sec)	N/A	0.046	5.068	0.088	0.267	60.517	9.861	0.367	7.785

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	159	159	159	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.089	2.599	0.000	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	148	148	148	0	0	0	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	2.939	0.000	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	34	34	36	28	30	127	32	30	30
N.S.	1	1.00	1.06	0.82	0.88	3.74	0.94	0.88	0.88
time (sec)	N/A	0.040	10.594	0.088	0.285	0.278	7.327	0.352	9.043

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [64] had the largest ratio of [.2812000000000000006]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	24	0.042
2	A	2	1	1.00	24	0.042
3	A	2	1	1.00	24	0.042
4	A	2	1	1.00	22	0.045
5	A	3	3	1.00	24	0.125
6	A	3	3	1.00	24	0.125
7	A	3	3	1.00	24	0.125
8	A	4	4	1.00	24	0.167
9	A	2	1	1.00	26	0.038
10	A	2	1	1.00	26	0.038
11	A	2	1	1.00	24	0.042
12	A	4	3	1.00	26	0.115
13	A	4	4	1.00	26	0.154
14	A	4	3	1.00	26	0.115
15	A	4	3	1.00	26	0.115
16	A	2	1	1.00	26	0.038
17	A	2	1	1.00	26	0.038
18	A	2	1	1.00	24	0.042
19	A	5	3	1.00	26	0.115
20	A	5	4	1.00	26	0.154
21	A	5	4	1.00	26	0.154
22	A	5	3	1.00	26	0.115
23	A	7	5	1.00	30	0.167
24	A	6	5	1.00	30	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	5	5	1.00	30	0.167
26	A	5	5	1.00	30	0.167
27	A	4	4	1.00	30	0.133
28	A	5	5	1.00	30	0.167
29	A	7	5	1.00	30	0.167
30	A	6	5	1.00	30	0.167
31	A	6	6	1.00	30	0.200
32	A	6	5	1.00	30	0.167
33	A	5	4	1.00	30	0.133
34	A	6	5	1.00	30	0.167
35	A	7	5	1.00	30	0.167
36	A	6	5	1.00	30	0.167
37	A	5	5	1.00	30	0.167
38	A	4	4	1.00	30	0.133
39	A	3	3	1.00	30	0.100
40	A	4	4	1.00	30	0.133
41	A	5	4	1.00	30	0.133
42	A	7	6	1.00	30	0.200
43	A	6	6	1.00	30	0.200
44	A	5	5	1.00	30	0.167
45	A	3	3	1.00	30	0.100
46	A	4	4	1.00	30	0.133
47	A	5	4	1.00	30	0.133
48	A	3	3	1.00	30	0.100
49	A	8	7	1.00	31	0.226
50	A	8	7	1.00	31	0.226
51	A	8	7	1.00	32	0.219
52	A	4	4	1.00	30	0.133
53	A	5	5	1.00	30	0.167
54	A	6	5	1.00	30	0.167
55	A	2	2	1.00	87	0.023
56	A	5	5	1.00	81	0.062
57	A	6	6	1.00	28	0.214
58	A	14	8	1.00	30	0.267
59	A	9	7	1.00	30	0.233

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
60	A	5	5	1.00	28	0.179
61	A	2	2	1.00	21	0.095
62	A	5	3	1.00	30	0.100
63	A	7	5	1.00	30	0.167
64	A	14	9	1.28	32	0.281
65	A	7	7	1.00	32	0.219
66	A	6	6	1.00	32	0.188
67	A	1	1	1.00	32	0.031
68	A	3	3	1.00	32	0.094
69	A	6	6	1.00	32	0.188
70	A	9	8	1.00	32	0.250
71	A	14	9	1.19	32	0.281
72	A	7	7	1.00	32	0.219
73	A	6	6	1.00	32	0.188
74	A	3	3	1.00	32	0.094
75	A	6	6	1.00	32	0.188
76	A	9	8	1.00	32	0.250
77	A	12	8	1.00	32	0.250
78	A	6	6	1.00	32	0.188
79	A	1	1	1.00	32	0.031
80	A	3	2	1.00	32	0.062
81	A	5	5	1.00	32	0.156
82	A	8	7	1.00	32	0.219
83	A	14	9	1.00	32	0.281
84	A	3	3	1.00	32	0.094
85	A	3	3	1.00	32	0.094
86	A	5	5	1.00	32	0.156
87	A	8	7	1.00	32	0.219
88	A	11	6	1.00	32	0.188
89	A	7	7	0.99	28	0.250
90	A	6	6	1.00	28	0.214
91	A	1	1	1.00	28	0.036
92	A	3	3	1.00	28	0.107
93	A	6	6	1.00	28	0.214
94	A	6	6	1.00	32	0.188

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
95	A	1	1	1.00	32	0.031
96	A	1	1	1.00	32	0.031
97	A	3	3	1.00	30	0.100
98	A	4	4	1.00	28	0.143
99	A	11	9	1.00	33	0.273
100	A	8	7	1.00	32	0.219
101	A	11	9	1.00	33	0.273
102	A	8	7	1.00	32	0.219
103	N/A	0	0	1.00	34	0.000
104	A	7	7	1.00	34	0.206
105	A	2	2	1.00	34	0.059
106	A	2	2	1.00	34	0.059
107	N/A	0	0	1.00	34	0.000
108	A	8	8	1.00	34	0.235
109	A	5	5	1.00	34	0.147
110	N/A	0	0	1.00	34	0.000
111	A	7	7	1.00	34	0.206
112	N/A	0	0	1.00	34	0.000
113	A	2	2	1.00	34	0.059
114	A	2	2	1.00	34	0.059
115	N/A	0	0	1.00	34	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\begin{aligned}\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx = {} & ace^4x+\frac{1}{3}e^3(bce+ade+4acf)x^3\\ & +\frac{1}{5}e^2(2af(2de+3cf)+be(de+4cf))x^5\\ & +\frac{2}{7}ef(af(3de+2cf)+be(2de+3cf))x^7\\ & +\frac{1}{9}f^2(af(4de+cf)+2be(3de+2cf))x^9\\ & +\frac{1}{11}f^3(4bde+bcf+adf)x^{11}+\frac{1}{13}bdf^4x^{13}\end{aligned}$$

```
[Out] a*c*e^4*x+1/3*e^3*(4*a*c*f+a*d*e+b*c*e)*x^3+1/5*e^2*(2*a*f*(3*c*f+2*d*e)+b*
e*(4*c*f+d*e))*x^5+2/7*e*f*(a*f*(2*c*f+3*d*e)+b*e*(3*c*f+2*d*e))*x^7+1/9*f^
2*(a*f*(c*f+4*d*e)+2*b*e*(2*c*f+3*d*e))*x^9+1/11*f^3*(a*d*f+b*c*f+4*b*d*e)*
x^11+1/13*b*d*f^4*x^13
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used

$= \{535\}$

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^4\,dx = \frac{1}{3}e^3x^3(4acf+ade+bce) + \frac{1}{5}e^2x^5(2af(3cf+2de)+be(4cf+de)) + \frac{1}{11}f^3x^{11}(adf+bcf+4bde) + \frac{1}{9}f^2x^9(af(cf+4de)+2be(2cf+3de)) + \frac{2}{7}efx^7(af(2cf+3de)+be(3cf+2de)) + ace^4x + \frac{1}{13}bdf^4x^{13}$$

```
[In] Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x]
```

```
[Out] a*c*e^4*x + (e^3*(b*c*e + a*d*e + 4*a*c*f)*x^3)/3 + (e^2*(2*a*f*(2*d*e + 3*
c*f) + b*e*(d*e + 4*c*f))*x^5)/5 + (2*e*f*(a*f*(3*d*e + 2*c*f) + b*e*(2*d*e
 + 3*c*f))*x^7)/7 + (f^2*(a*f*(4*d*e + c*f) + 2*b*e*(3*d*e + 2*c*f))*x^9)/9
 + (f^3*(4*b*d*e + b*c*f + a*d*f)*x^11)/11 + (b*d*f^4*x^13)/13
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\text{integral} = \int \left(ace^4 + e^3(bce+ade+4acf)x^2 + e^2(2af(2de+3cf)+be(de+4cf))x^4 + 2ef(af(3de+2cf)+be(2de+3cf))x^6 + f^2(af(4de+cf)+2be(3de+2cf))x^8 + f^3(4bde+bcf+adf)x^{10} + bdf^4x^{12}\right)\,dx$$

$$= ace^4x + \frac{1}{3}e^3(bce+ade+4acf)x^3 + \frac{1}{5}e^2(2af(2de+3cf)+be(de+4cf))x^5 + \frac{2}{7}ef(af(3de+2cf)+be(2de+3cf))x^7 + \frac{1}{9}f^2(af(4de+cf)+2be(3de+2cf))x^9 + \frac{1}{11}f^3(4bde+bcf+adf)x^{11} + \frac{1}{13}bdf^4x^{13}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx = ace^4x + \frac{1}{3}e^3(bce+ade+4acf)x^3 + \frac{1}{5}e^2(2af(2de+3cf)+be(de+4cf))x^5 + \frac{2}{7}ef(af(3de+2cf)+be(2de+3cf))x^7 + \frac{1}{9}f^2(af(4de+cf)+2be(3de+2cf))x^9 + \frac{1}{11}f^3(4bde+bcf+adf)x^{11} + \frac{1}{13}bdf^4x^{13}$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x]
```

```
[Out] a*c*e^4*x + (e^3*(b*c*e + a*d*e + 4*a*c*f)*x^3)/3 + (e^2*(2*a*f*(2*d*e + 3*
c*f) + b*e*(d*e + 4*c*f))*x^5)/5 + (2*e*f*(a*f*(3*d*e + 2*c*f) + b*e*(2*d*e
 + 3*c*f))*x^7)/7 + (f^2*(a*f*(4*d*e + c*f) + 2*b*e*(3*d*e + 2*c*f))*x^9)/9
 + (f^3*(4*b*d*e + b*c*f + a*d*f)*x^11)/11 + (b*d*f^4*x^13)/13
```

Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.02

method	result
default	$\frac{bd\,f^4x^{13}}{13}+\frac{((ad+bc)f^4+4bde\,f^3)x^{11}}{11}+\frac{(ac\,f^4+4(ad+bc)e\,f^3+6bd\,e^2f^2)x^9}{9}+\frac{(4ace\,f^3+6(ad+bc)e^2f^2+4bd\,e^3f)x^7}{7}+$ (6
norman	$\frac{bd\,f^4x^{13}}{13}+\left(\frac{1}{11}ad\,f^4+\frac{1}{11}bc\,f^4+\frac{4}{11}bde\,f^3\right)x^{11}+\left(\frac{1}{9}ac\,f^4+\frac{4}{9}ade\,f^3+\frac{4}{9}bce\,f^3+\frac{2}{3}bd\,e^2f^2\right)x^9+$
gosper	$\frac{1}{13}bd\,f^4x^{13}+\frac{1}{11}x^{11}ad\,f^4+\frac{1}{11}x^{11}bc\,f^4+\frac{4}{11}x^{11}bde\,f^3+\frac{1}{9}x^9ac\,f^4+\frac{4}{9}x^9ade\,f^3+\frac{4}{9}x^9bce\,f^3+\frac{2}{3}$
risch	$\frac{1}{13}bd\,f^4x^{13}+\frac{1}{11}x^{11}ad\,f^4+\frac{1}{11}x^{11}bc\,f^4+\frac{4}{11}x^{11}bde\,f^3+\frac{1}{9}x^9ac\,f^4+\frac{4}{9}x^9ade\,f^3+\frac{4}{9}x^9bce\,f^3+\frac{2}{3}$
parallelrisch	$\frac{1}{13}bd\,f^4x^{13}+\frac{1}{11}x^{11}ad\,f^4+\frac{1}{11}x^{11}bc\,f^4+\frac{4}{11}x^{11}bde\,f^3+\frac{1}{9}x^9ac\,f^4+\frac{4}{9}x^9ade\,f^3+\frac{4}{9}x^9bce\,f^3+\frac{2}{3}$

```
[In] int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b*d*f^4*x^13+1/11*((a*d+b*c)*f^4+4*b*d*e*f^3)*x^11+1/9*(a*c*f^4+4*(a*d
+b*c)*e*f^3+6*b*d*e^2*f^2)*x^9+1/7*(4*a*c*e*f^3+6*(a*d+b*c)*e^2*f^2+4*b*d*e
^3*f)*x^7+1/5*(6*a*c*e^2*f^2+4*(a*d+b*c)*e^3*f+b*d*e^4)*x^5+1/3*(4*a*c*e^3*
f+(a*d+b*c)*e^4)*x^3+a*c*e^4*x
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx = \frac{1}{13}bdf^4x^{13} + \frac{1}{11}(4bdef^3 + (bc+ad)f^4)x^{11} + \frac{1}{9}(6bde^2f^2 + acf^4 + 4(bc+ad)ef^3)x^9 + \frac{2}{7}(2bde^3f + 2acef^3 + 3(bc+ad)e^2f^2)x^7 + ace^4x + \frac{1}{5}(bde^4 + 6ace^2f^2 + 4(bc+ad)e^3f)x^5 + \frac{1}{3}(4ace^3f + (bc+ad)e^4)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="fricas")
```

```
[Out] 1/13*b*d*f^4*x^13 + 1/11*(4*b*d*e*f^3 + (b*c + a*d)*f^4)*x^11 + 1/9*(6*b*d*
e^2*f^2 + a*c*f^4 + 4*(b*c + a*d)*e*f^3)*x^9 + 2/7*(2*b*d*e^3*f + 2*a*c*e*f
^3 + 3*(b*c + a*d)*e^2*f^2)*x^7 + a*c*e^4*x + 1/5*(b*d*e^4 + 6*a*c*e^2*f^2
+ 4*(b*c + a*d)*e^3*f)*x^5 + 1/3*(4*a*c*e^3*f + (b*c + a*d)*e^4)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.37

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^4\,dx = ace^4x + \frac{bdf^4x^{13}}{13} + x^{11}\left(\frac{adf^4}{11} + \frac{bcf^4}{11} + \frac{4bdef^3}{11}\right) + x^9\left(\frac{acf^4}{9} + \frac{4adef^3}{9} + \frac{4bcef^3}{9} + \frac{2bde^2f^2}{3}\right) + x^7 \cdot \left(\frac{4acef^3}{7} + \frac{6ade^2f^2}{7} + \frac{6bce^2f^2}{7} + \frac{4bde^3f}{7}\right) + x^5 \cdot \left(\frac{6ace^2f^2}{5} + \frac{4ade^3f}{5} + \frac{4bce^3f}{5} + \frac{bde^4}{5}\right) + x^3 \cdot \left(\frac{4ace^3f}{3} + \frac{ade^4}{3} + \frac{bce^4}{3}\right)$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**4,x)
```

```
[Out] a*c*e**4*x + b*d*f**4*x**13/13 + x**11*(a*d*f**4/11 + b*c*f**4/11 + 4*b*d*e
*f**3/11) + x**9*(a*c*f**4/9 + 4*a*d*e*f**3/9 + 4*b*c*e*f**3/9 + 2*b*d*e**2
*f**2/3) + x**7*(4*a*c*e*f**3/7 + 6*a*d*e**2*f**2/7 + 6*b*c*e**2*f**2/7 + 4
*b*d*e**3*f/7) + x**5*(6*a*c*e**2*f**2/5 + 4*a*d*e**3*f/5 + 4*b*c*e**3*f/5
+ b*d*e**4/5) + x**3*(4*a*c*e**3*f/3 + a*d*e**4/3 + b*c*e**4/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.02

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^4 \, dx = \frac{1}{13} bdf^4x^{13} + \frac{1}{11}\left(4\, bdef^3 + (bc+ad)f^4\right)x^{11} + \frac{1}{9}\left(6\, bde^2f^2 + acf^4 + 4\,(bc+ad)ef^3\right)x^9 + \frac{2}{7}\left(2\, bde^3f + 2\, acef^3 + 3\,(bc+ad)e^2f^2\right)x^7 + ace^4x + \frac{1}{5}\left(bde^4 + 6\, ace^2f^2 + 4\,(bc+ad)e^3f\right)x^5 + \frac{1}{3}\left(4\, ace^3f + (bc+ad)e^4\right)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="maxima")
```

```
[Out] 1/13*b*d*f^4*x^13 + 1/11*(4*b*d*e*f^3 + (b*c + a*d)*f^4)*x^11 + 1/9*(6*b*d*
e^2*f^2 + a*c*f^4 + 4*(b*c + a*d)*e*f^3)*x^9 + 2/7*(2*b*d*e^3*f + 2*a*c*e*f
^3 + 3*(b*c + a*d)*e^2*f^2)*x^7 + a*c*e^4*x + 1/5*(b*d*e^4 + 6*a*c*e^2*f^2
+ 4*(b*c + a*d)*e^3*f)*x^5 + 1/3*(4*a*c*e^3*f + (b*c + a*d)*e^4)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.27

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^4 \, dx = \frac{1}{13} bdf^4x^{13} + \frac{4}{11} bdef^3x^{11} + \frac{1}{11} bcf^4x^{11} + \frac{1}{11} adf^4x^{11} + \frac{2}{3} bde^2f^2x^9 + \frac{4}{9} bcef^3x^9 + \frac{4}{9} adef^3x^9 + \frac{1}{9} acf^4x^9 + \frac{4}{7} bde^3fx^7 + \frac{6}{7} bce^2f^2x^7 + \frac{6}{7} ade^2f^2x^7 + \frac{4}{7} acef^3x^7 + \frac{1}{5} bde^4x^5 + \frac{4}{5} bce^3fx^5 + \frac{4}{5} ade^3fx^5 + \frac{6}{5} ace^2f^2x^5 + \frac{1}{3} bce^4x^3 + \frac{1}{3} ade^4x^3 + \frac{4}{3} ace^3fx^3 + ace^4x$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^4,x, algorithm="giac")
```

```
[Out] 1/13*b*d*f^4*x^13 + 4/11*b*d*e*f^3*x^11 + 1/11*b*c*f^4*x^11 + 1/11*a*d*f^4*
x^11 + 2/3*b*d*e^2*f^2*x^9 + 4/9*b*c*e*f^3*x^9 + 4/9*a*d*e*f^3*x^9 + 1/9*a*
c*f^4*x^9 + 4/7*b*d*e^3*f*x^7 + 6/7*b*c*e^2*f^2*x^7 + 6/7*a*d*e^2*f^2*x^7 +
 4/7*a*c*e*f^3*x^7 + 1/5*b*d*e^4*x^5 + 4/5*b*c*e^3*f*x^5 + 4/5*a*d*e^3*f*x^
5 + 6/5*a*c*e^2*f^2*x^5 + 1/3*b*c*e^4*x^3 + 1/3*a*d*e^4*x^3 + 4/3*a*c*e^3*f
*x^3 + a*c*e^4*x
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.06

$$\int (a+bx^2)\left(c+dx^2\right)\left(e+fx^2\right)^4\,dx = x^3\left(\frac{a\,d\,e^4}{3}+\frac{b\,c\,e^4}{3}+\frac{4\,a\,c\,e^3\,f}{3}\right)+x^{11}\left(\frac{a\,d\,f^4}{11}+\frac{b\,c\,f^4}{11}+\frac{4\,b\,d\,e\,f^3}{11}\right)+x^5\left(\frac{b\,d\,e^4}{5}+\frac{4\,a\,d\,e^3\,f}{5}+\frac{4\,b\,c\,e^3\,f}{5}+\frac{6\,a\,c\,e^2\,f^2}{5}\right)+x^9\left(\frac{a\,c\,f^4}{9}+\frac{4\,a\,d\,e\,f^3}{9}+\frac{4\,b\,c\,e\,f^3}{9}+\frac{2\,b\,d\,e^2\,f^2}{3}\right)+\frac{2\,e\,f\,x^7\,(2\,a\,c\,f^2+2\,b\,d\,e^2+3\,a\,d\,e\,f+3\,b\,c\,e\,f)}{7}+a\,c\,e^4\,x+\frac{b\,d\,f^4\,x^{13}}{13}$$

```
[In] int((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^4,x)
```

```
[Out] x^3*((a*d*e^4)/3 + (b*c*e^4)/3 + (4*a*c*e^3*f)/3) + x^11*((a*d*f^4)/11 + (b
*c*f^4)/11 + (4*b*d*e*f^3)/11) + x^5*((b*d*e^4)/5 + (4*a*d*e^3*f)/5 + (4*b*
c*e^3*f)/5 + (6*a*c*e^2*f^2)/5) + x^9*((a*c*f^4)/9 + (4*a*d*e*f^3)/9 + (4*b
*c*e*f^3)/9 + (2*b*d*e^2*f^2)/3) + (2*e*f*x^7*(2*a*c*f^2 + 2*b*d*e^2 + 3*a*
d*e*f + 3*b*c*e*f))/7 + a*c*e^4*x + (b*d*f^4*x^13)/13
```

3.2 $\int (a+bx^2)(c+dx^2)(e+fx^2)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\begin{aligned}\int (a+bx^2)(c+dx^2)(e+fx^2)^3 \, dx = {} & ace^3x+\frac{1}{3}e^2(bce+ade+3acf)x^3 \\ & +\frac{1}{5}e(3af(de+cf)+be(de+3cf))x^5 \\ & +\frac{1}{7}f(3be(de+cf)+af(3de+cf))x^7 \\ & +\frac{1}{9}f^2(3bde+bcf+adf)x^9+\frac{1}{11}bdf^3x^{11}\end{aligned}$$

```
[Out] a*c*e^3*x+1/3*e^2*(3*a*c*f+a*d*e+b*c*e)*x^3+1/5*e*(3*a*f*(c*f+d*e)+b*e*(3*c
*f+d*e))*x^5+1/7*f*(3*b*e*(c*f+d*e)+a*f*(c*f+3*d*e))*x^7+1/9*f^2*(a*d*f+b*c
*f+3*b*d*e)*x^9+1/11*b*d*f^3*x^11
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {535}

$$\begin{aligned}\int (a+bx^2)(c+dx^2)(e+fx^2)^3 \, dx = {} & \frac{1}{3}e^2x^3(3acf+ade+bce)+\frac{1}{9}f^2x^9(adf+bcf+3bde) \\ & +\frac{1}{7}fx^7(af(cf+3de)+3be(cf+de)) \\ & +\frac{1}{5}ex^5(3af(cf+de)+be(3cf+de)) \\ & +ace^3x+\frac{1}{11}bdf^3x^{11}\end{aligned}$$

```
[In] Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x]
```

```
[Out] a*c*e^3*x + (e^2*(b*c*e + a*d*e + 3*a*c*f)*x^3)/3 + (e*(3*a*f*(d*e + c*f) +
 b*e*(d*e + 3*c*f))*x^5)/5 + (f*(3*b*e*(d*e + c*f) + a*f*(3*d*e + c*f))*x^7
)/7 + (f^2*(3*b*d*e + b*c*f + a*d*f)*x^9)/9 + (b*d*f^3*x^11)/11
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ace^3 + e^2(bce + ade + 3acf)x^2 + e(3af(de + cf) + be(de + 3cf))x^4 \right.\\ &\quad \left. + f(3be(de + cf) + af(3de + cf))x^6 + f^2(3bde + bcf + adf)x^8 + bdf^3x^{10}\right)\,dx\\ &= ace^3x + \frac{1}{3}e^2(bce + ade + 3acf)x^3 + \frac{1}{5}e(3af(de + cf) + be(de + 3cf))x^5\\ &\quad + \frac{1}{7}f(3be(de + cf) + af(3de + cf))x^7 + \frac{1}{9}f^2(3bde + bcf + adf)x^9 + \frac{1}{11}bdf^3x^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(a + bx^2\right)\left(c + dx^2\right)\left(e + fx^2\right)^3\,dx = ace^3x &+ \frac{1}{3}e^2(bce + ade + 3acf)x^3\\ &+ \frac{1}{5}e(3af(de + cf) + be(de + 3cf))x^5\\ &+ \frac{1}{7}f(3be(de + cf) + af(3de + cf))x^7\\ &+ \frac{1}{9}f^2(3bde + bcf + adf)x^9 + \frac{1}{11}bdf^3x^{11}\end{aligned}$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x]
```

```
[Out] a*c*e^3*x + (e^2*(b*c*e + a*d*e + 3*a*c*f)*x^3)/3 + (e*(3*a*f*(d*e + c*f) +
 b*e*(d*e + 3*c*f))*x^5)/5 + (f*(3*b*e*(d*e + c*f) + a*f*(3*d*e + c*f))*x^7
)/7 + (f^2*(3*b*d*e + b*c*f + a*d*f)*x^9)/9 + (b*d*f^3*x^11)/11
```

Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.04

method	result
default	$\frac{bd\,f^3x^{11}}{11}+\frac{((ad+bc)f^3+3bde\,f^2)x^9}{9}+\frac{(ac\,f^3+3(ad+bc)e\,f^2+3bd\,e^2f)x^7}{7}+\frac{(3ace\,f^2+3(ad+bc)e^2f+bd\,e^3)x^5}{5}+$ (3ac e^2
norman	$\frac{bd\,f^3x^{11}}{11}+\left(\frac{1}{9}ad\,f^3+\frac{1}{9}bc\,f^3+\frac{1}{3}bde\,f^2\right)x^9+\left(\frac{1}{7}ac\,f^3+\frac{3}{7}ade\,f^2+\frac{3}{7}bce\,f^2+\frac{3}{7}bd\,e^2f\right)x^7+$ ($\frac{3}{5}$a
gosper	$\frac{1}{11}bd\,f^3x^{11}+\frac{1}{9}x^9ad\,f^3+\frac{1}{9}x^9bc\,f^3+\frac{1}{3}x^9bde\,f^2+\frac{1}{7}x^7ac\,f^3+\frac{3}{7}x^7ade\,f^2+\frac{3}{7}x^7bce\,f^2+\frac{3}{7}x^7bd$ e
risch	$\frac{1}{11}bd\,f^3x^{11}+\frac{1}{9}x^9ad\,f^3+\frac{1}{9}x^9bc\,f^3+\frac{1}{3}x^9bde\,f^2+\frac{1}{7}x^7ac\,f^3+\frac{3}{7}x^7ade\,f^2+\frac{3}{7}x^7bce\,f^2+\frac{3}{7}x^7bd$ e
parallelrisch	$\frac{1}{11}bd\,f^3x^{11}+\frac{1}{9}x^9ad\,f^3+\frac{1}{9}x^9bc\,f^3+\frac{1}{3}x^9bde\,f^2+\frac{1}{7}x^7ac\,f^3+\frac{3}{7}x^7ade\,f^2+\frac{3}{7}x^7bce\,f^2+\frac{3}{7}x^7bd$ e

```
[In] int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b*d*f^3*x^11+1/9*((a*d+b*c)*f^3+3*b*d*e*f^2)*x^9+1/7*(a*c*f^3+3*(a*d+b
*c)*e*f^2+3*b*d*e^2*f)*x^7+1/5*(3*a*c*e*f^2+3*(a*d+b*c)*e^2*f+b*d*e^3)*x^5+
1/3*(3*a*c*e^2*f+(a*d+b*c)*e^3)*x^3+a*c*e^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^3\,dx &= \frac{1}{11}\,bdf^3x^{11}+\frac{1}{9}\left(3\,bdef^2+(bc+ad)f^3\right)x^9\\&\quad+\frac{1}{7}\left(3\,bde^2f+acf^3+3\,(bc+ad)ef^2\right)x^7\\&\quad+ace^3x+\frac{1}{5}\left(bde^3+3\,acef^2+3\,(bc+ad)e^2f\right)x^5\\&\quad+\frac{1}{3}\left(3\,ace^2f+(bc+ad)e^3\right)x^3\end{aligned}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] 1/11*b*d*f^3*x^11 + 1/9*(3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^9 + 1/7*(3*b*d*e^
2*f + a*c*f^3 + 3*(b*c + a*d)*e*f^2)*x^7 + a*c*e^3*x + 1/5*(b*d*e^3 + 3*a*c
*e*f^2 + 3*(b*c + a*d)*e^2*f)*x^5 + 1/3*(3*a*c*e^2*f + (b*c + a*d)*e^3)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.33

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx = ace^3x + \frac{bdf^3x^{11}}{11} + x^9\left(\frac{adf^3}{9} + \frac{bcf^3}{9} + \frac{bdef^2}{3}\right) + x^7\left(\frac{acf^3}{7} + \frac{3adef^2}{7} + \frac{3bcef^2}{7} + \frac{3bde^2f}{7}\right) + x^5\cdot\left(\frac{3acef^2}{5} + \frac{3ade^2f}{5} + \frac{3bce^2f}{5} + \frac{bde^3}{5}\right) + x^3\left(ace^2f + \frac{ade^3}{3} + \frac{bce^3}{3}\right)$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**3,x)
```

```
[Out] a*c*e**3*x + b*d*f**3*x**11/11 + x**9*(a*d*f**3/9 + b*c*f**3/9 + b*d*e*f**2
/3) + x**7*(a*c*f**3/7 + 3*a*d*e*f**2/7 + 3*b*c*e*f**2/7 + 3*b*d*e**2*f/7)
+ x**5*(3*a*c*e*f**2/5 + 3*a*d*e**2*f/5 + 3*b*c*e**2*f/5 + b*d*e**3/5) + x*
*3*(a*c*e**2*f + a*d*e**3/3 + b*c*e**3/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3\,dx = \frac{1}{11}\,bdf^3x^{11} + \frac{1}{9}\left(3\,bdef^2 + (bc+ad)f^3\right)x^9 + \frac{1}{7}\left(3\,bde^2f + acf^3 + 3\,(bc+ad)ef^2\right)x^7 + ace^3x + \frac{1}{5}\left(bde^3 + 3\,acef^2 + 3\,(bc+ad)e^2f\right)x^5 + \frac{1}{3}\left(3\,ace^2f + (bc+ad)e^3\right)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] 1/11*b*d*f^3*x^11 + 1/9*(3*b*d*e*f^2 + (b*c + a*d)*f^3)*x^9 + 1/7*(3*b*d*e^
2*f + a*c*f^3 + 3*(b*c + a*d)*e*f^2)*x^7 + a*c*e^3*x + 1/5*(b*d*e^3 + 3*a*c
*e*f^2 + 3*(b*c + a*d)*e^2*f)*x^5 + 1/3*(3*a*c*e^2*f + (b*c + a*d)*e^3)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.27

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3 \, dx = \frac{1}{11}bdf^3x^{11} + \frac{1}{3}bdef^2x^9 + \frac{1}{9}bcf^3x^9 + \frac{1}{9}adf^3x^9 + \frac{3}{7}bde^2fx^7 + \frac{3}{7}bcef^2x^7 + \frac{3}{7}adef^2x^7 + \frac{1}{7}acf^3x^7 + \frac{1}{5}bde^3x^5 + \frac{3}{5}bce^2fx^5 + \frac{3}{5}ade^2fx^5 + \frac{3}{5}acef^2x^5 + \frac{1}{3}bce^3x^3 + \frac{1}{3}ade^3x^3 + ace^2fx^3 + ace^3x$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/11*b*d*f^3*x^11 + 1/3*b*d*e*f^2*x^9 + 1/9*b*c*f^3*x^9 + 1/9*a*d*f^3*x^9 +
 3/7*b*d*e^2*f*x^7 + 3/7*b*c*e*f^2*x^7 + 3/7*a*d*e*f^2*x^7 + 1/7*a*c*f^3*x^
7 + 1/5*b*d*e^3*x^5 + 3/5*b*c*e^2*f*x^5 + 3/5*a*d*e^2*f*x^5 + 3/5*a*c*e*f^2
*x^5 + 1/3*b*c*e^3*x^3 + 1/3*a*d*e^3*x^3 + a*c*e^2*f*x^3 + a*c*e^3*x
```

Mupad [B] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^3 \, dx = x^5\left(\frac{b\,d\,e^3}{5} + \frac{3\,a\,c\,e\,f^2}{5} + \frac{3\,a\,d\,e^2\,f}{5} + \frac{3\,b\,c\,e^2\,f}{5}\right) + x^7\left(\frac{a\,c\,f^3}{7} + \frac{3\,a\,d\,e\,f^2}{7} + \frac{3\,b\,c\,e\,f^2}{7} + \frac{3\,b\,d\,e^2\,f}{7}\right) + x^3\left(\frac{a\,d\,e^3}{3} + \frac{b\,c\,e^3}{3} + a\,c\,e^2\,f\right) + x^9\left(\frac{a\,d\,f^3}{9} + \frac{b\,c\,f^3}{9} + \frac{b\,d\,e\,f^2}{3}\right) + a\,c\,e^3\,x + \frac{b\,d\,f^3\,x^{11}}{11}$$

```
[In] int((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^3,x)
```

```
[Out] x^5*((b*d*e^3)/5 + (3*a*c*e*f^2)/5 + (3*a*d*e^2*f)/5 + (3*b*c*e^2*f)/5) + x
^7*((a*c*f^3)/7 + (3*a*d*e*f^2)/7 + (3*b*c*e*f^2)/7 + (3*b*d*e^2*f)/7) + x^
3*((a*d*e^3)/3 + (b*c*e^3)/3 + a*c*e^2*f) + x^9*((a*d*f^3)/9 + (b*c*f^3)/9
+ (b*d*e*f^2)/3) + a*c*e^3*x + (b*d*f^3*x^11)/11
```

3.3 $\int (a+bx^2)(c+dx^2)(e+fx^2)^2 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2 \, dx = ace^2x + \frac{1}{3}e(bce+ade+2acf)x^3 + \frac{1}{5}(af(2de+cf)+be(de+2cf))x^5 + \frac{1}{7}f(2bde+bcf+adf)x^7 + \frac{1}{9}bdf^2x^9$$

```
[Out] a*c*e^2*x+1/3*e*(2*a*c*f+a*d*e+b*c*e)*x^3+1/5*(a*f*(c*f+2*d*e)+b*e*(2*c*f+d
*e))*x^5+1/7*f*(a*d*f+b*c*f+2*b*d*e)*x^7+1/9*b*d*f^2*x^9
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {535}

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2 \, dx = \frac{1}{7}fx^7(adf+bcf+2bde) + \frac{1}{5}x^5(af(cf+2de)+be(2cf+de)) + \frac{1}{3}ex^3(2acf+ade+bce) + ace^2x + \frac{1}{9}bdf^2x^9$$

```
[In] Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x]
```

```
[Out] a*c*e^2*x + (e*(b*c*e + a*d*e + 2*a*c*f)*x^3)/3 + ((a*f*(2*d*e + c*f) + b*e
*(d*e + 2*c*f))*x^5)/5 + (f*(2*b*d*e + b*c*f + a*d*f)*x^7)/7 + (b*d*f^2*x^9
)/9
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ace^2 + e(bce + ade + 2acf)x^2 + (af(2de + cf) + be(de + 2cf))x^4 \right. \\ &\qquad \left. + f(2bde + bcf + adf)x^6 + bdf^2x^8\right)\, dx \\ &= ace^2x + \frac{1}{3}e(bce + ade + 2acf)x^3 + \frac{1}{5}(af(2de + cf) + be(de + 2cf))x^5 \\ &\quad + \frac{1}{7}f(2bde + bcf + adf)x^7 + \frac{1}{9}bdf^2x^9\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\begin{aligned}\int \left(a + bx^2\right)\left(c + dx^2\right)\left(e + fx^2\right)^2\, dx &= ace^2x + \frac{1}{3}e(bce + ade + 2acf)x^3 \\ &\quad + \frac{1}{5}\left(bde^2 + 2bcef + 2adef + acf^2\right)x^5 \\ &\quad + \frac{1}{7}f(2bde + bcf + adf)x^7 + \frac{1}{9}bdf^2x^9\end{aligned}$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x]
```

```
[Out] a*c*e^2*x + (e*(b*c*e + a*d*e + 2*a*c*f)*x^3)/3 + ((b*d*e^2 + 2*b*c*e*f + 2
*a*d*e*f + a*c*f^2)*x^5)/5 + (f*(2*b*d*e + b*c*f + a*d*f)*x^7)/7 + (b*d*f^2
*x^9)/9
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

method	result
default	$\frac{bd\,f^2x^9}{9}+\frac{((ad+bc)f^2+2ebdf)x^7}{7}+\frac{(ac\,f^2+2(ad+bc)ef+bd\,e^2)x^5}{5}+\frac{(2acef+(ad+bc)e^2)x^3}{3}+ac\,e^2x$
norman	$\frac{bd\,f^2x^9}{9}+\left(\frac{1}{7}ad\,f^2+\frac{1}{7}bc\,f^2+\frac{2}{7}ebdf\right)x^7+\left(\frac{1}{5}ac\,f^2+\frac{2}{5}adef+\frac{2}{5}bcef+\frac{1}{5}bd\,e^2\right)x^5+\left(\frac{2}{3}acef+\right.$
gosper	$\frac{1}{9}bd\,f^2x^9+\frac{1}{7}x^7ad\,f^2+\frac{1}{7}x^7bc\,f^2+\frac{2}{7}x^7ebdf+\frac{1}{5}x^5ac\,f^2+\frac{2}{5}x^5adef+\frac{2}{5}x^5bcef+\frac{1}{5}x^5bd\,e^2+$
risch	$\frac{1}{9}bd\,f^2x^9+\frac{1}{7}x^7ad\,f^2+\frac{1}{7}x^7bc\,f^2+\frac{2}{7}x^7ebdf+\frac{1}{5}x^5ac\,f^2+\frac{2}{5}x^5adef+\frac{2}{5}x^5bcef+\frac{1}{5}x^5bd\,e^2+$
parallelrisch	$\frac{1}{9}bd\,f^2x^9+\frac{1}{7}x^7ad\,f^2+\frac{1}{7}x^7bc\,f^2+\frac{2}{7}x^7ebdf+\frac{1}{5}x^5ac\,f^2+\frac{2}{5}x^5adef+\frac{2}{5}x^5bcef+\frac{1}{5}x^5bd\,e^2+$

```
[In] int((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b*d*f^2*x^9+1/7*((a*d+b*c)*f^2+2*e*b*d*f)*x^7+1/5*(a*c*f^2+2*(a*d+b*c)*
e*f+b*d*e^2)*x^5+1/3*(2*a*c*e*f+(a*d+b*c)*e^2)*x^3+a*c*e^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^2\,dx=\frac{1}{9}\,bdf^2x^9+\frac{1}{7}\left(2\,bdef+(bc+ad)f^2\right)x^7+\frac{1}{5}\left(bde^2+acf^2+2\,(bc+ad)ef\right)x^5+ace^2x+\frac{1}{3}\left(2\,acef+(bc+ad)e^2\right)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] 1/9*b*d*f^2*x^9 + 1/7*(2*b*d*e*f + (b*c + a*d)*f^2)*x^7 + 1/5*(b*d*e^2 + a*
c*f^2 + 2*(b*c + a*d)*e*f)*x^5 + a*c*e^2*x + 1/3*(2*a*c*e*f + (b*c + a*d)*e
^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)^2\,dx=ace^2x+\frac{bdf^2x^9}{9}+x^7\left(\frac{adf^2}{7}+\frac{bcf^2}{7}+\frac{2bdef}{7}\right)+x^5\left(\frac{acf^2}{5}+\frac{2adef}{5}+\frac{2bcef}{5}+\frac{bde^2}{5}\right)+x^3\cdot\left(\frac{2acef}{3}+\frac{ade^2}{3}+\frac{bce^2}{3}\right)$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e)**2,x)
```

```
[Out] a*c*e**2*x + b*d*f**2*x**9/9 + x**7*(a*d*f**2/7 + b*c*f**2/7 + 2*b*d*e*f/7)
 + x**5*(a*c*f**2/5 + 2*a*d*e*f/5 + 2*b*c*e*f/5 + b*d*e**2/5) + x**3*(2*a*c
*e*f/3 + a*d*e**2/3 + b*c*e**2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = \frac{1}{9}bdf^2x^9 + \frac{1}{7}(2bdef + (bc+ad)f^2)x^7 + \frac{1}{5}(bde^2 + acf^2 + 2(bc+ad)ef)x^5 + ace^2x + \frac{1}{3}(2acef + (bc+ad)e^2)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] 1/9*b*d*f^2*x^9 + 1/7*(2*b*d*e*f + (b*c + a*d)*f^2)*x^7 + 1/5*(b*d*e^2 + a*
c*f^2 + 2*(b*c + a*d)*e*f)*x^5 + a*c*e^2*x + 1/3*(2*a*c*e*f + (b*c + a*d)*e
^2)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = \frac{1}{9}bdf^2x^9 + \frac{2}{7}bdefx^7 + \frac{1}{7}bcf^2x^7 + \frac{1}{7}adf^2x^7 + \frac{1}{5}bde^2x^5 + \frac{2}{5}bcefx^5 + \frac{2}{5}adefx^5 + \frac{1}{5}acf^2x^5 + \frac{1}{3}bce^2x^3 + \frac{1}{3}ade^2x^3 + \frac{2}{3}acefx^3 + ace^2x$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] 1/9*b*d*f^2*x^9 + 2/7*b*d*e*f*x^7 + 1/7*b*c*f^2*x^7 + 1/7*a*d*f^2*x^7 + 1/5
*b*d*e^2*x^5 + 2/5*b*c*e*f*x^5 + 2/5*a*d*e*f*x^5 + 1/5*a*c*f^2*x^5 + 1/3*b*
c*e^2*x^3 + 1/3*a*d*e^2*x^3 + 2/3*a*c*e*f*x^3 + a*c*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int (a+bx^2)(c+dx^2)(e+fx^2)^2\,dx = x^5\left(\frac{a\,c\,f^2}{5}+\frac{b\,d\,e^2}{5}+\frac{2\,a\,d\,e\,f}{5}+\frac{2\,b\,c\,e\,f}{5}\right) + x^3\left(\frac{a\,d\,e^2}{3}+\frac{b\,c\,e^2}{3}+\frac{2\,a\,c\,e\,f}{3}\right) + x^7\left(\frac{a\,d\,f^2}{7}+\frac{b\,c\,f^2}{7}+\frac{2\,b\,d\,e\,f}{7}\right)+a\,c\,e^2\,x+\frac{b\,d\,f^2\,x^9}{9}$$

```
[In] int((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^2,x)
```

```
[Out] x^5*((a*c*f^2)/5 + (b*d*e^2)/5 + (2*a*d*e*f)/5 + (2*b*c*e*f)/5) + x^3*((a*d
*e^2)/3 + (b*c*e^2)/3 + (2*a*c*e*f)/3) + x^7*((a*d*f^2)/7 + (b*c*f^2)/7 + (
2*b*d*e*f)/7) + a*c*e^2*x + (b*d*f^2*x^9)/9
```

3.4 $\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 56

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = acex+\frac{1}{3}(bce+ade+acf)x^3 + \frac{1}{5}(bde+bcf+adf)x^5+\frac{1}{7}bdfx^7$$

```
[Out] a*c*e*x+1/3*(a*c*f+a*d*e+b*c*e)*x^3+1/5*(a*d*f+b*c*f+b*d*e)*x^5+1/7*b*d*f*x
^7
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{535\}$

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = \frac{1}{5}x^5(adf+bcf+bde) + \frac{1}{3}x^3(acf+ade+bce)+acex+\frac{1}{7}bdfx^7$$

```
[In] Int[(a + b*x^2)*(c + d*x^2)*(e + f*x^2),x]
```

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b*d*e + b*c*f + a*d*f)*x^5)/5
 + (b*d*f*x^7)/7
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ace + (bce + ade + acf)x^2 + (bde + bcf + adf)x^4 + bdfx^6\right) dx \\ &= acex + \frac{1}{3}(bce + ade + acf)x^3 + \frac{1}{5}(bde + bcf + adf)x^5 + \frac{1}{7}bdfx^7\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \left(a + bx^2\right)\left(c + dx^2\right)\left(e + fx^2\right) dx = acex + \frac{1}{3}(bce + ade + acf)x^3 + \frac{1}{5}(bde + bcf + adf)x^5 + \frac{1}{7}bdfx^7$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)*(e + f*x^2),x]
```

```
[Out] a*c*e*x + ((b*c*e + a*d*e + a*c*f)*x^3)/3 + ((b*d*e + b*c*f + a*d*f)*x^5)/5
 + (b*d*f*x^7)/7
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{bdf\,x^7}{7} + \frac{((ad+bc)f+bde)x^5}{5} + \frac{(acf+(ad+bc)e)x^3}{3} + acex$	53
norman	$\frac{bdf\,x^7}{7} + \left(\frac{1}{5}adf + \frac{1}{5}bcf + \frac{1}{5}bde\right)x^5 + \left(\frac{1}{3}acf + \frac{1}{3}ade + \frac{1}{3}bce\right)x^3 + acex$	55
gosper	$\frac{1}{7}bdf\,x^7 + \frac{1}{5}x^5adf + \frac{1}{5}x^5bcf + \frac{1}{5}x^5bde + \frac{1}{3}x^3acf + \frac{1}{3}x^3ade + \frac{1}{3}x^3bce + acex$	63
risch	$\frac{1}{7}bdf\,x^7 + \frac{1}{5}x^5adf + \frac{1}{5}x^5bcf + \frac{1}{5}x^5bde + \frac{1}{3}x^3acf + \frac{1}{3}x^3ade + \frac{1}{3}x^3bce + acex$	63
parallelrisch	$\frac{1}{7}bdf\,x^7 + \frac{1}{5}x^5adf + \frac{1}{5}x^5bcf + \frac{1}{5}x^5bde + \frac{1}{3}x^3acf + \frac{1}{3}x^3ade + \frac{1}{3}x^3bce + acex$	63

```
[In] int((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*b*d*f*x^7+1/5*((a*d+b*c)*f+b*d*e)*x^5+1/3*(a*c*f+(a*d+b*c)*e)*x^3+a*c*e
*x
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)\,dx=\frac{1}{7}bdfx^7+\frac{1}{5}(bde+(bc+ad)f)x^5+acex+\frac{1}{3}(acf+(bc+ad)e)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="fricas")
```

```
[Out] 1/7*b*d*f*x^7 + 1/5*(b*d*e + (b*c + a*d)*f)*x^5 + a*c*e*x + 1/3*(a*c*f + (b
*c + a*d)*e)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)\,dx=acex+\frac{bdfx^7}{7}+x^5\left(\frac{adf}{5}+\frac{bcf}{5}+\frac{bde}{5}\right)+x^3\left(\frac{acf}{3}+\frac{ade}{3}+\frac{bce}{3}\right)$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)*(f*x**2+e),x)
```

```
[Out] a*c*e*x + b*d*f*x**7/7 + x**5*(a*d*f/5 + b*c*f/5 + b*d*e/5) + x**3*(a*c*f/3
 + a*d*e/3 + b*c*e/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\int \left(a+bx^2\right)\left(c+dx^2\right)\left(e+fx^2\right)\,dx=\frac{1}{7}bdfx^7+\frac{1}{5}(bde+(bc+ad)f)x^5+acex+\frac{1}{3}(acf+(bc+ad)e)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="maxima")
```

```
[Out] 1/7*b*d*f*x^7 + 1/5*(b*d*e + (b*c + a*d)*f)*x^5 + a*c*e*x + 1/3*(a*c*f + (b
*c + a*d)*e)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.11

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = \frac{1}{7}bdfx^7 + \frac{1}{5}bdex^5 + \frac{1}{5}bcfx^5 + \frac{1}{5}adfx^5 + \frac{1}{3}bcex^3 + \frac{1}{3}adex^3 + \frac{1}{3}acfx^3 + acex$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)*(f*x^2+e),x, algorithm="giac")
```

```
[Out] 1/7*b*d*f*x^7 + 1/5*b*d*e*x^5 + 1/5*b*c*f*x^5 + 1/5*a*d*f*x^5 + 1/3*b*c*e*x
^3 + 1/3*a*d*e*x^3 + 1/3*a*c*f*x^3 + a*c*e*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int (a+bx^2)(c+dx^2)(e+fx^2)\,dx = \frac{b\,d\,f\,x^7}{7} + \left(\frac{a\,d\,f}{5} + \frac{b\,c\,f}{5} + \frac{b\,d\,e}{5}\right)x^5 + \left(\frac{a\,c\,f}{3} + \frac{a\,d\,e}{3} + \frac{b\,c\,e}{3}\right)x^3 + a\,c\,e\,x$$

```
[In] int((a + b*x^2)*(c + d*x^2)*(e + f*x^2),x)
```

```
[Out] x^3*((a*c*f)/3 + (a*d*e)/3 + (b*c*e)/3) + x^5*((a*d*f)/5 + (b*c*f)/5 + (b*d
*e)/5) + a*c*e*x + (b*d*f*x^7)/7
```

3.5 $\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 81

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2} dx = -\frac{(3bde-3bcf-2adf)x}{3f^2} + \frac{dx(a+bx^2)}{3f} + \frac{(be-af)(de-cf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}}$$

```
[Out] -1/3*(-2*a*d*f-3*b*c*f+3*b*d*e)*x/f^2+1/3*d*x*(b*x^2+a)/f+(-a*f+b*e)*(-c*f+
d*e)*arctan(x*f^(1/2)/e^(1/2))/f^(5/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {542, 396, 211}

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2} dx = \frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)}{\sqrt{e}f^{5/2}} - \frac{x(-2adf-3bcf+3bde)}{3f^2} + \frac{dx(a+bx^2)}{3f}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x]
```

```
[Out] -1/3*((3*b*d*e - 3*b*c*f - 2*a*d*f)*x)/f^2 + (d*x*(a + b*x^2))/(3*f) + ((b*
e - a*f)*(d*e - c*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{dx(a+bx^2)}{3f} + \frac{\int \frac{-a(de-3cf)-(3bde-3bcf-2adf)x^2}{e+fx^2}\,dx}{3f} \\
&= -\frac{(3bde-3bcf-2adf)x}{3f^2} + \frac{dx(a+bx^2)}{3f} + \frac{((be-af)(de-cf))\int \frac{1}{e+fx^2}\,dx}{f^2} \\
&= -\frac{(3bde-3bcf-2adf)x}{3f^2} + \frac{dx(a+bx^2)}{3f} + \frac{(be-af)(de-cf)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = \frac{(-bde+bcf+adf)x}{f^2} + \frac{bdx^3}{3f} + \frac{(be-af)(de-cf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{5/2}}$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x]
[Out] ((-(b*d*e) + b*c*f + a*d*f)*x)/f^2 + (b*d*x^3)/(3*f) + ((b*e - a*f)*(d*e -
c*f)*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(5/2))
```

Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

method	result
default	$\frac{\frac{1}{3}bdf\,x^3+adfx+bcfx-bdex}{f^2}+\frac{\left(ac\,f^2-adef-bcef+bd\,e^2\right)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{f^2\sqrt{ef}}$
risch	$\frac{bd\,x^3}{3f}+\frac{adx}{f}+\frac{bcx}{f}-\frac{bdex}{f^2}-\frac{\ln\left(fx+\sqrt{-ef}\right)ac}{2\sqrt{-ef}}+\frac{\ln\left(fx+\sqrt{-ef}\right)ade}{2f\sqrt{-ef}}+\frac{\ln\left(fx+\sqrt{-ef}\right)bce}{2f\sqrt{-ef}}-\frac{\ln\left(fx+\sqrt{-ef}\right)bd\,e^2}{2f^2\sqrt{-ef}}+\ln(-f$

[In] int((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x,method=_RETURNVERBOSE)

[Out] 1/f^2*(1/3*b*d*f*x^3+a*d*f*x+b*c*f*x-b*d*e*x)+(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/f^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.36

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx$$
$$=\left[\frac{2\,bdef^2x^3-3\,(bde^2+acf^2-(bc+ad)ef)\sqrt{-ef}\log\left(\frac{fx^2-2\sqrt{-ef}x-e}{fx^2+e}\right)-6\,(bde^2f-(bc+ad)ef^2)x}{6\,ef^3},\ \frac{bdef^2\cdots}{\cdots}\right.$$

[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="fricas")

[Out] [1/6*(2*b*d*e*f^2*x^3 - 3*(b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) - 6*(b*d*e^2*f - (b*c + a*d)*e*f^2)*x)/(e*f^3), 1/3*(b*d*e*f^2*x^3 + 3*(b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) - 3*(b*d*e^2*f - (b*c + a*d)*e*f^2)*x)/(e*f^3)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(73) = 146$.

Time = 0.31 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.54

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx=\frac{bdx^3}{3f}+x\left(\frac{ad}{f}+\frac{bc}{f}-\frac{bde}{f^2}\right)$$
$$-\frac{\sqrt{-\frac{1}{ef^5}}(af-be)(cf-de)\log\left(-\frac{ef^2\sqrt{-\frac{1}{ef^5}}(af-be)(cf-de)}{acf^2-adef-bcef+bde^2}+x\right)}{2}$$
$$+\frac{\sqrt{-\frac{1}{ef^5}}(af-be)(cf-de)\log\left(\frac{ef^2\sqrt{-\frac{1}{ef^5}}(af-be)(cf-de)}{acf^2-adef-bcef+bde^2}+x\right)}{2}$$

[In] integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e),x)

[Out] b*d*x**3/(3*f) + x*(a*d/f + b*c/f - b*d*e/f**2) - sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)*log(-e*f**2*sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)/(a*c*f**2 - a*d*e*f - b*c*e*f + b*d*e**2) + x)/2 + sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)*log(e*f**2*sqrt(-1/(e*f**5))*(a*f - b*e)*(c*f - d*e)/(a*c*f**2 - a*d*e*f - b*c*e*f + b*d*e**2) + x)/2

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(e>0)', see `assume?` for more details)Is e

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = \frac{(bde^2 - bcef - adef + acf^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{\sqrt{ef}\,f^2} + \frac{bdf^2x^3 - 3\,bdefx + 3\,bcf^2x + 3\,adf^2x}{3\,f^3}$$

[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e),x, algorithm="giac")

[Out] (b*d*e^2 - b*c*e*f - a*d*e*f + a*c*f^2)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*f^2) + 1/3*(b*d*f^2*x^3 - 3*b*d*e*f*x + 3*b*c*f^2*x + 3*a*d*f^2*x)/f^3

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.33

$$\int \frac{(a+bx^2)(c+dx^2)}{e+fx^2}\,dx = x\left(\frac{a\,d+b\,c}{f}-\frac{b\,d\,e}{f^2}\right)+\frac{b\,d\,x^3}{3\,f} + \frac{\operatorname{atan}\left(\frac{\sqrt{f}\,x\,(a\,f-b\,e)\,(c\,f-d\,e)}{\sqrt{e}\,(a\,c\,f^2+b\,d\,e^2-a\,d\,e\,f-b\,c\,e\,f)}\right)\,(a\,f-b\,e)\,(c\,f-d\,e)}{\sqrt{e}\,f^{5/2}}$$

```
[In] int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2),x)
```

```
[Out] x*((a*d + b*c)/f - (b*d*e)/f^2) + (b*d*x^3)/(3*f) + (atan((f^(1/2)*x*(a*f -
 b*e)*(c*f - d*e))/(e^(1/2)*(a*c*f^2 + b*d*e^2 - a*d*e*f - b*c*e*f)))*(a*f
- b*e)*(c*f - d*e))/(e^(1/2)*f^(5/2))
```

3.6 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2} dx$

Optimal result

Integrand size = 24, antiderivative size = 108

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2} dx = \frac{b(3de-cf)x}{2ef^2} - \frac{(de-cf)x(a+bx^2)}{2ef(e+fx^2)} - \frac{(be(3de-cf)-af(de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{5/2}}$$

```
[Out] 1/2*b*(-c*f+3*d*e)*x/e/f^2-1/2*(-c*f+d*e)*x*(b*x^2+a)/e/f/(f*x^2+e)-1/2*(b*
e*(-c*f+3*d*e)-a*f*(c*f+d*e))*arctan(x*f^(1/2)/e^(1/2))/e^(3/2)/f^(5/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {540, 396, 211}

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2} dx = -\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(3de-cf)-af(cf+de))}{2e^{3/2}f^{5/2}} - \frac{x(a+bx^2)(de-cf)}{2ef(e+fx^2)} + \frac{bx(3de-cf)}{2ef^2}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x]
```

```
[Out] (b*(3*d*e - c*f)*x)/(2*e*f^2) - ((d*e - c*f)*x*(a + b*x^2))/(2*e*f*(e + f*x
^2)) - ((b*e*(3*d*e - c*f) - a*f*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(2*e^(3/2)*f^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(de-cf)x(a+bx^2)}{2ef\,(e+fx^2)} - \frac{\int \frac{-a(de+cf)-b(3de-cf)x^2}{e+fx^2}\,dx}{2ef} \\
&= \frac{b(3de-cf)x}{2ef^2} - \frac{(de-cf)x(a+bx^2)}{2ef\,(e+fx^2)} - \frac{(be(3de-cf)-af(de+cf))\int \frac{1}{e+fx^2}\,dx}{2ef^2} \\
&= \frac{b(3de-cf)x}{2ef^2} - \frac{(de-cf)x(a+bx^2)}{2ef\,(e+fx^2)} - \frac{(be(3de-cf)-af(de+cf))\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)\,(c+dx^2)}{(e+fx^2)^2}\,dx = \frac{bdx}{f^2} + \frac{(be-af)(de-cf)x}{2ef^2\,(e+fx^2)} - \frac{(be(3de-cf)-af(de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{5/2}}$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x]
```

```
[Out] (b*d*x)/f^2 + ((b*e - a*f)*(d*e - c*f)*x)/(2*e*f^2*(e + f*x^2)) - ((b*e*(3*
d*e - c*f) - a*f*(d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(5/
2))
```

Maple [A] (verified)

Time = 3.27 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.90

method	result
default	$\frac{bdx}{f^2}+\frac{\frac{\left(ac\,f^2-adef-bcef+bd\,e^2\right)x}{2e\left(f\,x^2+e\right)}+\frac{\left(ac\,f^2+adef+bcef-3bd\,e^2\right)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2e\sqrt{ef}}}{f^2}$
risch	$\frac{bdx}{f^2}+\frac{(ac\,f^2-adef-bcef+bd\,e^2)x}{2e\,f^2(f\,x^2+e)}-\frac{\ln(fx+\sqrt{-ef})ac}{4\sqrt{-ef}\,e}-\frac{\ln(fx+\sqrt{-ef})ad}{4f\sqrt{-ef}}-\frac{\ln(fx+\sqrt{-ef})bc}{4f\sqrt{-ef}}+\frac{3e\ln(fx+\sqrt{-ef})bd}{4f^2\sqrt{-ef}}+$

[In] int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x,method=_RETURNVERBOSE)

[Out] b*d/f^2*x+1/f^2*(1/2*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e*x/(f*x^2+e)+1/2*(a*c*f^2+a*d*e*f+b*c*e*f-3*b*d*e^2)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.94

$$\int \frac{(a+bx^2)\,(c+dx^2)}{(e+fx^2)^2}\,dx$$
$$=\left[\frac{4\,bde^2f^2x^3+(3\,bde^3-acef^2-(bc+ad)e^2f+(3\,bde^2f-acf^3-(bc+ad)ef^2)x^2)\sqrt{-ef}\log\left(\frac{fx^2-2\sqrt{-e}}{fx^2+e}\right.}{4\,(e^2f^4x^2+e^3f^3)}\right.$$

[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="fricas")

[Out] [1/4*(4*b*d*e^2*f^2*x^3 + (3*b*d*e^3 - a*c*e*f^2 - (b*c + a*d)*e^2*f + (3*b*d*e^2*f - a*c*f^3 - (b*c + a*d)*e*f^2)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 2*(3*b*d*e^3*f + a*c*e*f^3 - (b*c + a*d)*e^2*f^2)*x)/(e^2*f^4*x^2 + e^3*f^3), 1/2*(2*b*d*e^2*f^2*x^3 - (3*b*d*e^3 - a*c*e*f^2 - (b*c + a*d)*e^2*f + (3*b*d*e^2*f - a*c*f^3 - (b*c + a*d)*e*f^2)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + (3*b*d*e^3*f + a*c*e*f^3 - (b*c + a*d)*e^2*f^2)*x)/(e^2*f^4*x^2 + e^3*f^3)]

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.76

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx = \frac{bdx}{f^2} + \frac{x(acf^2-adef-bcef+bde^2)}{2e^2f^2+2ef^3x^2} - \frac{\sqrt{-\frac{1}{e^3f^5}}(acf^2+adef+bcef-3bde^2)\log\left(-e^2f^2\sqrt{-\frac{1}{e^3f^5}}+x\right)}{4} + \frac{\sqrt{-\frac{1}{e^3f^5}}(acf^2+adef+bcef-3bde^2)\log\left(e^2f^2\sqrt{-\frac{1}{e^3f^5}}+x\right)}{4}$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**2,x)
```

```
[Out] b*d*x/f**2 + x*(a*c*f**2 - a*d*e*f - b*c*e*f + b*d*e**2)/(2*e**2*f**2 + 2*e
*f**3*x**2) - sqrt(-1/(e**3*f**5))*(a*c*f**2 + a*d*e*f + b*c*e*f - 3*b*d*e*
*2)*log(-e**2*f**2*sqrt(-1/(e**3*f**5)) + x)/4 + sqrt(-1/(e**3*f**5))*(a*c*
f**2 + a*d*e*f + b*c*e*f - 3*b*d*e**2)*log(e**2*f**2*sqrt(-1/(e**3*f**5)) +
 x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.95

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx = \frac{bdx}{f^2} - \frac{(3\,bde^2 - bcef - adef - acf^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2\sqrt{ef}ef^2} + \frac{bde^2x - bcefx - adefx + acf^2x}{2\,(fx^2+e)ef^2}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] b*d*x/f^2 - 1/2*(3*b*d*e^2 - b*c*e*f - a*d*e*f - a*c*f^2)*arctan(f*x/sqrt(e
*f))/(sqrt(e*f)*e*f^2) + 1/2*(b*d*e^2*x - b*c*e*f*x - a*d*e*f*x + a*c*f^2*x
)/((f*x^2 + e)*e*f^2)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^2}\,dx = \frac{b\,d\,x}{f^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,\left(a\,c\,f^2 - 3\,b\,d\,e^2 + a\,d\,e\,f + b\,c\,e\,f\right)}{2\,e^{3/2}\,f^{5/2}} + \frac{x\,(a\,c\,f^2 + b\,d\,e^2 - a\,d\,e\,f - b\,c\,e\,f)}{2\,e\,\left(f^3\,x^2 + e\,f^2\right)}$$

```
[In] int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^2,x)
```

```
[Out] (b*d*x)/f^2 + (atan((f^(1/2)*x)/e^(1/2))*(a*c*f^2 - 3*b*d*e^2 + a*d*e*f + b
*c*e*f))/(2*e^(3/2)*f^(5/2)) + (x*(a*c*f^2 + b*d*e^2 - a*d*e*f - b*c*e*f))/
(2*e*(e*f^2 + f^3*x^2))
```

3.7 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3} \, dx = -\frac{(de-cf)x(a+bx^2)}{4ef(e+fx^2)^2} - \frac{(be(3de+cf)-af(de+3cf))x}{8e^2f^2(e+fx^2)} + \frac{(be(3de+cf)+af(de+3cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{5/2}}$$

```
[Out] -1/4*(-c*f+d*e)*x*(b*x^2+a)/e/f/(f*x^2+e)^2-1/8*(b*e*(c*f+3*d*e)-a*f*(3*c*f
+d*e))*x/e^2/f^2/(f*x^2+e)+1/8*(b*e*(c*f+3*d*e)+a*f*(3*c*f+d*e))*arctan(x*f
^(1/2)/e^(1/2))/e^(5/2)/f^(5/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {540, 393, 211}

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3} \, dx = \frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(3cf+de)+be(cf+3de))}{8e^{5/2}f^{5/2}} - \frac{x(be(cf+3de)-af(3cf+de))}{8e^2f^2(e+fx^2)} - \frac{x(a+bx^2)(de-cf)}{4ef(e+fx^2)^2}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x]
```

```
[Out] -1/4*((d*e - c*f)*x*(a + b*x^2))/(e*f*(e + f*x^2)^2) - ((b*e*(3*d*e + c*f)
- a*f*(d*e + 3*c*f))*x)/(8*e^2*f^2*(e + f*x^2)) + ((b*e*(3*d*e + c*f) + a*f
*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(de-cf)x(a+bx^2)}{4ef\left(e+fx^2\right)^2}-\frac{\int \frac{-a(de+3cf)-b(3de+cf)x^2}{(e+fx^2)^2}\,dx}{4ef}\\
&= -\frac{(de-cf)x(a+bx^2)}{4ef\left(e+fx^2\right)^2}-\frac{(be(3de+cf)-af(de+3cf))x}{8e^2f^2\left(e+fx^2\right)}\\
&\quad+\frac{(be(3de+cf)+af(de+3cf))\int \frac{1}{e+fx^2}\,dx}{8e^2f^2}\\
&= -\frac{(de-cf)x(a+bx^2)}{4ef\left(e+fx^2\right)^2}-\frac{(be(3de+cf)-af(de+3cf))x}{8e^2f^2\left(e+fx^2\right)}\\
&\quad+\frac{(be(3de+cf)+af(de+3cf))\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{5/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx = \frac{(be-af)(de-cf)x}{4ef^2(e+fx^2)^2} + \frac{(be(-5de+cf)+af(de+3cf))x}{8e^2f^2(e+fx^2)} + \frac{(be(3de+cf)+af(de+3cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{5/2}}$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x]
```

```
[Out] ((b*e - a*f)*(d*e - c*f)*x)/(4*e*f^2*(e + f*x^2)^2) + ((b*e*(-5*d*e + c*f)
+ a*f*(d*e + 3*c*f))*x)/(8*e^2*f^2*(e + f*x^2)) + ((b*e*(3*d*e + c*f) + a*f
*(d*e + 3*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(5/2))
```

Maple [A] (verified)

Time = 3.27 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02

method	result
default	$\frac{\frac{(3acf^2+adef+bcef-5bde^2)x^3}{8e^2f}+\frac{(5acf^2-adef-bcef-3bde^2)x}{8ef^2}}{(fx^2+e)^2}+\frac{(3acf^2+adef+bcef+3bde^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{8e^2f^2\sqrt{ef}}$
risch	$\frac{\frac{(3acf^2+adef+bcef-5bde^2)x^3}{8e^2f}+\frac{(5acf^2-adef-bcef-3bde^2)x}{8ef^2}}{(fx^2+e)^2}-\frac{3\ln(fx+\sqrt{-ef})ac}{16\sqrt{-ef}\,e^2}-\frac{\ln(fx+\sqrt{-ef})ad}{16\sqrt{-ef}\,fe}-\frac{\ln(fx+\sqrt{-ef})bc}{16\sqrt{-ef}\,fe}-3$

```
[In] int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*a*c*f^2+a*d*e*f+b*c*e*f-5*b*d*e^2)/e^2/f*x^3+1/8*(5*a*c*f^2-a*d*e*f
-b*c*e*f-3*b*d*e^2)/e/f^2*x)/(f*x^2+e)^2+1/8*(3*a*c*f^2+a*d*e*f+b*c*e*f+3*b
*d*e^2)/e^2/f^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 471, normalized size of antiderivative = 3.62

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx$$
$$= \left[-\frac{2\,(5\,bde^3f^2 - 3\,acef^4 - (bc+ad)e^2f^3)x^3 + (3\,bde^4 + 3\,ace^2f^2 + (bc+ad)e^3f + (3\,bde^2f^2 + 3\,acf^4 + \cdots}{1\cdots} \right.$$
$$-\frac{(5\,bde^3f^2 - 3\,acef^4 - (bc+ad)e^2f^3)x^3 - (3\,bde^4 + 3\,ace^2f^2 + (bc+ad)e^3f + (3\,bde^2f^2 + 3\,acf^4 + (bc \cdots}{8\,(e^3f^5 \cdots}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*(5*b*d*e^3*f^2 - 3*a*c*e*f^4 - (b*c + a*d)*e^2*f^3)*x^3 + (3*b*d*
e^4 + 3*a*c*e^2*f^2 + (b*c + a*d)*e^3*f + (3*b*d*e^2*f^2 + 3*a*c*f^4 + (b*c
 + a*d)*e*f^3)*x^4 + 2*(3*b*d*e^3*f + 3*a*c*e*f^3 + (b*c + a*d)*e^2*f^2)*x^
2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 2*(3*b*d*e^4*
f - 5*a*c*e^2*f^3 + (b*c + a*d)*e^3*f^2)*x)/(e^3*f^5*x^4 + 2*e^4*f^4*x^2 +
e^5*f^3), -1/8*((5*b*d*e^3*f^2 - 3*a*c*e*f^4 - (b*c + a*d)*e^2*f^3)*x^3 - (
3*b*d*e^4 + 3*a*c*e^2*f^2 + (b*c + a*d)*e^3*f + (3*b*d*e^2*f^2 + 3*a*c*f^4
+ (b*c + a*d)*e*f^3)*x^4 + 2*(3*b*d*e^3*f + 3*a*c*e*f^3 + (b*c + a*d)*e^2*f
^2)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + (3*b*d*e^4*f - 5*a*c*e^2*f^3 + (
b*c + a*d)*e^3*f^2)*x)/(e^3*f^5*x^4 + 2*e^4*f^4*x^2 + e^5*f^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. 2(122) = 244.

Time = 1.39 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx = -\frac{\sqrt{-\frac{1}{e^5f^5}}\cdot(3acf^2+adef+bcef+3bde^2)\log\left(-e^3f^2\sqrt{-\frac{1}{e^5f^5}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{e^5f^5}}\cdot(3acf^2+adef+bcef+3bde^2)\log\left(e^3f^2\sqrt{-\frac{1}{e^5f^5}}+x\right)}{16} + \frac{x^3\cdot(3acf^3+adef^2+bcef^2-5bde^2f)+x(5acef^2-ade^2f-bce^2f-3bde^3)}{8e^4f^2+16e^3f^3x^2+8e^2f^4x^4}$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**3,x)
```

```
[Out] -sqrt(-1/(e**5*f**5))*(3*a*c*f**2 + a*d*e*f + b*c*e*f + 3*b*d*e**2)*log(-e*
*3*f**2*sqrt(-1/(e**5*f**5)) + x)/16 + sqrt(-1/(e**5*f**5))*(3*a*c*f**2 + a
```

```
*d*e*f + b*c*e*f + 3*b*d*e**2)*log(e**3*f**2*sqrt(-1/(e**5*f**5)) + x)/16 +
 (x**3*(3*a*c*f**3 + a*d*e*f**2 + b*c*e*f**2 - 5*b*d*e**2*f) + x*(5*a*c*e*f
**2 - a*d*e**2*f - b*c*e**2*f - 3*b*d*e**3))/(8*e**4*f**2 + 16*e**3*f**3*x*
*2 + 8*e**2*f**4*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.09

$$\begin{aligned} \int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx &= \frac{(3\,bde^2+bcef+adef+3\,acf^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{8\,\sqrt{ef}e^2f^2} \\ &\quad - \frac{5\,bde^2fx^3-bcef^2x^3-adef^2x^3-3\,acf^3x^3+3\,bde^3x+bce^2fx+ade^2fx-5\,acef^2x}{8\,(fx^2+e)^2e^2f^2} \end{aligned}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*b*d*e^2 + b*c*e*f + a*d*e*f + 3*a*c*f^2)*arctan(f*x/sqrt(e*f))/(sqrt
(e*f)*e^2*f^2) - 1/8*(5*b*d*e^2*f*x^3 - b*c*e*f^2*x^3 - a*d*e*f^2*x^3 - 3*a
*c*f^3*x^3 + 3*b*d*e^3*x + b*c*e^2*f*x + a*d*e^2*f*x - 5*a*c*e*f^2*x)/((f*x
^2 + e)^2*e^2*f^2)
```

Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^3}\,dx = \frac{\operatorname{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,(3\,a\,c\,f^2+3\,b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{8\,e^{5/2}\,f^{5/2}} - \frac{\frac{x\,(3\,b\,d\,e^2-5\,a\,c\,f^2+a\,d\,e\,f+b\,c\,e\,f)}{8\,e\,f^2}-\frac{x^3\,(3\,a\,c\,f^2-5\,b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{8\,e^2\,f}}{e^2+2\,e\,f\,x^2+f^2\,x^4}$$

```
[In] int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^3,x)
```

```
[Out] (atan((f^(1/2)*x)/e^(1/2))*(3*a*c*f^2 + 3*b*d*e^2 + a*d*e*f + b*c*e*f))/(8*
e^(5/2)*f^(5/2)) - ((x*(3*b*d*e^2 - 5*a*c*f^2 + a*d*e*f + b*c*e*f))/(8*e*f^
2) - (x^3*(3*a*c*f^2 - 5*b*d*e^2 + a*d*e*f + b*c*e*f))/(8*e^2*f))/(e^2 + f^
2*x^4 + 2*e*f*x^2)
```

3.8 $\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4} dx$

Optimal result

Integrand size = 24, antiderivative size = 171

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4} dx = -\frac{(de-cf)x(a+bx^2)}{6ef(e+fx^2)^3} - \frac{(3be(de+cf)-af(de+5cf))x}{24e^2f^2(e+fx^2)^2} + \frac{(be(de+cf)+af(de+5cf))x}{16e^3f^2(e+fx^2)} + \frac{(be(de+cf)+af(de+5cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{5/2}}$$

```
[Out] -1/6*(-c*f+d*e)*x*(b*x^2+a)/e/f/(f*x^2+e)^3-1/24*(3*b*e*(c*f+d*e)-a*f*(5*c*
f+d*e))*x/e^2/f^2/(f*x^2+e)^2+1/16*(b*e*(c*f+d*e)+a*f*(5*c*f+d*e))*x/e^3/f^
2/(f*x^2+e)+1/16*(b*e*(c*f+d*e)+a*f*(5*c*f+d*e))*arctan(x*f^(1/2)/e^(1/2))/
e^(7/2)/f^(5/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {540, 393, 205, 211}

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4} dx = \frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(5cf+de)+be(cf+de))}{16e^{7/2}f^{5/2}} + \frac{x(af(5cf+de)+be(cf+de))}{16e^3f^2(e+fx^2)} - \frac{x(3be(cf+de)-af(5cf+de))}{24e^2f^2(e+fx^2)^2} - \frac{x(a+bx^2)(de-cf)}{6ef(e+fx^2)^3}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x]
```

```
[Out] -1/6*((d*e - c*f)*x*(a + b*x^2))/(e*f*(e + f*x^2)^3) - ((3*b*e*(d*e + c*f)
- a*f*(d*e + 5*c*f))*x)/(24*e^2*f^2*(e + f*x^2)^2) + ((b*e*(d*e + c*f) + a*
f*(d*e + 5*c*f))*x)/(16*e^3*f^2*(e + f*x^2)) + ((b*e*(d*e + c*f) + a*f*(d*e
 + 5*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7/2)*f^(5/2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(de-cf)x(a+bx^2)}{6ef\left(e+fx^2\right)^3}-\frac{\int\frac{-a(de+5cf)-3b(de+cf)x^2}{\left(e+fx^2\right)^3}\,dx}{6ef}\\
&= -\frac{(de-cf)x(a+bx^2)}{6ef\left(e+fx^2\right)^3}-\frac{(3be(de+cf)-af(de+5cf))x}{24e^2f^2\left(e+fx^2\right)^2}\\
&\quad+\frac{(be(de+cf)+af(de+5cf))\int\frac{1}{\left(e+fx^2\right)^2}\,dx}{8e^2f^2}
\end{aligned}$$

$$= -\frac{(de-cf)x(a+bx^2)}{6ef(e+fx^2)^3} - \frac{(3be(de+cf)-af(de+5cf))x}{24e^2f^2(e+fx^2)^2} + \frac{(be(de+cf)+af(de+5cf))x}{16e^3f^2(e+fx^2)} + \frac{(be(de+cf)+af(de+5cf))\int \frac{1}{e+fx^2}\,dx}{16e^3f^2}$$

$$= -\frac{(de-cf)x(a+bx^2)}{6ef(e+fx^2)^3} - \frac{(3be(de+cf)-af(de+5cf))x}{24e^2f^2(e+fx^2)^2} + \frac{(be(de+cf)+af(de+5cf))x}{16e^3f^2(e+fx^2)} + \frac{(be(de+cf)+af(de+5cf))\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{5/2}}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = \frac{(be-af)(de-cf)x}{6ef^2(e+fx^2)^3} + \frac{(be(-7de+cf)+af(de+5cf))x}{24e^2f^2(e+fx^2)^2} + \frac{(be(de+cf)+af(de+5cf))x}{16e^3f^2(e+fx^2)} + \frac{(be(de+cf)+af(de+5cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{5/2}}$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x]
```

```
[Out] ((b*e - a*f)*(d*e - c*f)*x)/(6*e*f^2*(e + f*x^2)^3) + ((b*e*(-7*d*e + c*f)
+ a*f*(d*e + 5*c*f))*x)/(24*e^2*f^2*(e + f*x^2)^2) + ((b*e*(d*e + c*f) + a*
f*(d*e + 5*c*f))*x)/(16*e^3*f^2*(e + f*x^2)) + ((b*e*(d*e + c*f) + a*f*(d*e
 + 5*c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^(7/2)*f^(5/2))
```

Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{(5acf^2+adef+bcef+bde^2)x^5}{16e^3}+\frac{(5acf^2+adef+bcef-bde^2)x^3}{6e^2f}+\frac{(11acf^2-adef-bcef-bde^2)x}{16ef^2}}{(fx^2+e)^3}+\frac{(5acf^2+adef+bcef+bde^2)\arctan\left(\frac{fx}{\sqrt{e\ldots}}\right)}{16e^3f^2\sqrt{ef}}$
risch	$\frac{\frac{(5acf^2+adef+bcef+bde^2)x^5}{16e^3}+\frac{(5acf^2+adef+bcef-bde^2)x^3}{6e^2f}+\frac{(11acf^2-adef-bcef-bde^2)x}{16ef^2}}{(fx^2+e)^3}-\frac{5\ln(fx+\sqrt{-ef})ac}{32\sqrt{-ef}\,e^3}-\frac{\ln(fx+\sqrt{-ef})a\ldots}{32\sqrt{-ef}\,f\,e^2}$

```
[In] int((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(5*a*c*f^2+a*d*e*f+b*c*e*f+b*d*e^2)/e^3*x^5+1/6*(5*a*c*f^2+a*d*e*f+b*
c*e*f-b*d*e^2)/e^2/f*x^3+1/16*(11*a*c*f^2-a*d*e*f-b*c*e*f-b*d*e^2)/e/f^2*x)
```

```
/(f*x^2+e)^3+1/16*(5*a*c*f^2+a*d*e*f+b*c*e*f+b*d*e^2)/e^3/f^2/(e*f)^(1/2)*a
rctan(f*x/(e*f)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(155) = 310$.

Time = 0.29 (sec) , antiderivative size = 642, normalized size of antiderivative = 3.75

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = \left[\frac{6\,(bde^3f^3+5\,acef^5+(bc+ad)e^2f^4)x^5-16\,(bde^4f^2-5\,ace^2f^4-(bc+ad)e^3f^3)x^3-3\,(bde^5+5\,ace^3f}{}\right.$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(6*(b*d*e^3*f^3 + 5*a*c*e*f^5 + (b*c + a*d)*e^2*f^4)*x^5 - 16*(b*d*e^
4*f^2 - 5*a*c*e^2*f^4 - (b*c + a*d)*e^3*f^3)*x^3 - 3*(b*d*e^5 + 5*a*c*e^3*f
^2 + (b*d*e^2*f^3 + 5*a*c*f^5 + (b*c + a*d)*e*f^4)*x^6 + (b*c + a*d)*e^4*f
+ 3*(b*d*e^3*f^2 + 5*a*c*e*f^4 + (b*c + a*d)*e^2*f^3)*x^4 + 3*(b*d*e^4*f +
5*a*c*e^2*f^3 + (b*c + a*d)*e^3*f^2)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e
*f)*x - e)/(f*x^2 + e)) - 6*(b*d*e^5*f - 11*a*c*e^3*f^3 + (b*c + a*d)*e^4*f
^2)*x)/(e^4*f^6*x^6 + 3*e^5*f^5*x^4 + 3*e^6*f^4*x^2 + e^7*f^3), 1/48*(3*(b*
d*e^3*f^3 + 5*a*c*e*f^5 + (b*c + a*d)*e^2*f^4)*x^5 - 8*(b*d*e^4*f^2 - 5*a*c
*e^2*f^4 - (b*c + a*d)*e^3*f^3)*x^3 + 3*(b*d*e^5 + 5*a*c*e^3*f^2 + (b*d*e^2
*f^3 + 5*a*c*f^5 + (b*c + a*d)*e*f^4)*x^6 + (b*c + a*d)*e^4*f + 3*(b*d*e^3*
f^2 + 5*a*c*e*f^4 + (b*c + a*d)*e^2*f^3)*x^4 + 3*(b*d*e^4*f + 5*a*c*e^2*f^3
 + (b*c + a*d)*e^3*f^2)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) - 3*(b*d*e^5*f
 - 11*a*c*e^3*f^3 + (b*c + a*d)*e^4*f^2)*x)/(e^4*f^6*x^6 + 3*e^5*f^5*x^4 +
3*e^6*f^4*x^2 + e^7*f^3)]
```

Sympy [A] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = -\frac{\sqrt{-\frac{1}{e^7f^5}}\cdot(5acf^2+adef+bcef+bde^2)\log\left(-e^4f^2\sqrt{-\frac{1}{e^7f^5}}+x\right)}{32} + \frac{\sqrt{-\frac{1}{e^7f^5}}\cdot(5acf^2+adef+bcef+bde^2)\log\left(e^4f^2\sqrt{-\frac{1}{e^7f^5}}+x\right)}{32} + \frac{x^5\cdot(15acf^4+3adef^3+3bcef^3+3bde^2f^2)+x^3\cdot(40acef^3+8ade^2f^2+8bce^2f^2-8bde^3f)+x(33ace^2}{48e^6f^2+144e^5f^3x^2+144e^4f^4x^4+48e^3f^5x^6}$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)/(f*x**2+e)**4,x)
```

```
[Out] -sqrt(-1/(e**7*f**5))*(5*a*c*f**2 + a*d*e*f + b*c*e*f + b*d*e**2)*log(-e**4
*f**2*sqrt(-1/(e**7*f**5)) + x)/32 + sqrt(-1/(e**7*f**5))*(5*a*c*f**2 + a*d
*e*f + b*c*e*f + b*d*e**2)*log(e**4*f**2*sqrt(-1/(e**7*f**5)) + x)/32 + (x*
*5*(15*a*c*f**4 + 3*a*d*e*f**3 + 3*b*c*e*f**3 + 3*b*d*e**2*f**2) + x**3*(40
*a*c*e*f**3 + 8*a*d*e**2*f**2 + 8*b*c*e**2*f**2 - 8*b*d*e**3*f) + x*(33*a*c
*e**2*f**2 - 3*a*d*e**3*f - 3*b*c*e**3*f - 3*b*d*e**4))/(48*e**6*f**2 + 144
*e**5*f**3*x**2 + 144*e**4*f**4*x**4 + 48*e**3*f**5*x**6)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = \frac{(bde^2+bcef+adef+5\,acf^2)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{16\,\sqrt{ef}e^3f^2} + \frac{3\,bde^2f^2x^5+3\,bcef^3x^5+3\,adef^3x^5+15\,acf^4x^5-8\,bde^3fx^3+8\,bce^2f^2x^3+8\,ade^2f^2x^3+40\,acef^3x^3-}{48\,(fx^2+e)^3e^3f^2}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)/(f*x^2+e)^4,x, algorithm="giac")
```

```
[Out] 1/16*(b*d*e^2 + b*c*e*f + a*d*e*f + 5*a*c*f^2)*arctan(f*x/sqrt(e*f))/(sqrt(
e*f)*e^3*f^2) + 1/48*(3*b*d*e^2*f^2*x^5 + 3*b*c*e*f^3*x^5 + 3*a*d*e*f^3*x^5
 + 15*a*c*f^4*x^5 - 8*b*d*e^3*f*x^3 + 8*b*c*e^2*f^2*x^3 + 8*a*d*e^2*f^2*x^3
 + 40*a*c*e*f^3*x^3 - 3*b*d*e^4*x - 3*b*c*e^3*f*x - 3*a*d*e^3*f*x + 33*a*c*
e^2*f^2*x)/((f*x^2 + e)^3*e^3*f^2)
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)(c+dx^2)}{(e+fx^2)^4}\,dx = \frac{\frac{x^5\,(5\,a\,c\,f^2+b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{16\,e^3}-\frac{x\,(b\,d\,e^2-11\,a\,c\,f^2+a\,d\,e\,f+b\,c\,e\,f)}{16\,e\,f^2}+\frac{x^3\,(5\,a\,c\,f^2-b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{6\,e^2\,f}}{e^3+3\,e^2\,f\,x^2+3\,e\,f^2\,x^4+f^3\,x^6} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,(5\,a\,c\,f^2+b\,d\,e^2+a\,d\,e\,f+b\,c\,e\,f)}{16\,e^{7/2}\,f^{5/2}}$$

```
[In] int(((a + b*x^2)*(c + d*x^2))/(e + f*x^2)^4,x)
```

```
[Out] ((x^5*(5*a*c*f^2 + b*d*e^2 + a*d*e*f + b*c*e*f))/(16*e^3) - (x*(b*d*e^2 - 1
1*a*c*f^2 + a*d*e*f + b*c*e*f))/(16*e*f^2) + (x^3*(5*a*c*f^2 - b*d*e^2 + a*
d*e*f + b*c*e*f))/(6*e^2*f))/(e^3 + f^3*x^6 + 3*e^2*f*x^2 + 3*e*f^2*x^4) +
(atan((f^(1/2)*x)/e^(1/2))*(5*a*c*f^2 + b*d*e^2 + a*d*e*f + b*c*e*f))/(16*e
^(7/2)*f^(5/2))
```

3.9 $\int (a+bx^2)(c+dx^2)^2(e+fx^2)^3\,dx$

Optimal result

Integrand size = 26, antiderivative size = 226

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^3\,dx = ac^2e^3x+\frac{1}{3}ce^2(bce+2ade+3acf)x^3 \\ +\frac{1}{5}e\left(bce(2de+3cf)+a\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^5 \\ +\frac{1}{7}\left(af\left(3d^2e^2+6cdef+c^2f^2\right) \\ +be\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^7 \\ +\frac{1}{9}f\left(adf(3de+2cf)+b\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^9 \\ +\frac{1}{11}df^2(3bde+2bcf+adf)x^{11}+\frac{1}{13}bd^2f^3x^{13}$$

```
[Out] a*c^2*e^3*x+1/3*c*e^2*(3*a*c*f+2*a*d*e+b*c*e)*x^3+1/5*e*(b*c*e*(3*c*f+2*d*e
)+a*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^5+1/7*(a*f*(c^2*f^2+6*c*d*e*f+3*d^2*e^
2)+b*e*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^7+1/9*f*(a*d*f*(2*c*f+3*d*e)+b*(c^2
*f^2+6*c*d*e*f+3*d^2*e^2))*x^9+1/11*d*f^2*(a*d*f+2*b*c*f+3*b*d*e)*x^11+1/13
*b*d^2*f^3*x^13
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used

$= \{535\}$

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx = \frac{1}{9}fx^9\left(adf(2cf+3de)+b\left(c^2f^2+6cdef+3d^2e^2\right)\right) + \frac{1}{7}x^7\left(af\left(c^2f^2+6cdef+3d^2e^2\right) + be\left(3c^2f^2+6cdef+d^2e^2\right)\right) + \frac{1}{5}ex^5\left(a\left(3c^2f^2+6cdef+d^2e^2\right)+bce(3cf+2de)\right) + \frac{1}{3}ce^2x^3(3acf+2ade+bce) + \frac{1}{11}df^2x^{11}(adf+2bcf+3bde) + ac^2e^3x + \frac{1}{13}bd^2f^3x^{13}$$

```
[In] Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

```
[Out] a*c^2*e^3*x + (c*e^2*(b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b*c*e*(2*d*e
+ 3*c*f) + a*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^5)/5 + ((a*f*(3*d^2*e^2 +
 6*c*d*e*f + c^2*f^2) + b*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + (f*
(a*d*f*(3*d*e + 2*c*f) + b*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^9)/9 + (d*f
^2*(3*b*d*e + 2*b*c*f + a*d*f)*x^11)/11 + (b*d^2*f^3*x^13)/13
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\text{integral} = \int \left(ac^2e^3+ce^2(bce+2ade+3acf)x^2+e\left(bce(2de+3cf)+a\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^4 + \left(af\left(3d^2e^2+6cdef+c^2f^2\right)+be\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^6 + f\left(adf(3de+2cf)+b\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^8+df^2(3bde+2bcf+adf)x^{10} + bd^2f^3x^{12}\right)\,dx$$

$$= ac^2e^3x+\frac{1}{3}ce^2(bce+2ade+3acf)x^3+\frac{1}{5}e\left(bce(2de+3cf)+a\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^5 + \frac{1}{7}\left(af\left(3d^2e^2+6cdef+c^2f^2\right)+be\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^7 + \frac{1}{9}f\left(adf(3de+2cf)+b\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^9 + \frac{1}{11}df^2(3bde+2bcf+adf)x^{11}+\frac{1}{13}bd^2f^3x^{13}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx = ac^2e^3x+\frac{1}{3}ce^2(bce+2ade+3acf)x^3+\frac{1}{5}e\left(bce(2de+3cf)+a\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^5+\frac{1}{7}\left(af\left(3d^2e^2+6cdef+c^2f^2\right)+be\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^7+\frac{1}{9}f\left(adf(3de+2cf)+b\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^9+\frac{1}{11}df^2(3bde+2bcf+adf)x^{11}+\frac{1}{13}bd^2f^3x^{13}$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x]
```

```
[Out] a*c^2*e^3*x + (c*e^2*(b*c*e + 2*a*d*e + 3*a*c*f)*x^3)/3 + (e*(b*c*e*(2*d*e
+ 3*c*f) + a*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^5)/5 + ((a*f*(3*d^2*e^2 +
 6*c*d*e*f + c^2*f^2) + b*e*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + (f*
(a*d*f*(3*d*e + 2*c*f) + b*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^9)/9 + (d*f
^2*(3*b*d*e + 2*b*c*f + a*d*f)*x^11)/11 + (b*d^2*f^3*x^13)/13
```

Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.05

method	result
default	$\frac{b\,d^2 f^3 x^{13}}{13}+\frac{\left(\left(a\,d^2+2bcd\right) f^3+3b\,d^2 e\,f^2\right) x^{11}}{11}+\frac{\left(\left(2acd+b\,c^2\right) f^3+3\left(a\,d^2+2bcd\right) e\,f^2+3b\,d^2 e^2 f\right) x^9}{9}+\frac{\left(c^2 a\,f^3+3\left(2acd+b\,c^2\right)\right.}{}$
norman	$\frac{b\,d^2 f^3 x^{13}}{13}+\left(\frac{1}{11}a\,d^2 f^3+\frac{2}{11}bcd\,f^3+\frac{3}{11}b\,d^2 e\,f^2\right) x^{11}+\left(\frac{2}{9}acd\,f^3+\frac{1}{3}a\,d^2 e\,f^2+\frac{1}{9}b\,c^2 f^3+\frac{2}{3}bcde\,f^2\right.$
gosper	$\frac{1}{13}b\,d^2 f^3 x^{13}+\frac{1}{11}x^{11}a\,d^2 f^3+\frac{2}{11}x^{11}bcd\,f^3+\frac{3}{11}x^{11}b\,d^2 e\,f^2+\frac{2}{9}x^9 acd\,f^3+\frac{1}{3}x^9 a\,d^2 e\,f^2+\frac{1}{9}x^9 b\,c^2$
risch	$\frac{1}{13}b\,d^2 f^3 x^{13}+\frac{1}{11}x^{11}a\,d^2 f^3+\frac{2}{11}x^{11}bcd\,f^3+\frac{3}{11}x^{11}b\,d^2 e\,f^2+\frac{2}{9}x^9 acd\,f^3+\frac{1}{3}x^9 a\,d^2 e\,f^2+\frac{1}{9}x^9 b\,c^2$
parallelrisch	$\frac{1}{13}b\,d^2 f^3 x^{13}+\frac{1}{11}x^{11}a\,d^2 f^3+\frac{2}{11}x^{11}bcd\,f^3+\frac{3}{11}x^{11}b\,d^2 e\,f^2+\frac{2}{9}x^9 acd\,f^3+\frac{1}{3}x^9 a\,d^2 e\,f^2+\frac{1}{9}x^9 b\,c^2$

```
[In] int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b*d^2*f^3*x^13+1/11*((a*d^2+2*b*c*d)*f^3+3*b*d^2*e*f^2)*x^11+1/9*((2*a
*c*d+b*c^2)*f^3+3*(a*d^2+2*b*c*d)*e*f^2+3*b*d^2*e^2*f)*x^9+1/7*(c^2*a*f^3+3
*(2*a*c*d+b*c^2)*e*f^2+3*(a*d^2+2*b*c*d)*e^2*f+b*d^2*e^3)*x^7+1/5*(3*c^2*a*
e*f^2+3*(2*a*c*d+b*c^2)*e^2*f+(a*d^2+2*b*c*d)*e^3)*x^5+1/3*(3*c^2*a*e^2*f+(
2*a*c*d+b*c^2)*e^3)*x^3+a*c^2*e^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.04

$$\begin{aligned}
&\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx \\
&= \frac{1}{13}bd^2f^3x^{13}+\frac{1}{11}\left(3\,bd^2ef^2+\left(2\,bcd+ad^2\right)f^3\right)x^{11} \\
&\quad+\frac{1}{9}\left(3\,bd^2e^2f+3\left(2\,bcd+ad^2\right)ef^2+\left(bc^2+2\,acd\right)f^3\right)x^9 \\
&\quad+\frac{1}{7}\left(bd^2e^3+ac^2f^3+3\left(2\,bcd+ad^2\right)e^2f+3\left(bc^2+2\,acd\right)ef^2\right)x^7 \\
&\quad+ac^2e^3x+\frac{1}{5}\left(3\,ac^2ef^2+\left(2\,bcd+ad^2\right)e^3+3\left(bc^2+2\,acd\right)e^2f\right)x^5 \\
&\quad+\frac{1}{3}\left(3\,ac^2e^2f+\left(bc^2+2\,acd\right)e^3\right)x^3
\end{aligned}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] 1/13*b*d^2*f^3*x^13 + 1/11*(3*b*d^2*e*f^2 + (2*b*c*d + a*d^2)*f^3)*x^11 + 1
/9*(3*b*d^2*e^2*f + 3*(2*b*c*d + a*d^2)*e*f^2 + (b*c^2 + 2*a*c*d)*f^3)*x^9
+ 1/7*(b*d^2*e^3 + a*c^2*f^3 + 3*(2*b*c*d + a*d^2)*e^2*f + 3*(b*c^2 + 2*a*c
*d)*e*f^2)*x^7 + a*c^2*e^3*x + 1/5*(3*a*c^2*e*f^2 + (2*b*c*d + a*d^2)*e^3 +
 3*(b*c^2 + 2*a*c*d)*e^2*f)*x^5 + 1/3*(3*a*c^2*e^2*f + (b*c^2 + 2*a*c*d)*e^
3)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.35

$$\begin{aligned}
\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx &= ac^2e^3x+\frac{bd^2f^3x^{13}}{13} \\
&\quad+x^{11}\left(\frac{ad^2f^3}{11}+\frac{2bcdf^3}{11}+\frac{3bd^2ef^2}{11}\right)+x^9 \\
&\quad\cdot\left(\frac{2acdf^3}{9}+\frac{ad^2ef^2}{3}+\frac{bc^2f^3}{9}+\frac{2bcdef^2}{3}+\frac{bd^2e^2f}{3}\right) \\
&\quad+x^7\left(\frac{ac^2f^3}{7}+\frac{6acdef^2}{7}+\frac{3ad^2e^2f}{7}+\frac{3bc^2ef^2}{7}\right. \\
&\quad\left.+\frac{6bcde^2f}{7}+\frac{bd^2e^3}{7}\right)+x^5 \\
&\quad\cdot\left(\frac{3ac^2ef^2}{5}+\frac{6acde^2f}{5}+\frac{ad^2e^3}{5}+\frac{3bc^2e^2f}{5}+\frac{2bcde^3}{5}\right) \\
&\quad+x^3\left(ac^2e^2f+\frac{2acde^3}{3}+\frac{bc^2e^3}{3}\right)
\end{aligned}$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e)**3,x)
```

```
[Out] a*c**2*e**3*x + b*d**2*f**3*x**13/13 + x**11*(a*d**2*f**3/11 + 2*b*c*d*f**3
/11 + 3*b*d**2*e*f**2/11) + x**9*(2*a*c*d*f**3/9 + a*d**2*e*f**2/3 + b*c**2
*f**3/9 + 2*b*c*d*e*f**2/3 + b*d**2*e**2*f/3) + x**7*(a*c**2*f**3/7 + 6*a*c
*d*e*f**2/7 + 3*a*d**2*e**2*f/7 + 3*b*c**2*e*f**2/7 + 6*b*c*d*e**2*f/7 + b*
d**2*e**3/7) + x**5*(3*a*c**2*e*f**2/5 + 6*a*c*d*e**2*f/5 + a*d**2*e**3/5 +
 3*b*c**2*e**2*f/5 + 2*b*c*d*e**3/5) + x**3*(a*c**2*e**2*f + 2*a*c*d*e**3/3
 + b*c**2*e**3/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.04

$$\begin{aligned}
&\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx \\
&= \frac{1}{13}\,bd^2f^3x^{13} + \frac{1}{11}\left(3\,bd^2ef^2 + \left(2\,bcd + ad^2\right)f^3\right)x^{11} \\
&\quad + \frac{1}{9}\left(3\,bd^2e^2f + 3\left(2\,bcd + ad^2\right)ef^2 + \left(bc^2 + 2\,acd\right)f^3\right)x^9 \\
&\quad + \frac{1}{7}\left(bd^2e^3 + ac^2f^3 + 3\left(2\,bcd + ad^2\right)e^2f + 3\left(bc^2 + 2\,acd\right)ef^2\right)x^7 \\
&\quad + ac^2e^3x + \frac{1}{5}\left(3\,ac^2ef^2 + \left(2\,bcd + ad^2\right)e^3 + 3\left(bc^2 + 2\,acd\right)e^2f\right)x^5 \\
&\quad + \frac{1}{3}\left(3\,ac^2e^2f + \left(bc^2 + 2\,acd\right)e^3\right)x^3
\end{aligned}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^2*f^3*x^13 + 1/11*(3*b*d^2*e*f^2 + (2*b*c*d + a*d^2)*f^3)*x^11 + 1
/9*(3*b*d^2*e^2*f + 3*(2*b*c*d + a*d^2)*e*f^2 + (b*c^2 + 2*a*c*d)*f^3)*x^9
+ 1/7*(b*d^2*e^3 + a*c^2*f^3 + 3*(2*b*c*d + a*d^2)*e^2*f + 3*(b*c^2 + 2*a*c
*d)*e*f^2)*x^7 + a*c^2*e^3*x + 1/5*(3*a*c^2*e*f^2 + (2*b*c*d + a*d^2)*e^3 +
 3*(b*c^2 + 2*a*c*d)*e^2*f)*x^5 + 1/3*(3*a*c^2*e^2*f + (b*c^2 + 2*a*c*d)*e^
3)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.28

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx = \frac{1}{13}\,bd^2f^3x^{13} + \frac{3}{11}\,bd^2ef^2x^{11} + \frac{2}{11}\,bcdf^3x^{11} + \frac{1}{11}\,ad^2f^3x^{11} + \frac{1}{3}\,bd^2e^2fx^9 + \frac{2}{3}\,bcdef^2x^9 + \frac{1}{3}\,ad^2ef^2x^9 + \frac{1}{9}\,bc^2f^3x^9 + \frac{2}{9}\,acdf^3x^9 + \frac{1}{7}\,bd^2e^3x^7 + \frac{6}{7}\,bcde^2fx^7 + \frac{3}{7}\,ad^2e^2fx^7 + \frac{3}{7}\,bc^2ef^2x^7 + \frac{6}{7}\,acdef^2x^7 + \frac{1}{7}\,ac^2f^3x^7 + \frac{2}{5}\,bcde^3x^5 + \frac{1}{5}\,ad^2e^3x^5 + \frac{3}{5}\,bc^2e^2fx^5 + \frac{6}{5}\,acde^2fx^5 + \frac{3}{5}\,ac^2ef^2x^5 + \frac{1}{3}\,bc^2e^3x^3 + \frac{2}{3}\,acde^3x^3 + ac^2e^2fx^3 + ac^2e^3x$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/13*b*d^2*f^3*x^13 + 3/11*b*d^2*e*f^2*x^11 + 2/11*b*c*d*f^3*x^11 + 1/11*a*
d^2*f^3*x^11 + 1/3*b*d^2*e^2*f*x^9 + 2/3*b*c*d*e*f^2*x^9 + 1/3*a*d^2*e*f^2*
x^9 + 1/9*b*c^2*f^3*x^9 + 2/9*a*c*d*f^3*x^9 + 1/7*b*d^2*e^3*x^7 + 6/7*b*c*d
*e^2*f*x^7 + 3/7*a*d^2*e^2*f*x^7 + 3/7*b*c^2*e*f^2*x^7 + 6/7*a*c*d*e*f^2*x^
7 + 1/7*a*c^2*f^3*x^7 + 2/5*b*c*d*e^3*x^5 + 1/5*a*d^2*e^3*x^5 + 3/5*b*c^2*e
^2*f*x^5 + 6/5*a*c*d*e^2*f*x^5 + 3/5*a*c^2*e*f^2*x^5 + 1/3*b*c^2*e^3*x^3 +
2/3*a*c*d*e^3*x^3 + a*c^2*e^2*f*x^3 + a*c^2*e^3*x
```

Mupad [B] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.03

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^3\,dx = x^5\left(\frac{3\,b\,c^2\,e^2\,f}{5} + \frac{3\,a\,c^2\,e\,f^2}{5} + \frac{2\,b\,c\,d\,e^3}{5} + \frac{6\,a\,c\,d\,e^2\,f}{5} + \frac{a\,d^2\,e^3}{5}\right) + x^9\left(\frac{b\,c^2\,f^3}{9} + \frac{2\,b\,c\,d\,e\,f^2}{3} + \frac{2\,a\,c\,d\,f^3}{9} + \frac{b\,d^2\,e^2\,f}{3} + \frac{a\,d^2\,e\,f^2}{3}\right) + x^7\left(\frac{3\,b\,c^2\,e\,f^2}{7} + \frac{a\,c^2\,f^3}{7} + \frac{6\,b\,c\,d\,e^2\,f}{7} + \frac{6\,a\,c\,d\,e\,f^2}{7} + \frac{b\,d^2\,e^3}{7} + \frac{3\,a\,d^2\,e^2\,f}{7}\right) + \frac{b\,d^2\,f^3\,x^{13}}{13} + \frac{c\,e^2\,x^3\,\left(3\,a\,c\,f+2\,a\,d\,e+b\,c\,e\right)}{3} + \frac{d\,f^2\,x^{11}\,\left(a\,d\,f+2\,b\,c\,f+3\,b\,d\,e\right)}{11} + a\,c^2\,e^3\,x$$

```
[In] int((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^3,x)
```

```
[Out] x^5*((a*d^2*e^3)/5 + (2*b*c*d*e^3)/5 + (3*a*c^2*e*f^2)/5 + (3*b*c^2*e^2*f)/
5 + (6*a*c*d*e^2*f)/5) + x^9*((b*c^2*f^3)/9 + (2*a*c*d*f^3)/9 + (a*d^2*e*f^
2)/3 + (b*d^2*e^2*f)/3 + (2*b*c*d*e*f^2)/3) + x^7*((a*c^2*f^3)/7 + (b*d^2*e
^3)/7 + (3*a*d^2*e^2*f)/7 + (3*b*c^2*e*f^2)/7 + (6*a*c*d*e*f^2)/7 + (6*b*c*
d*e^2*f)/7) + (b*d^2*f^3*x^13)/13 + (c*e^2*x^3*(3*a*c*f + 2*a*d*e + b*c*e))
/3 + (d*f^2*x^11*(a*d*f + 2*b*c*f + 3*b*d*e))/11 + a*c^2*e^3*x
```

3.10 $\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 158

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = ac^2e^2x+\frac{1}{3}ce(bce+2a(de+cf))x^3 + \frac{1}{5}\left(2bce(de+cf)+a\left(d^2e^2+4cdef+c^2f^2\right)\right)x^5 + \frac{1}{7}\left(2adf(de+cf)+b\left(d^2e^2+4cdef+c^2f^2\right)\right)x^7 + \frac{1}{9}df(adf+2b(de+cf))x^9+\frac{1}{11}bd^2f^2x^{11}$$

```
[Out] a*c^2*e^2*x+1/3*c*e*(b*c*e+2*a*(c*f+d*e))*x^3+1/5*(2*b*c*e*(c*f+d*e)+a*(c^2
*f^2+4*c*d*e*f+d^2*e^2))*x^5+1/7*(2*a*d*f*(c*f+d*e)+b*(c^2*f^2+4*c*d*e*f+d^
2*e^2))*x^7+1/9*d*f*(a*d*f+2*b*(c*f+d*e))*x^9+1/11*b*d^2*f^2*x^11
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {535}

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = \frac{1}{7}x^7\left(2adf(cf+de)+b\left(c^2f^2+4cdef+d^2e^2\right)\right) + \frac{1}{5}x^5\left(a\left(c^2f^2+4cdef+d^2e^2\right)+2bce(cf+de)\right) + \frac{1}{9}dfx^9(adf+2b(cf+de)) + \frac{1}{3}cex^3(2a(cf+de)+bce)+ac^2e^2x+\frac{1}{11}bd^2f^2x^{11}$$

```
[In] Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

```
[Out] a*c^2*e^2*x + (c*e*(b*c*e + 2*a*(d*e + c*f))*x^3)/3 + ((2*b*c*e*(d*e + c*f)
 + a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + ((2*a*d*f*(d*e + c*f) + b*(d
^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^7)/7 + (d*f*(a*d*f + 2*b*(d*e + c*f))*x^9)
/9 + (b*d^2*f^2*x^11)/11
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ac^2e^2 + ce(bce + 2a(de + cf))x^2 + \left(2bce(de + cf) + a\left(d^2e^2 + 4cdef + c^2f^2\right)\right)x^4 \right. \\ &\quad + \left(2adf(de + cf) + b\left(d^2e^2 + 4cdef + c^2f^2\right)\right)x^6 + df(adf + 2b(de + cf))x^8 \\ &\quad \left. + bd^2f^2x^{10}\right) dx \\ &= ac^2e^2x + \frac{1}{3}ce(bce + 2a(de + cf))x^3 + \frac{1}{5}\left(2bce(de + cf) + a\left(d^2e^2 + 4cdef + c^2f^2\right)\right)x^5 \\ &\quad + \frac{1}{7}\left(2adf(de + cf) + b\left(d^2e^2 + 4cdef + c^2f^2\right)\right)x^7 + \frac{1}{9}df(adf + 2b(de + cf))x^9 + \frac{1}{11}bd^2f^2x^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(a + bx^2\right)\left(c + dx^2\right)^2\left(e + fx^2\right)^2 dx &= ac^2e^2x + \frac{1}{3}ce(bce + 2a(de + cf))x^3 \\ &\quad + \frac{1}{5}\left(2bce(de + cf) + a\left(d^2e^2 + 4cdef + c^2f^2\right)\right)x^5 \\ &\quad + \frac{1}{7}\left(2adf(de + cf) + b\left(d^2e^2 + 4cdef + c^2f^2\right)\right)x^7 \\ &\quad + \frac{1}{9}df(adf + 2b(de + cf))x^9 + \frac{1}{11}bd^2f^2x^{11}\end{aligned}$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x]
```

```
[Out] a*c^2*e^2*x + (c*e*(b*c*e + 2*a*(d*e + c*f))*x^3)/3 + ((2*b*c*e*(d*e + c*f)
 + a*(d^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^5)/5 + ((2*a*d*f*(d*e + c*f) + b*(d
^2*e^2 + 4*c*d*e*f + c^2*f^2))*x^7)/7 + (d*f*(a*d*f + 2*b*(d*e + c*f))*x^9)
/9 + (b*d^2*f^2*x^11)/11
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.07

method	result
default	$\frac{b\,d^2 f^2 x^{11}}{11}+\frac{\left(\left(a\,d^2+2bcd\right) f^2+2b\,d^2 e f\right) x^9}{9}+\frac{\left(\left(2acd+b\,c^2\right) f^2+2\left(a\,d^2+2bcd\right) e f+b\,d^2 e^2\right) x^7}{7}+\frac{\left(c^2 a\,f^2+2\left(2acd+b\,c^2\right) e f+\left(\right.}{5}$
norman	$\frac{b\,d^2 f^2 x^{11}}{11}+\left(\frac{1}{9}a\,d^2 f^2+\frac{2}{9}bcd\,f^2+\frac{2}{9}b\,d^2 e f\right) x^9+\left(\frac{2}{7}acd\,f^2+\frac{2}{7}a\,d^2 e f+\frac{1}{7}b\,c^2 f^2+\frac{4}{7}bcdef+\frac{1}{7}b\right.$
gosper	$\frac{1}{11}b\,d^2 f^2 x^{11}+\frac{1}{9}x^9 a\,d^2 f^2+\frac{2}{9}x^9 bcd\,f^2+\frac{2}{9}x^9 b\,d^2 e f+\frac{2}{7}x^7 acd\,f^2+\frac{2}{7}x^7 a\,d^2 e f+\frac{1}{7}x^7 b\,c^2 f^2+\frac{4}{7}$
risch	$\frac{1}{11}b\,d^2 f^2 x^{11}+\frac{1}{9}x^9 a\,d^2 f^2+\frac{2}{9}x^9 bcd\,f^2+\frac{2}{9}x^9 b\,d^2 e f+\frac{2}{7}x^7 acd\,f^2+\frac{2}{7}x^7 a\,d^2 e f+\frac{1}{7}x^7 b\,c^2 f^2+\frac{4}{7}$
parallelrisch	$\frac{1}{11}b\,d^2 f^2 x^{11}+\frac{1}{9}x^9 a\,d^2 f^2+\frac{2}{9}x^9 bcd\,f^2+\frac{2}{9}x^9 b\,d^2 e f+\frac{2}{7}x^7 acd\,f^2+\frac{2}{7}x^7 a\,d^2 e f+\frac{1}{7}x^7 b\,c^2 f^2+\frac{4}{7}$

```
[In] int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b*d^2*f^2*x^11+1/9*((a*d^2+2*b*c*d)*f^2+2*b*d^2*e*f)*x^9+1/7*((2*a*c*d
+b*c^2)*f^2+2*(a*d^2+2*b*c*d)*e*f+b*d^2*e^2)*x^7+1/5*(c^2*a*f^2+2*(2*a*c*d+
b*c^2)*e*f+(a*d^2+2*b*c*d)*e^2)*x^5+1/3*(2*c^2*a*e*f+(2*a*c*d+b*c^2)*e^2)*x
^3+a*c^2*e^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06

$$\begin{aligned}\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)^2\,dx &= \frac{1}{11}bd^2f^2x^{11}+\frac{1}{9}\left(2\,bd^2ef+\left(2\,bcd+ad^2\right)f^2\right)x^9 \\ &\quad+\frac{1}{7}\left(bd^2e^2+2\left(2\,bcd+ad^2\right)ef+\left(bc^2+2\,acd\right)f^2\right)x^7+ac^2e^2x \\ &\quad+\frac{1}{5}\left(ac^2f^2+\left(2\,bcd+ad^2\right)e^2+2\left(bc^2+2\,acd\right)ef\right)x^5+\frac{1}{3}\left(2\,ac^2ef+\left(bc^2+2\,acd\right)e^2\right)x^3\end{aligned}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] 1/11*b*d^2*f^2*x^11 + 1/9*(2*b*d^2*e*f + (2*b*c*d + a*d^2)*f^2)*x^9 + 1/7*(
b*d^2*e^2 + 2*(2*b*c*d + a*d^2)*e*f + (b*c^2 + 2*a*c*d)*f^2)*x^7 + a*c^2*e^
2*x + 1/5*(a*c^2*f^2 + (2*b*c*d + a*d^2)*e^2 + 2*(b*c^2 + 2*a*c*d)*e*f)*x^5
 + 1/3*(2*a*c^2*e*f + (b*c^2 + 2*a*c*d)*e^2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.37

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = ac^2e^2x + \frac{bd^2f^2x^{11}}{11} + x^9\left(\frac{ad^2f^2}{9} + \frac{2bcdf^2}{9} + \frac{2bd^2ef}{9}\right) + x^7\cdot\left(\frac{2acdf^2}{7} + \frac{2ad^2ef}{7} + \frac{bc^2f^2}{7} + \frac{4bcdef}{7} + \frac{bd^2e^2}{7}\right) + x^5\left(\frac{ac^2f^2}{5} + \frac{4acdef}{5} + \frac{ad^2e^2}{5} + \frac{2bc^2ef}{5} + \frac{2bcde^2}{5}\right) + x^3\cdot\left(\frac{2ac^2ef}{3} + \frac{2acde^2}{3} + \frac{bc^2e^2}{3}\right)$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e)**2,x)
```

```
[Out] a*c**2*e**2*x + b*d**2*f**2*x**11/11 + x**9*(a*d**2*f**2/9 + 2*b*c*d*f**2/9
 + 2*b*d**2*e*f/9) + x**7*(2*a*c*d*f**2/7 + 2*a*d**2*e*f/7 + b*c**2*f**2/7
+ 4*b*c*d*e*f/7 + b*d**2*e**2/7) + x**5*(a*c**2*f**2/5 + 4*a*c*d*e*f/5 + a*
d**2*e**2/5 + 2*b*c**2*e*f/5 + 2*b*c*d*e**2/5) + x**3*(2*a*c**2*e*f/3 + 2*a
*c*d*e**2/3 + b*c**2*e**2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.06

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = \frac{1}{11}bd^2f^2x^{11} + \frac{1}{9}\left(2bd^2ef + (2bcd+ad^2)f^2\right)x^9 + \frac{1}{7}\left(bd^2e^2 + 2(2bcd+ad^2)ef + (bc^2+2acd)f^2\right)x^7 + ac^2e^2x + \frac{1}{5}\left(ac^2f^2 + (2bcd+ad^2)e^2 + 2(bc^2+2acd)ef\right)x^5 + \frac{1}{3}\left(2ac^2ef + (bc^2+2acd)e^2\right)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] 1/11*b*d^2*f^2*x^11 + 1/9*(2*b*d^2*e*f + (2*b*c*d + a*d^2)*f^2)*x^9 + 1/7*(
b*d^2*e^2 + 2*(2*b*c*d + a*d^2)*e*f + (b*c^2 + 2*a*c*d)*f^2)*x^7 + a*c^2*e^
2*x + 1/5*(a*c^2*f^2 + (2*b*c*d + a*d^2)*e^2 + 2*(b*c^2 + 2*a*c*d)*e*f)*x^5
 + 1/3*(2*a*c^2*e*f + (b*c^2 + 2*a*c*d)*e^2)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.28

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = \frac{1}{11}bd^2f^2x^{11} + \frac{2}{9}bd^2efx^9 + \frac{2}{9}bcdf^2x^9 + \frac{1}{9}ad^2f^2x^9 + \frac{1}{7}bd^2e^2x^7 + \frac{4}{7}bcdefx^7 + \frac{2}{7}ad^2efx^7 + \frac{1}{7}bc^2f^2x^7 + \frac{2}{7}acdf^2x^7 + \frac{2}{5}bcde^2x^5 + \frac{1}{5}ad^2e^2x^5 + \frac{2}{5}bc^2efx^5 + \frac{4}{5}acdefx^5 + \frac{1}{5}ac^2f^2x^5 + \frac{1}{3}bc^2e^2x^3 + \frac{2}{3}acde^2x^3 + \frac{2}{3}ac^2efx^3 + ac^2e^2x$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] 1/11*b*d^2*f^2*x^11 + 2/9*b*d^2*e*f*x^9 + 2/9*b*c*d*f^2*x^9 + 1/9*a*d^2*f^2
*x^9 + 1/7*b*d^2*e^2*x^7 + 4/7*b*c*d*e*f*x^7 + 2/7*a*d^2*e*f*x^7 + 1/7*b*c^
2*f^2*x^7 + 2/7*a*c*d*f^2*x^7 + 2/5*b*c*d*e^2*x^5 + 1/5*a*d^2*e^2*x^5 + 2/5
*b*c^2*e*f*x^5 + 4/5*a*c*d*e*f*x^5 + 1/5*a*c^2*f^2*x^5 + 1/3*b*c^2*e^2*x^3
+ 2/3*a*c*d*e^2*x^3 + 2/3*a*c^2*e*f*x^3 + a*c^2*e^2*x
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)^2\,dx = x^5\left(\frac{2\,b\,c^2\,e\,f}{5} + \frac{a\,c^2\,f^2}{5} + \frac{2\,b\,c\,d\,e^2}{5} + \frac{4\,a\,c\,d\,e\,f}{5} + \frac{a\,d^2\,e^2}{5}\right) + x^7\left(\frac{b\,c^2\,f^2}{7} + \frac{4\,b\,c\,d\,e\,f}{7} + \frac{2\,a\,c\,d\,f^2}{7} + \frac{b\,d^2\,e^2}{7} + \frac{2\,a\,d^2\,e\,f}{7}\right) + \frac{b\,d^2\,f^2\,x^{11}}{11} + a\,c^2\,e^2\,x + \frac{c\,e\,x^3\,(2\,a\,c\,f + 2\,a\,d\,e + b\,c\,e)}{3} + \frac{d\,f\,x^9\,(a\,d\,f + 2\,b\,c\,f + 2\,b\,d\,e)}{9}$$

```
[In] int((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2)^2,x)
```

```
[Out] x^5*((a*c^2*f^2)/5 + (a*d^2*e^2)/5 + (2*b*c*d*e^2)/5 + (2*b*c^2*e*f)/5 + (4
*a*c*d*e*f)/5) + x^7*((b*c^2*f^2)/7 + (b*d^2*e^2)/7 + (2*a*c*d*f^2)/7 + (2*
a*d^2*e*f)/7 + (4*b*c*d*e*f)/7) + (b*d^2*f^2*x^11)/11 + a*c^2*e^2*x + (c*e*
x^3*(2*a*c*f + 2*a*d*e + b*c*e))/3 + (d*f*x^9*(a*d*f + 2*b*c*f + 2*b*d*e))/
9
```

3.11 $\int (a+bx^2)(c+dx^2)^2(e+fx^2)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)\,dx = ac^2ex + \frac{1}{3}c(bce+2ade+acf)x^3 + \frac{1}{5}(bc(2de+cf)+ad(de+2cf))x^5 + \frac{1}{7}d(bde+2bcf+adf)x^7 + \frac{1}{9}bd^2fx^9$$

```
[Out] a*c^2*e*x+1/3*c*(a*c*f+2*a*d*e+b*c*e)*x^3+1/5*(b*c*(c*f+2*d*e)+a*d*(2*c*f+d
*e))*x^5+1/7*d*(a*d*f+2*b*c*f+b*d*e)*x^7+1/9*b*d^2*f*x^9
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {535}

$$\int (a+bx^2)(c+dx^2)^2(e+fx^2)\,dx = \frac{1}{7}dx^7(adf+2bcf+bde) + \frac{1}{5}x^5(ad(2cf+de)+bc(cf+2de)) + \frac{1}{3}cx^3(acf+2ade+bce) + ac^2ex + \frac{1}{9}bd^2fx^9$$

```
[In] Int[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2),x]
```

```
[Out] a*c^2*e*x + (c*(b*c*e + 2*a*d*e + a*c*f)*x^3)/3 + ((b*c*(2*d*e + c*f) + a*d
*(d*e + 2*c*f))*x^5)/5 + (d*(b*d*e + 2*b*c*f + a*d*f)*x^7)/7 + (b*d^2*f*x^9
)/9
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ac^2e + c(bce + 2ade + acf)x^2 + (bc(2de + cf) + ad(de + 2cf))x^4 \right.\\ &\qquad \left. + d(bde + 2bcf + adf)x^6 + bd^2fx^8\right)\,dx\\ &= ac^2ex + \frac{1}{3}c(bce + 2ade + acf)x^3 + \frac{1}{5}(bc(2de + cf) + ad(de + 2cf))x^5\\ &\quad + \frac{1}{7}d(bde + 2bcf + adf)x^7 + \frac{1}{9}bd^2fx^9\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.02

$$\begin{aligned}\int (a + bx^2)(c + dx^2)^2(e + fx^2)\,dx = ac^2ex + \frac{1}{3}c(bce + 2ade + acf)x^3\\ + \frac{1}{5}\left(2bcde + ad^2e + bc^2f + 2acdf\right)x^5\\ + \frac{1}{7}d(bde + 2bcf + adf)x^7 + \frac{1}{9}bd^2fx^9\end{aligned}$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^2*(e + f*x^2),x]
```

```
[Out] a*c^2*e*x + (c*(b*c*e + 2*a*d*e + a*c*f)*x^3)/3 + ((2*b*c*d*e + a*d^2*e + b
*c^2*f + 2*a*c*d*f)*x^5)/5 + (d*(b*d*e + 2*b*c*f + a*d*f)*x^7)/7 + (b*d^2*f
*x^9)/9
```

Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

method	result
norman	$\frac{b\,d^2 f\,x^9}{9}+\left(\frac{1}{7}a\,d^2 f+\frac{2}{7}bcfd+\frac{1}{7}b\,d^2 e\right)x^7+\left(\frac{2}{5}acdf+\frac{1}{5}ae\,d^2+\frac{1}{5}c^2 bf+\frac{2}{5}bcde\right)x^5+\left(\frac{1}{3}c^2 af+\frac{2}{3}\right.$
default	$\frac{b\,d^2 f\,x^9}{9}+\frac{\left(\left(a\,d^2+2bcd\right)f+b\,d^2 e\right)x^7}{7}+\frac{\left(\left(2acd+b\,c^2\right)f+\left(a\,d^2+2bcd\right)e\right)x^5}{5}+\frac{\left(c^2 af+\left(2acd+b\,c^2\right)e\right)x^3}{3}+a\,c^2 ex$
gosper	$\frac{1}{9}b\,d^2 f\,x^9+\frac{1}{7}x^7 a\,d^2 f+\frac{2}{7}x^7 bcfd+\frac{1}{7}x^7 b\,d^2 e+\frac{2}{5}x^5 acdf+\frac{1}{5}x^5 ae\,d^2+\frac{1}{5}x^5 c^2 bf+\frac{2}{5}x^5 bcde+\frac{1}{3}x$
risch	$\frac{1}{9}b\,d^2 f\,x^9+\frac{1}{7}x^7 a\,d^2 f+\frac{2}{7}x^7 bcfd+\frac{1}{7}x^7 b\,d^2 e+\frac{2}{5}x^5 acdf+\frac{1}{5}x^5 ae\,d^2+\frac{1}{5}x^5 c^2 bf+\frac{2}{5}x^5 bcde+\frac{1}{3}x$
parallelrisch	$\frac{1}{9}b\,d^2 f\,x^9+\frac{1}{7}x^7 a\,d^2 f+\frac{2}{7}x^7 bcfd+\frac{1}{7}x^7 b\,d^2 e+\frac{2}{5}x^5 acdf+\frac{1}{5}x^5 ae\,d^2+\frac{1}{5}x^5 c^2 bf+\frac{2}{5}x^5 bcde+\frac{1}{3}x$

```
[In] int((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*b*d^2*f*x^9+(1/7*a*d^2*f+2/7*b*c*f*d+1/7*b*d^2*e)*x^7+(2/5*a*c*d*f+1/5*
a*e*d^2+1/5*c^2*b*f+2/5*b*c*d*e)*x^5+(1/3*c^2*a*f+2/3*a*c*d*e+1/3*b*c^2*e)*
x^3+a*c^2*e*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx=\frac{1}{9}\,bd^2fx^9+\frac{1}{7}\,\left(bd^2e+\left(2\,bcd+ad^2\right)f\right)x^7+\frac{1}{5}\,\left(\left(2\,bcd+ad^2\right)e+\left(bc^2+2\,acd\right)f\right)x^5+ac^2ex+\frac{1}{3}\,\left(ac^2f+\left(bc^2+2\,acd\right)e\right)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="fricas")
```

```
[Out] 1/9*b*d^2*f*x^9 + 1/7*(b*d^2*e + (2*b*c*d + a*d^2)*f)*x^7 + 1/5*((2*b*c*d +
 a*d^2)*e + (b*c^2 + 2*a*c*d)*f)*x^5 + a*c^2*e*x + 1/3*(a*c^2*f + (b*c^2 +
2*a*c*d)*e)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.29

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx=ac^2ex+\frac{bd^2fx^9}{9}+x^7\left(\frac{ad^2f}{7}+\frac{2bcdf}{7}+\frac{bd^2e}{7}\right)+x^5\cdot\left(\frac{2acdf}{5}+\frac{ad^2e}{5}+\frac{bc^2f}{5}+\frac{2bcde}{5}\right)+x^3\left(\frac{ac^2f}{3}+\frac{2acde}{3}+\frac{bc^2e}{3}\right)$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2*(f*x**2+e),x)
```

```
[Out] a*c**2*e*x + b*d**2*f*x**9/9 + x**7*(a*d**2*f/7 + 2*b*c*d*f/7 + b*d**2*e/7)
 + x**5*(2*a*c*d*f/5 + a*d**2*e/5 + b*c**2*f/5 + 2*b*c*d*e/5) + x**3*(a*c**
2*f/3 + 2*a*c*d*e/3 + b*c**2*e/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx = \frac{1}{9}\,bd^2fx^9+\frac{1}{7}\left(bd^2e+\left(2\,bcd+ad^2\right)f\right)x^7 + \frac{1}{5}\left(\left(2\,bcd+ad^2\right)e+\left(bc^2+2\,acd\right)f\right)x^5 + ac^2ex+\frac{1}{3}\left(ac^2f+\left(bc^2+2\,acd\right)e\right)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="maxima")
```

```
[Out] 1/9*b*d^2*f*x^9 + 1/7*(b*d^2*e + (2*b*c*d + a*d^2)*f)*x^7 + 1/5*((2*b*c*d +
 a*d^2)*e + (b*c^2 + 2*a*c*d)*f)*x^5 + a*c^2*e*x + 1/3*(a*c^2*f + (b*c^2 +
2*a*c*d)*e)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.21

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx = \frac{1}{9}\,bd^2fx^9+\frac{1}{7}\,bd^2ex^7+\frac{2}{7}\,bcdfx^7+\frac{1}{7}\,ad^2fx^7 + \frac{2}{5}\,bcdex^5+\frac{1}{5}\,ad^2ex^5+\frac{1}{5}\,bc^2fx^5+\frac{2}{5}\,acdfx^5 + \frac{1}{3}\,bc^2ex^3+\frac{2}{3}\,acdex^3+\frac{1}{3}\,ac^2fx^3+ac^2ex$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2*(f*x^2+e),x, algorithm="giac")
```

```
[Out] 1/9*b*d^2*f*x^9 + 1/7*b*d^2*e*x^7 + 2/7*b*c*d*f*x^7 + 1/7*a*d^2*f*x^7 + 2/5
*b*c*d*e*x^5 + 1/5*a*d^2*e*x^5 + 1/5*b*c^2*f*x^5 + 2/5*a*c*d*f*x^5 + 1/3*b*
c^2*e*x^3 + 2/3*a*c*d*e*x^3 + 1/3*a*c^2*f*x^3 + a*c^2*e*x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int (a+bx^2)\left(c+dx^2\right)^2\left(e+fx^2\right)\,dx = x^5\left(\frac{a\,d^2\,e}{5}+\frac{b\,c^2\,f}{5}+\frac{2\,a\,c\,d\,f}{5}+\frac{2\,b\,c\,d\,e}{5}\right)+x^3\left(\frac{a\,c^2\,f}{3}+\frac{b\,c^2\,e}{3}+\frac{2\,a\,c\,d\,e}{3}\right)+x^7\left(\frac{a\,d^2\,f}{7}+\frac{b\,d^2\,e}{7}+\frac{2\,b\,c\,d\,f}{7}\right)+a\,c^2\,e\,x+\frac{b\,d^2\,f\,x^9}{9}$$

```
[In] int((a + b*x^2)*(c + d*x^2)^2*(e + f*x^2),x)
```

```
[Out] x^5*((a*d^2*e)/5 + (b*c^2*f)/5 + (2*a*c*d*f)/5 + (2*b*c*d*e)/5) + x^3*((a*c
^2*f)/3 + (b*c^2*e)/3 + (2*a*c*d*e)/3) + x^7*((a*d^2*f)/7 + (b*d^2*e)/7 + (
2*b*c*d*f)/7) + a*c^2*e*x + (b*d^2*f*x^9)/9
```

3.12 $\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 142

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2} \, dx = -\frac{(5adf(3de-5cf)-b(15d^2e^2-25cdef+8c^2f^2))\,x}{15f^3} - \frac{(5bde-4bcf-5adf)x(c+dx^2)}{15f^2} + \frac{bx(c+dx^2)^2}{5f} - \frac{(be-af)(de-cf)^2 \arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{7/2}}$$

```
[Out] -1/15*(5*a*d*f*(-5*c*f+3*d*e)-b*(8*c^2*f^2-25*c*d*e*f+15*d^2*e^2))*x/f^3-1/
15*(-5*a*d*f-4*b*c*f+5*b*d*e)*x*(d*x^2+c)/f^2+1/5*b*x*(d*x^2+c)^2/f-(-a*f+b
*e)*(-c*f+d*e)^2*arctan(x*f^(1/2)/e^(1/2))/f^(7/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {542, 396, 211}

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2} \, dx = -\frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2}{\sqrt{e}f^{7/2}} - \frac{x(5adf(3de-5cf)-b(8c^2f^2-25cdef+15d^2e^2))}{15f^3} - \frac{x(c+dx^2)(-5adf-4bcf+5bde)}{15f^2} + \frac{bx(c+dx^2)^2}{5f}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x]
```

```
[Out] -1/15*((5*a*d*f*(3*d*e - 5*c*f) - b*(15*d^2*e^2 - 25*c*d*e*f + 8*c^2*f^2))*
x)/f^3 - ((5*b*d*e - 4*b*c*f - 5*a*d*f)*x*(c + d*x^2))/(15*f^2) + (b*x*(c +
 d*x^2)^2)/(5*f) - ((b*e - a*f)*(d*e - c*f)^2*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/
(Sqrt[e]*f^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx(c+dx^2)^2}{5f} + \frac{\int \frac{(c+dx^2)\left(-c(be-5af)+(-5bde+4bcf+5adf)x^2\right)}{e+fx^2}\,dx}{5f} \\
&= -\frac{(5bde-4bcf-5adf)x(c+dx^2)}{15f^2} + \frac{bx(c+dx^2)^2}{5f} \\
&\quad + \frac{\int \frac{c(be(5de-7cf)-5af(de-3cf))-\left(5adf(3de-5cf)-b\left(15d^2e^2-25cdef+8c^2f^2\right)\right)x^2}{e+fx^2}\,dx}{15f^2} \\
&= -\frac{\left(5adf(3de-5cf)-b\left(15d^2e^2-25cdef+8c^2f^2\right)\right)x}{15f^3} \\
&\quad - \frac{(5bde-4bcf-5adf)x(c+dx^2)}{15f^2} + \frac{bx(c+dx^2)^2}{5f} \\
&\quad - \frac{\left((be-af)(de-cf)^2\right)\int \frac{1}{e+fx^2}\,dx}{f^3}
\end{aligned}
$$

$$= -\frac{(5adf(3de-5cf)-b(15d^2e^2-25cdef+8c^2f^2))\,x}{15f^3} - \frac{(5bde-4bcf-5adf)x(c+dx^2)}{15f^2} + \frac{bx(c+dx^2)^2}{5f} - \frac{(be-af)(de-cf)^2\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{7/2}}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = \frac{(b(de-cf)^2+adf(-de+2cf))\,x}{f^3} + \frac{d(-bde+2bcf+adf)x^3}{3f^2} + \frac{bd^2x^5}{5f} - \frac{(be-af)(de-cf)^2\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{7/2}}$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x]
```

```
[Out] ((b*(d*e - c*f)^2 + a*d*f*(-(d*e) + 2*c*f))*x)/f^3 + (d*(-(b*d*e) + 2*b*c*f
 + a*d*f)*x^3)/(3*f^2) + (b*d^2*x^5)/(5*f) - ((b*e - a*f)*(d*e - c*f)^2*Arc
Tan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(7/2))
```

Maple [A] (verified)

Time = 3.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.20

method	result
default	$\frac{\frac{1}{5}b\,d^2x^5f^2+\frac{1}{3}a\,d^2f^2x^3+\frac{2}{3}bcd\,f^2x^3-\frac{1}{3}b\,d^2ef\,x^3+2acd\,f^2x-a\,d^2efx+b\,c^2f^2x-2bcdefx+b\,d^2e^2x}{f^3} + \frac{(c^2a\,f^3-2acde\,f^2+a\,d^2e^2f-b}{}$
risch	$\frac{b\,d^2x^5}{5f}+\frac{a\,d^2x^3}{3f}+\frac{2bcd\,x^3}{3f}-\frac{b\,d^2e\,x^3}{3f^2}+\frac{2acdx}{f}-\frac{a\,d^2ex}{f^2}+\frac{b\,c^2x}{f}-\frac{2bcdex}{f^2}+\frac{b\,d^2e^2x}{f^3}-\frac{\ln(fx+\sqrt{-ef})c^2a}{2\sqrt{-ef}}+\frac{\ln(fx+}{f}$

```
[In] int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f^3*(1/5*b*d^2*x^5*f^2+1/3*a*d^2*f^2*x^3+2/3*b*c*d*f^2*x^3-1/3*b*d^2*e*f*
x^3+2*a*c*d*f^2*x-a*d^2*e*f*x+b*c^2*f^2*x-2*b*c*d*e*f*x+b*d^2*e^2*x)+(a*c^2
*f^3-2*a*c*d*e*f^2+a*d^2*e^2*f-b*c^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)/f^3/(e*
f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 366, normalized size of antiderivative = 2.58

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx$$
$$= \left[\frac{6\,bd^2ef^3x^5 - 10\,(bd^2e^2f^2 - (2\,bcd + ad^2)ef^3)x^3 + 15\,(bd^2e^3 - ac^2f^3 - (2\,bcd + ad^2)e^2f + (bc^2 + 2\,acd)ef\right.}{30\,ef^4}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="fricas")
```

```
[Out] [1/30*(6*b*d^2*e*f^3*x^5 - 10*(b*d^2*e^2*f^2 - (2*b*c*d + a*d^2)*e*f^3)*x^3
 + 15*(b*d^2*e^3 - a*c^2*f^3 - (2*b*c*d + a*d^2)*e^2*f + (b*c^2 + 2*a*c*d)*
e*f^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 30*(b*d^2
*e^3*f - (2*b*c*d + a*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x)/(e*f^4), 1
/15*(3*b*d^2*e*f^3*x^5 - 5*(b*d^2*e^2*f^2 - (2*b*c*d + a*d^2)*e*f^3)*x^3 -
15*(b*d^2*e^3 - a*c^2*f^3 - (2*b*c*d + a*d^2)*e^2*f + (b*c^2 + 2*a*c*d)*e*f
^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + 15*(b*d^2*e^3*f - (2*b*c*d + a*d^2)*e
^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x)/(e*f^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(138) = 276.

Time = 0.52 (sec) , antiderivative size = 347, normalized size of antiderivative = 2.44

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx$$
$$= \frac{bd^2x^5}{5f} + x^3\left(\frac{ad^2}{3f} + \frac{2bcd}{3f} - \frac{bd^2e}{3f^2}\right) + x\left(\frac{2acd}{f} - \frac{ad^2e}{f^2} + \frac{bc^2}{f} - \frac{2bcde}{f^2} + \frac{bd^2e^2}{f^3}\right)$$
$$- \frac{\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2\log\left(-\frac{ef^3\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2}{ac^2f^3-2acdef^2+ad^2e^2f-bc^2ef^2+2bcde^2f-bd^2e^3}+x\right)}{2}$$
$$+ \frac{\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2\log\left(\frac{ef^3\sqrt{-\frac{1}{ef^7}}(af-be)(cf-de)^2}{ac^2f^3-2acdef^2+ad^2e^2f-bc^2ef^2+2bcde^2f-bd^2e^3}+x\right)}{2}$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e),x)
```

```
[Out] b*d**2*x**5/(5*f) + x**3*(a*d**2/(3*f) + 2*b*c*d/(3*f) - b*d**2*e/(3*f**2))
 + x*(2*a*c*d/f - a*d**2*e/f**2 + b*c**2/f - 2*b*c*d*e/f**2 + b*d**2*e**2/f
**3) - sqrt(-1/(e*f**7))*(a*f - b*e)*(c*f - d*e)**2*log(-e*f**3*sqrt(-1/(e*
```

```
f**7))*(a*f - b*e)*(c*f - d*e)**2/(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**2*e*
*2*f - b*c**2*e*f**2 + 2*b*c*d*e**2*f - b*d**2*e**3) + x)/2 + sqrt(-1/(e*f*
*7))*(a*f - b*e)*(c*f - d*e)**2*log(e*f**3*sqrt(-1/(e*f**7))*(a*f - b*e)*(c
*f - d*e)**2/(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**2*e**2*f - b*c**2*e*f**2
+ 2*b*c*d*e**2*f - b*d**2*e**3) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx = -\frac{(bd^2e^3 - 2\,bcde^2f - ad^2e^2f + bc^2ef^2 + 2\,acdef^2 - ac^2f^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{\sqrt{ef}\,f^3} + \frac{3\,bd^2f^4x^5 - 5\,bd^2ef^3x^3 + 10\,bcdf^4x^3 + 5\,ad^2f^4x^3 + 15\,bd^2e^2f^2x - 30\,bcdef^3x - 15\,ad^2ef^3x + 15\,bc^2f^4x}{15\,f^5}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e),x, algorithm="giac")
```

```
[Out] -(b*d^2*e^3 - 2*b*c*d*e^2*f - a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 - a
*c^2*f^3)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*f^3) + 1/15*(3*b*d^2*f^4*x^5 - 5
*b*d^2*e*f^3*x^3 + 10*b*c*d*f^4*x^3 + 5*a*d^2*f^4*x^3 + 15*b*d^2*e^2*f^2*x
- 30*b*c*d*e*f^3*x - 15*a*d^2*e*f^3*x + 15*b*c^2*f^4*x + 30*a*c*d*f^4*x)/f^
5
```

Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.43

$$\int \frac{(a+bx^2)(c+dx^2)^2}{e+fx^2}\,dx$$
$$= x^3\left(\frac{a\,d^2+2\,b\,c\,d}{3\,f}-\frac{b\,d^2\,e}{3\,f^2}\right)+x\left(\frac{b\,c^2+2\,a\,d\,c}{f}-\frac{e\left(\frac{a\,d^2+2\,b\,c\,d}{f}-\frac{b\,d^2\,e}{f^2}\right)}{f}\right)+\frac{b\,d^2\,x^5}{5\,f}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(a\,f-b\,e)\,(c\,f-d\,e)^2}{\sqrt{e}\,(-b\,c^2\,e\,f^2+a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f-2\,a\,c\,d\,e\,f^2-b\,d^2\,e^3+a\,d^2\,e^2\,f)}\right)(a\,f-b\,e)\,(c\,f-d\,e)^2}{\sqrt{e}\,f^{7/2}}$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2),x)
```

```
[Out] x^3*((a*d^2 + 2*b*c*d)/(3*f) - (b*d^2*e)/(3*f^2)) + x*((b*c^2 + 2*a*c*d)/f
- (e*((a*d^2 + 2*b*c*d)/f - (b*d^2*e)/f^2))/f) + (b*d^2*x^5)/(5*f) + (atan(
(f^(1/2)*x*(a*f - b*e)*(c*f - d*e)^2)/(e^(1/2)*(a*c^2*f^3 - b*d^2*e^3 + a*d
^2*e^2*f - b*c^2*e*f^2 - 2*a*c*d*e*f^2 + 2*b*c*d*e^2*f)))*(a*f - b*e)*(c*f
- d*e)^2)/(e^(1/2)*f^(7/2))
```

3.13 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 164

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = -\frac{d(be(15de-13cf)-3af(3de-cf))x}{6ef^3} + \frac{d(5be-3af)x(c+dx^2)}{6ef^2} - \frac{(be-af)x(c+dx^2)^2}{2ef(e+fx^2)} + \frac{(de-cf)(be(5de-cf)-af(3de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{7/2}}$$

```
[Out] -1/6*d*(b*e*(-13*c*f+15*d*e)-3*a*f*(-c*f+3*d*e))*x/e/f^3+1/6*d*(-3*a*f+5*b*
e)*x*(d*x^2+c)/e/f^2-1/2*(-a*f+b*e)*x*(d*x^2+c)^2/e/f/(f*x^2+e)+1/2*(-c*f+d
*e)*(b*e*(-c*f+5*d*e)-a*f*(c*f+3*d*e))*arctan(x*f^(1/2)/e^(1/2))/e^(3/2)/f^
(7/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {540, 542, 396, 211}

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = \frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)(be(5de-cf)-af(cf+3de))}{2e^{3/2}f^{7/2}} - \frac{dx(be(15de-13cf)-3af(3de-cf))}{6ef^3} + \frac{dx(c+dx^2)(5be-3af)}{6ef^2} - \frac{x(c+dx^2)^2(be-af)}{2ef(e+fx^2)}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

```
[Out] -1/6*(d*(b*e*(15*d*e - 13*c*f) - 3*a*f*(3*d*e - c*f))*x)/(e*f^3) + (d*(5*b*
e - 3*a*f)*x*(c + d*x^2))/(6*e*f^2) - ((b*e - a*f)*x*(c + d*x^2)^2)/(2*e*f*
(e + f*x^2)) + ((d*e - c*f)*(b*e*(5*d*e - c*f) - a*f*(3*d*e + c*f))*ArcTan[
(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(be-af)x\left(c+dx^2\right)^2}{2ef\left(e+fx^2\right)}-\frac{\int \frac{\left(c+dx^2\right)\left(-c(be+af)-d(5be-3af)x^2\right)}{e+fx^2}\,dx}{2ef}\\
&= \frac{d(5be-3af)x\left(c+dx^2\right)}{6ef^2}-\frac{(be-af)x\left(c+dx^2\right)^2}{2ef\left(e+fx^2\right)}\\
&\quad -\frac{\int \frac{c(be(5de-3cf)-3af(de+cf))+d(be(15de-13cf)-3af(3de-cf))x^2}{e+fx^2}\,dx}{6ef^2}
\end{aligned}$$

$$= -\frac{d(be(15de-13cf)-3af(3de-cf))x}{6ef^3} + \frac{d(5be-3af)x(c+dx^2)}{6ef^2} - \frac{(be-af)x(c+dx^2)^2}{2ef\,(e+fx^2)} + \frac{((de-cf)(be(5de-cf)-af(3de+cf)))\int \frac{1}{e+fx^2}\,dx}{2ef^3}$$

$$= -\frac{d(be(15de-13cf)-3af(3de-cf))x}{6ef^3} + \frac{d(5be-3af)x(c+dx^2)}{6ef^2} - \frac{(be-af)x(c+dx^2)^2}{2ef\,(e+fx^2)} + \frac{(de-cf)(be(5de-cf)-af(3de+cf))\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{7/2}}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)\,(c+dx^2)^2}{(e+fx^2)^2}\,dx = \frac{d(-2bde+2bcf+adf)x}{f^3} + \frac{bd^2x^3}{3f^2} - \frac{(be-af)(de-cf)^2x}{2ef^3\,(e+fx^2)} + \frac{(de-cf)(be(5de-cf)-af(3de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{7/2}}$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x]
```

```
[Out] (d*(-2*b*d*e + 2*b*c*f + a*d*f)*x)/f^3 + (b*d^2*x^3)/(3*f^2) - ((b*e - a*f)
*(d*e - c*f)^2*x)/(2*e*f^3*(e + f*x^2)) + ((d*e - c*f)*(b*e*(5*d*e - c*f) -
 a*f*(3*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(7/2))
```

Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.11

method	result
default	$\frac{d\left(\frac{1}{3}bdf\,x^3+adfx+2bcfx-2bdex\right)}{f^3}+\frac{\frac{\left(c^2a\,f^3-2acde\,f^2+a\,d^2e^2f-b\,c^2e\,f^2+2bcd\,e^2f-b\,d^2e^3\right)x}{2e\left(f\,x^2+e\right)}+\frac{\left(c^2a\,f^3+2acde\,f^2-3a\,d^2e^2f+b\,c^2e\,f^2-6bc\right.}{2e\sqrt{ef}}}{f^3}$
risch	$\frac{d^2b\,x^3}{3f^2}+\frac{d^2ax}{f^2}+\frac{2dbcx}{f^2}-\frac{2d^2bex}{f^3}+\frac{(c^2a\,f^3-2acde\,f^2+a\,d^2e^2f-b\,c^2e\,f^2+2bcd\,e^2f-b\,d^2e^3)x}{2e\,f^3(f\,x^2+e)}-\frac{\ln(fx+\sqrt{-ef})c^2a}{4\sqrt{-ef}\,e}-\ln(f$

```
[In] int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d/f^3*(1/3*b*d*f*x^3+a*d*f*x+2*b*c*f*x-2*b*d*e*x)+1/f^3*(1/2*(a*c^2*f^3-2*a
*c*d*e*f^2+a*d^2*e^2*f-b*c^2*e*f^2+2*b*c*d*e^2*f-b*d^2*e^3)/e*x/(f*x^2+e)+1
/2*(a*c^2*f^3+2*a*c*d*e*f^2-3*a*d^2*e^2*f+b*c^2*e*f^2-6*b*c*d*e^2*f+5*b*d^2
*e^3)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 552, normalized size of antiderivative = 3.37

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx$$
$$= \left[\frac{4\,bd^2e^2f^3x^5 - 4\,(5\,bd^2e^3f^2 - 3\,(2\,bcd + ad^2)e^2f^3)x^3 - 3\,(5\,bd^2e^4 + ac^2ef^3 - 3\,(2\,bcd + ad^2)e^3f + (bc^2 + 2\right.$$

[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="fricas")

[Out] [1/12*(4*b*d^2*e^2*f^3*x^5 - 4*(5*b*d^2*e^3*f^2 - 3*(2*b*c*d + a*d^2)*e^2*f^3)*x^3 - 3*(5*b*d^2*e^4 + a*c^2*e*f^3 - 3*(2*b*c*d + a*d^2)*e^3*f + (b*c^2 + 2*a*c*d)*e^2*f^2 + (5*b*d^2*e^3*f + a*c^2*f^4 - 3*(2*b*c*d + a*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) - 6*(5*b*d^2*e^4*f - a*c^2*e*f^4 - 3*(2*b*c*d + a*d^2)*e^3*f^2 + (b*c^2 + 2*a*c*d)*e^2*f^3)*x)/(e^2*f^5*x^2 + e^3*f^4), 1/6*(2*b*d^2*e^2*f^3*x^5 - 2*(5*b*d^2*e^3*f^2 - 3*(2*b*c*d + a*d^2)*e^2*f^3)*x^3 + 3*(5*b*d^2*e^4 + a*c^2*e*f^3 - 3*(2*b*c*d + a*d^2)*e^3*f + (b*c^2 + 2*a*c*d)*e^2*f^2 + (5*b*d^2*e^3*f + a*c^2*f^4 - 3*(2*b*c*d + a*d^2)*e^2*f^2 + (b*c^2 + 2*a*c*d)*e*f^3)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) - 3*(5*b*d^2*e^4*f - a*c^2*e*f^4 - 3*(2*b*c*d + a*d^2)*e^3*f^2 + (b*c^2 + 2*a*c*d)*e^2*f^3)*x)/(e^2*f^5*x^2 + e^3*f^4)]

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(151) = 302.

Time = 1.28 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.95

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = \frac{bd^2x^3}{3f^2} + x\left(\frac{ad^2}{f^2} + \frac{2bcd}{f^2} - \frac{2bd^2e}{f^3}\right)$$
$$+ \frac{x(ac^2f^3 - 2acdef^2 + ad^2e^2f - bc^2ef^2 + 2bcde^2f - bd^2e^3)}{2e^2f^3 + 2ef^4x^2}$$
$$- \frac{\sqrt{-\frac{1}{e^3f^7}}(cf-de)(acf^2+3adef+bcef-5bde^2)\log\left(-\frac{e^2f^3\sqrt{-\frac{1}{e^3f^7}}(cf-de)(acf^2+3adef+bcef-5bde^2)}{ac^2f^3+2acdef^2-3ad^2e^2f+bc^2ef^2-6bcde^2f+5bd^2e^3}+x\right)}{4}$$
$$+ \frac{\sqrt{-\frac{1}{e^3f^7}}(cf-de)(acf^2+3adef+bcef-5bde^2)\log\left(\frac{e^2f^3\sqrt{-\frac{1}{e^3f^7}}(cf-de)(acf^2+3adef+bcef-5bde^2)}{ac^2f^3+2acdef^2-3ad^2e^2f+bc^2ef^2-6bcde^2f+5bd^2e^3}+x\right)}{4}$$

[In] integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**2,x)

```
[Out] b*d**2*x**3/(3*f**2) + x*(a*d**2/f**2 + 2*b*c*d/f**2 - 2*b*d**2*e/f**3) + x
*(a*c**2*f**3 - 2*a*c*d*e*f**2 + a*d**2*e**2*f - b*c**2*e*f**2 + 2*b*c*d*e*
*2*f - b*d**2*e**3)/(2*e**2*f**3 + 2*e*f**4*x**2) - sqrt(-1/(e**3*f**7))*(c
*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)*log(-e**2*f**3*sqrt
(-1/(e**3*f**7))*(c*f - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)/
(a*c**2*f**3 + 2*a*c*d*e*f**2 - 3*a*d**2*e**2*f + b*c**2*e*f**2 - 6*b*c*d*e
**2*f + 5*b*d**2*e**3) + x)/4 + sqrt(-1/(e**3*f**7))*(c*f - d*e)*(a*c*f**2
+ 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)*log(e**2*f**3*sqrt(-1/(e**3*f**7))*(c*f
 - d*e)*(a*c*f**2 + 3*a*d*e*f + b*c*e*f - 5*b*d*e**2)/(a*c**2*f**3 + 2*a*c*
d*e*f**2 - 3*a*d**2*e**2*f + b*c**2*e*f**2 - 6*b*c*d*e**2*f + 5*b*d**2*e**3
) + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.24

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx &= \frac{(5\,bd^2e^3-6\,bcde^2f-3\,ad^2e^2f+bc^2ef^2+2\,acdef^2+ac^2f^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2\,\sqrt{ef}ef^3} \\ &\quad - \frac{bd^2e^3x-2\,bcde^2fx-ad^2e^2fx+bc^2ef^2x+2\,acdef^2x-ac^2f^3x}{2\,(fx^2+e)ef^3} \\ &\quad + \frac{bd^2f^4x^3-6\,bd^2ef^3x+6\,bcdf^4x+3\,ad^2f^4x}{3\,f^6}\end{aligned}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*b*d^2*e^3 - 6*b*c*d*e^2*f - 3*a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*
f^2 + a*c^2*f^3)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e*f^3) - 1/2*(b*d^2*e^3*x
 - 2*b*c*d*e^2*f*x - a*d^2*e^2*f*x + b*c^2*e*f^2*x + 2*a*c*d*e*f^2*x - a*c^
2*f^3*x)/((f*x^2 + e)*e*f^3) + 1/3*(b*d^2*f^4*x^3 - 6*b*d^2*e*f^3*x + 6*b*c
*d*f^4*x + 3*a*d^2*f^4*x)/f^6
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.57

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^2}\,dx = x\left(\frac{a\,d^2+2\,b\,c\,d}{f^2}-\frac{2\,b\,d^2\,e}{f^3}\right)+\frac{b\,d^2\,x^3}{3\,f^2} + \frac{x\,(-b\,c^2\,e\,f^2+a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f-2\,a\,c\,d\,e\,f^2-b\,d^2\,e^3+a\,d^2\,e^2\,f)}{2\,e\,(f^4\,x^2+e\,f^3)} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(c\,f-d\,e)\,(a\,c\,f^2-5\,b\,d\,e^2+3\,a\,d\,e\,f+b\,c\,e\,f)}{\sqrt{e}\,(b\,c^2\,e\,f^2+a\,c^2\,f^3-6\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2+5\,b\,d^2\,e^3-3\,a\,d^2\,e^2\,f)}\right)(c\,f-d\,e)\,(a\,c\,f^2-5\,b\,d\,e^2+3\,a\,d\,e\,f+b\,c\,e\,f}{2\,e^{3/2}\,f^{7/2}}$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^2,x)
```

```
[Out] x*((a*d^2 + 2*b*c*d)/f^2 - (2*b*d^2*e)/f^3) + (b*d^2*x^3)/(3*f^2) + (x*(a*c
^2*f^3 - b*d^2*e^3 + a*d^2*e^2*f - b*c^2*e*f^2 - 2*a*c*d*e*f^2 + 2*b*c*d*e^
2*f))/(2*e*(e*f^3 + f^4*x^2)) + (atan((f^(1/2)*x*(c*f - d*e)*(a*c*f^2 - 5*b
*d*e^2 + 3*a*d*e*f + b*c*e*f))/(e^(1/2)*(a*c^2*f^3 + 5*b*d^2*e^3 - 3*a*d^2*
e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 - 6*b*c*d*e^2*f)))*(c*f - d*e)*(a*c*f^2
 - 5*b*d*e^2 + 3*a*d*e*f + b*c*e*f))/(2*e^(3/2)*f^(7/2))
```

3.14 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 207

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx &= \frac{d(be(15de-cf)-3af(de+cf))x}{8e^2f^3} - \frac{(be-af)x(c+dx^2)^2}{4ef(e+fx^2)^2} \\ &\quad - \frac{(be(5de-cf)-af(de+3cf))x(c+dx^2)}{8e^2f^2(e+fx^2)} \\ &\quad - \frac{(be(15d^2e^2-6cdef-c^2f^2)-af(3d^2e^2+2cdef+3c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{7/2}}\end{aligned}$$

```
[Out] 1/8*d*(b*e*(-c*f+15*d*e)-3*a*f*(c*f+d*e))*x/e^2/f^3-1/4*(-a*f+b*e)*x*(d*x^2
+c)^2/e/f/(f*x^2+e)^2-1/8*(b*e*(-c*f+5*d*e)-a*f*(3*c*f+d*e))*x*(d*x^2+c)/e^
2/f^2/(f*x^2+e)-1/8*(b*e*(-c^2*f^2-6*c*d*e*f+15*d^2*e^2)-a*f*(3*c^2*f^2+2*c
*d*e*f+3*d^2*e^2))*arctan(x*f^(1/2)/e^(1/2))/e^(5/2)/f^(7/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used

$= \{540, 396, 211\}$

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3} dx$$
$$= -\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(be(-c^2f^2-6cdef+15d^2e^2)-af(3c^2f^2+2cdef+3d^2e^2))}{8e^{5/2}f^{7/2}}$$
$$+\frac{dx(be(15de-cf)-3af(cf+de))}{8e^2f^3}$$
$$-\frac{x(c+dx^2)(be(5de-cf)-af(3cf+de))}{8e^2f^2(e+fx^2)}-\frac{x(c+dx^2)^2(be-af)}{4ef(e+fx^2)^2}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

```
[Out] (d*(b*e*(15*d*e - c*f) - 3*a*f*(d*e + c*f))*x)/(8*e^2*f^3) - ((b*e - a*f)*x
*(c + d*x^2)^2)/(4*e*f*(e + f*x^2)^2) - ((b*e*(5*d*e - c*f) - a*f*(d*e + 3*
c*f))*x*(c + d*x^2))/(8*e^2*f^2*(e + f*x^2)) - ((b*e*(15*d^2*e^2 - 6*c*d*e*
f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c*d*e*f + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/
Sqrt[e]])/(8*e^(5/2)*f^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\text{integral} = -\frac{(be-af)x(c+dx^2)^2}{4ef(e+fx^2)^2}-\frac{\int \frac{(c+dx^2)(-c(be+3af)-d(5be-af)x^2)}{(e+fx^2)^2} dx}{4ef}$$

$$= -\frac{(be-af)x(c+dx^2)^2}{4ef(e+fx^2)^2} - \frac{(be(5de-cf)-af(de+3cf))x(c+dx^2)}{8e^2f^2(e+fx^2)} + \frac{\int \frac{-c(af(de-3cf)-be(5de+cf))+d(be(15de-cf)-3af(de+cf))x^2}{e+fx^2}dx}{8e^2f^2}$$

$$= \frac{d(be(15de-cf)-3af(de+cf))x}{8e^2f^3} - \frac{(be-af)x(c+dx^2)^2}{4ef(e+fx^2)^2} - \frac{(be(5de-cf)-af(de+3cf))x(c+dx^2)}{8e^2f^2(e+fx^2)} - \frac{(be(15d^2e^2-6cdef-c^2f^2)-af(3d^2e^2+2cdef+3c^2f^2))\int \frac{1}{e+fx^2}dx}{8e^2f^3}$$

$$= \frac{d(be(15de-cf)-3af(de+cf))x}{8e^2f^3} - \frac{(be-af)x(c+dx^2)^2}{4ef(e+fx^2)^2} - \frac{(be(5de-cf)-af(de+3cf))x(c+dx^2)}{8e^2f^2(e+fx^2)} - \frac{(be(15d^2e^2-6cdef-c^2f^2)-af(3d^2e^2+2cdef+3c^2f^2))\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{7/2}}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}dx = \frac{bd^2x}{f^3} - \frac{(be-af)(de-cf)^2x}{4ef^3(e+fx^2)^2} + \frac{(de-cf)(be(9de-cf)-af(5de+3cf))x}{8e^2f^3(e+fx^2)} - \frac{(be(15d^2e^2-6cdef-c^2f^2)-af(3d^2e^2+2cdef+3c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{7/2}}$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x]
```

```
[Out] (b*d^2*x)/f^3 - ((b*e - a*f)*(d*e - c*f)^2*x)/(4*e*f^3*(e + f*x^2)^2) + ((d
*e - c*f)*(b*e*(9*d*e - c*f) - a*f*(5*d*e + 3*c*f))*x)/(8*e^2*f^3*(e + f*x^
2)) - ((b*e*(15*d^2*e^2 - 6*c*d*e*f - c^2*f^2) - a*f*(3*d^2*e^2 + 2*c*d*e*f
 + 3*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(7/2))
```

Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.11

method	result
default	$\frac{b\,d^{2}x}{f^{3}}+\frac{\frac{\frac{f\left(3c^{2}a\,f^{3}+2acde\,f^{2}-5a\,d^{2}e^{2}f+b\,c^{2}e\,f^{2}-10bcd\,e^{2}f+9b\,d^{2}e^{3}\right)x^{3}}{8e^{2}}+\frac{\left(5c^{2}a\,f^{3}-2acde\,f^{2}-3a\,d^{2}e^{2}f-b\,c^{2}e\,f^{2}-6bcd\,e^{2}f+7b\,d^{2}e^{3}\right)x}{8e}}{\left(f\,x^{2}+e\right)^{2}}+\frac{\left(3c^{2}\right.}{}}{f^{3}}$
risch	$\frac{b\,d^{2}x}{f^{3}}+\frac{\frac{f\left(3c^{2}a\,f^{3}+2acde\,f^{2}-5a\,d^{2}e^{2}f+b\,c^{2}e\,f^{2}-10bcd\,e^{2}f+9b\,d^{2}e^{3}\right)x^{3}}{8e^{2}}+\frac{\left(5c^{2}a\,f^{3}-2acde\,f^{2}-3a\,d^{2}e^{2}f-b\,c^{2}e\,f^{2}-6bcd\,e^{2}f+7b\,d^{2}e^{3}\right)x}{8e}}{f^{3}\left(f\,x^{2}+e\right)^{2}}-\frac{3\,\mathrm{l}}{}$

```
[In] int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] b*d^2/f^3*x+1/f^3*((1/8*f*(3*a*c^2*f^3+2*a*c*d*e*f^2-5*a*d^2*e^2*f+b*c^2*e*
f^2-10*b*c*d*e^2*f+9*b*d^2*e^3)/e^2*x^3+1/8*(5*a*c^2*f^3-2*a*c*d*e*f^2-3*a*
d^2*e^2*f-b*c^2*e*f^2-6*b*c*d*e^2*f+7*b*d^2*e^3)/e*x)/(f*x^2+e)^2+1/8*(3*a*
c^2*f^3+2*a*c*d*e*f^2+3*a*d^2*e^2*f+b*c^2*e*f^2+6*b*c*d*e^2*f-15*b*d^2*e^3)
/e^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 777, normalized size of antiderivative = 3.75

$$\int \frac{(a+bx^2)\,(c+dx^2)^2}{(e+fx^2)^3}\,dx$$
$$=\left[\frac{16\,bd^2e^3f^3x^5+2\,(25\,bd^2e^4f^2+3\,ac^2ef^5-5\,(2\,bcd+ad^2)e^3f^3+(bc^2+2\,acd)e^2f^4)x^3+(15\,bd^2e^5-3\,ac^2e}{}\right.$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*b*d^2*e^3*f^3*x^5 + 2*(25*b*d^2*e^4*f^2 + 3*a*c^2*e*f^5 - 5*(2*b*
c*d + a*d^2)*e^3*f^3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*x^3 + (15*b*d^2*e^5 - 3*a
*c^2*e^2*f^3 - 3*(2*b*c*d + a*d^2)*e^4*f - (b*c^2 + 2*a*c*d)*e^3*f^2 + (15*
b*d^2*e^3*f^2 - 3*a*c^2*f^5 - 3*(2*b*c*d + a*d^2)*e^2*f^3 - (b*c^2 + 2*a*c*
d)*e*f^4)*x^4 + 2*(15*b*d^2*e^4*f - 3*a*c^2*e*f^4 - 3*(2*b*c*d + a*d^2)*e^3
*f^2 - (b*c^2 + 2*a*c*d)*e^2*f^3)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)
*x - e)/(f*x^2 + e)) + 2*(15*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 - 3*(2*b*c*d + a
*d^2)*e^4*f^2 - (b*c^2 + 2*a*c*d)*e^3*f^3)*x)/(e^3*f^6*x^4 + 2*e^4*f^5*x^2
+ e^5*f^4), 1/8*(8*b*d^2*e^3*f^3*x^5 + (25*b*d^2*e^4*f^2 + 3*a*c^2*e*f^5 -
5*(2*b*c*d + a*d^2)*e^3*f^3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*x^3 - (15*b*d^2*e^
5 - 3*a*c^2*e^2*f^3 - 3*(2*b*c*d + a*d^2)*e^4*f - (b*c^2 + 2*a*c*d)*e^3*f^2
 + (15*b*d^2*e^3*f^2 - 3*a*c^2*f^5 - 3*(2*b*c*d + a*d^2)*e^2*f^3 - (b*c^2 +
```

```
2*a*c*d)*e*f^4)*x^4 + 2*(15*b*d^2*e^4*f - 3*a*c^2*e*f^4 - 3*(2*b*c*d + a*d
^2)*e^3*f^2 - (b*c^2 + 2*a*c*d)*e^2*f^3)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/
e) + (15*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 - 3*(2*b*c*d + a*d^2)*e^4*f^2 - (b*c
^2 + 2*a*c*d)*e^3*f^3)*x)/(e^3*f^6*x^4 + 2*e^4*f^5*x^2 + e^5*f^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(199) = 398$.

Time = 6.09 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.93

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx = \frac{bd^2x}{f^3} - \frac{\sqrt{-\frac{1}{e^5f^7}}\cdot(3ac^2f^3+2acdef^2+3ad^2e^2f+bc^2ef^2+6bcde^2f-15bd^2e^3)\log\left(-e^3f^3\sqrt{-\frac{1}{e^5f^7}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{e^5f^7}}\cdot(3ac^2f^3+2acdef^2+3ad^2e^2f+bc^2ef^2+6bcde^2f-15bd^2e^3)\log\left(e^3f^3\sqrt{-\frac{1}{e^5f^7}}+x\right)}{16} + \frac{x^3\cdot(3ac^2f^4+2acdef^3-5ad^2e^2f^2+bc^2ef^3-10bcde^2f^2+9bd^2e^3f)+x(5ac^2ef^3-2acde^2f^2-3ad^2e^3}{8e^4f^3+16e^3f^4x^2+8e^2f^5x^4}$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**3,x)
```

```
[Out] b*d**2*x/f**3 - sqrt(-1/(e**5*f**7))*(3*a*c**2*f**3 + 2*a*c*d*e*f**2 + 3*a*
d**2*e**2*f + b*c**2*e*f**2 + 6*b*c*d*e**2*f - 15*b*d**2*e**3)*log(-e**3*f*
*3*sqrt(-1/(e**5*f**7)) + x)/16 + sqrt(-1/(e**5*f**7))*(3*a*c**2*f**3 + 2*a
*c*d*e*f**2 + 3*a*d**2*e**2*f + b*c**2*e*f**2 + 6*b*c*d*e**2*f - 15*b*d**2*
e**3)*log(e**3*f**3*sqrt(-1/(e**5*f**7)) + x)/16 + (x**3*(3*a*c**2*f**4 + 2
*a*c*d*e*f**3 - 5*a*d**2*e**2*f**2 + b*c**2*e*f**3 - 10*b*c*d*e**2*f**2 + 9
*b*d**2*e**3*f) + x*(5*a*c**2*e*f**3 - 2*a*c*d*e**2*f**2 - 3*a*d**2*e**3*f
- b*c**2*e**2*f**2 - 6*b*c*d*e**3*f + 7*b*d**2*e**4))/(8*e**4*f**3 + 16*e**
3*f**4*x**2 + 8*e**2*f**5*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.22

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx$$
$$= \frac{bd^2x}{f^3} - \frac{(15\,bd^2e^3 - 6\,bcde^2f - 3\,ad^2e^2f - bc^2ef^2 - 2\,acdef^2 - 3\,ac^2f^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{8\sqrt{ef}e^2f^3}$$
$$+ \frac{9\,bd^2e^3fx^3 - 10\,bcde^2f^2x^3 - 5\,ad^2e^2f^2x^3 + bc^2ef^3x^3 + 2\,acdef^3x^3 + 3\,ac^2f^4x^3 + 7\,bd^2e^4x - 6\,bcde^3fx - }{8\,(fx^2+e)^2e^2f^3}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] b*d^2*x/f^3 - 1/8*(15*b*d^2*e^3 - 6*b*c*d*e^2*f - 3*a*d^2*e^2*f - b*c^2*e*f
^2 - 2*a*c*d*e*f^2 - 3*a*c^2*f^3)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e^2*f^3)
 + 1/8*(9*b*d^2*e^3*f*x^3 - 10*b*c*d*e^2*f^2*x^3 - 5*a*d^2*e^2*f^2*x^3 + b*
c^2*e*f^3*x^3 + 2*a*c*d*e*f^3*x^3 + 3*a*c^2*f^4*x^3 + 7*b*d^2*e^4*x - 6*b*c
*d*e^3*f*x - 3*a*d^2*e^3*f*x - b*c^2*e^2*f^2*x - 2*a*c*d*e^2*f^2*x + 5*a*c^
2*e*f^3*x)/((f*x^2 + e)^2*e^2*f^3)
```

Mupad [B] (verification not implemented)

Time = 5.45 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^3}\,dx$$
$$= \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,(b\,c^2\,e\,f^2 + 3\,a\,c^2\,f^3 + 6\,b\,c\,d\,e^2\,f + 2\,a\,c\,d\,e\,f^2 - 15\,b\,d^2\,e^3 + 3\,a\,d^2\,e^2\,f)}{8\,e^{5/2}\,f^{7/2}}$$
$$- \frac{\frac{x\,(b\,c^2\,e\,f^2 - 5\,a\,c^2\,f^3 + 6\,b\,c\,d\,e^2\,f + 2\,a\,c\,d\,e\,f^2 - 7\,b\,d^2\,e^3 + 3\,a\,d^2\,e^2\,f)}{8\,e} - \frac{x^3\,(b\,c^2\,e\,f^3 + 3\,a\,c^2\,f^4 - 10\,b\,c\,d\,e^2\,f^2 + 2\,a\,c\,d\,e\,f^3 + 9\,b\,d^2\,e^3\,f - 5\,a\,d^2\,e^2\,}{8\,e^2}}{e^2\,f^3 + 2\,e\,f^4\,x^2 + f^5\,x^4}$$
$$+ \frac{b\,d^2\,x}{f^3}$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^3,x)
```

```
[Out] (atan((f^(1/2)*x)/e^(1/2))*(3*a*c^2*f^3 - 15*b*d^2*e^3 + 3*a*d^2*e^2*f + b*
c^2*e*f^2 + 2*a*c*d*e*f^2 + 6*b*c*d*e^2*f))/(8*e^(5/2)*f^(7/2)) - ((x*(3*a*
d^2*e^2*f - 7*b*d^2*e^3 - 5*a*c^2*f^3 + b*c^2*e*f^2 + 2*a*c*d*e*f^2 + 6*b*c
*d*e^2*f))/(8*e) - (x^3*(3*a*c^2*f^4 - 5*a*d^2*e^2*f^2 + b*c^2*e*f^3 + 9*b*
d^2*e^3*f - 10*b*c*d*e^2*f^2 + 2*a*c*d*e*f^3))/(8*e^2))/(e^2*f^3 + f^5*x^4
+ 2*e*f^4*x^2) + (b*d^2*x)/f^3
```

3.15 $\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 240

$$\begin{aligned}\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4} dx &= -\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3} - \frac{(de(5be+af)-cf(be+5af))x(c+dx^2)}{24e^2f^2(e+fx^2)^2} \\ &\quad - \frac{(af(3d^2e^2+4cdef-15c^2f^2)+be(15d^2e^2-4cdef-3c^2f^2))x}{48e^3f^3(e+fx^2)} \\ &\quad + \frac{(be(5d^2e^2+2cdef+c^2f^2)+af(d^2e^2+2cdef+5c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{7/2}}\end{aligned}$$

```
[Out] -1/6*(-a*f+b*e)*x*(d*x^2+c)^2/e/f/(f*x^2+e)^3-1/24*(d*e*(a*f+5*b*e)-c*f*(5*
a*f+b*e))*x*(d*x^2+c)/e^2/f^2/(f*x^2+e)^2-1/48*(a*f*(-15*c^2*f^2+4*c*d*e*f+
3*d^2*e^2)+b*e*(-3*c^2*f^2-4*c*d*e*f+15*d^2*e^2))*x/e^3/f^3/(f*x^2+e)+1/16*
(b*e*(c^2*f^2+2*c*d*e*f+5*d^2*e^2)+a*f*(5*c^2*f^2+2*c*d*e*f+d^2*e^2))*arcta
n(x*f^(1/2)/e^(1/2))/e^(7/2)/f^(7/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used

= {540, 393, 211}

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = \frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(af(5c^2f^2+2cdef+d^2e^2)+be(c^2f^2+2cdef+5d^2e^2))}{16e^{7/2}f^{7/2}} - \frac{x(af(-15c^2f^2+4cdef+3d^2e^2)+be(-3c^2f^2-4cdef+15d^2e^2))}{48e^3f^3(e+fx^2)} - \frac{x(c+dx^2)(de(af+5be)-cf(5af+be))}{24e^2f^2(e+fx^2)^2} - \frac{x(c+dx^2)^2(be-af)}{6ef(e+fx^2)^3}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x]
```

```
[Out] -1/6*((b*e - a*f)*x*(c + d*x^2)^2)/(e*f*(e + f*x^2)^3) - ((d*e*(5*b*e + a*f
) - c*f*(b*e + 5*a*f))*x*(c + d*x^2))/(24*e^2*f^2*(e + f*x^2)^2) - ((a*f*(3
*d^2*e^2 + 4*c*d*e*f - 15*c^2*f^2) + b*e*(15*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^
2))*x)/(48*e^3*f^3*(e + f*x^2)) + ((b*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2) +
 a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^
(7/2)*f^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\text{integral} = -\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3} - \frac{\int \frac{(c+dx^2)(-c(be+5af)-d(5be+af)x^2)}{(e+fx^2)^3}\,dx}{6ef}$$

$$= -\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3} - \frac{(de(5be+af)-cf(be+5af))x(c+dx^2)}{24e^2f^2(e+fx^2)^2} + \frac{\int \frac{c(de(5be+af)+3cf(be+5af))+d(be(15de+cf)+af(3de+5cf))x^2}{(e+fx^2)^2}\,dx}{24e^2f^2}$$

$$= -\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3} - \frac{(de(5be+af)-cf(be+5af))x(c+dx^2)}{24e^2f^2(e+fx^2)^2} - \frac{(af(3d^2e^2+4cdef-15c^2f^2)+be(15d^2e^2-4cdef-3c^2f^2))\,x}{48e^3f^3(e+fx^2)} + \frac{(be(5d^2e^2+2cdef+c^2f^2)+af(d^2e^2+2cdef+5c^2f^2))\int \frac{1}{e+fx^2}\,dx}{16e^3f^3}$$

$$= -\frac{(be-af)x(c+dx^2)^2}{6ef(e+fx^2)^3} - \frac{(de(5be+af)-cf(be+5af))x(c+dx^2)}{24e^2f^2(e+fx^2)^2} - \frac{(af(3d^2e^2+4cdef-15c^2f^2)+be(15d^2e^2-4cdef-3c^2f^2))\,x}{48e^3f^3(e+fx^2)} + \frac{(be(5d^2e^2+2cdef+c^2f^2)+af(d^2e^2+2cdef+5c^2f^2))\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{7/2}}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = -\frac{(be-af)(de-cf)^2x}{6ef^3(e+fx^2)^3} + \frac{(de-cf)(be(13de-cf)-af(7de+5cf))x}{24e^2f^3(e+fx^2)^2} + \frac{(be(-11d^2e^2+2cdef+c^2f^2)+af(d^2e^2+2cdef+5c^2f^2))\,x}{16e^3f^3(e+fx^2)} + \frac{(be(5d^2e^2+2cdef+c^2f^2)+af(d^2e^2+2cdef+5c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{7/2}}$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x]
[Out] -1/6*((b*e - a*f)*(d*e - c*f)^2*x)/(e*f^3*(e + f*x^2)^3) + ((d*e - c*f)*(b*
e*(13*d*e - c*f) - a*f*(7*d*e + 5*c*f))*x)/(24*e^2*f^3*(e + f*x^2)^2) + ((b
```

```
*e*(-11*d^2*e^2 + 2*c*d*e*f + c^2*f^2) + a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f
^2))*x)/(16*e^3*f^3*(e + f*x^2)) + ((b*e*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2)
+ a*f*(d^2*e^2 + 2*c*d*e*f + 5*c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e
^(7/2)*f^(7/2))
```

Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.20

method	result
default	$\frac{\frac{\left(5c^2 a f^3+2acde f^2+a d^2 e^2 f+b c^2 e f^2+2bcd e^2 f-11b d^2 e^3\right)x^5}{16e^3 f}+\frac{\left(5c^2 a f^3+2acde f^2-a d^2 e^2 f+b c^2 e f^2-2bcd e^2 f-5b d^2 e^3\right)x^3}{6e^2 f^2}+\frac{\left(11c^2 a f^3-2acde\right.}{}}{\left(f x^2+e\right)^3}$
risch	$\frac{\frac{\left(5c^2 a f^3+2acde f^2+a d^2 e^2 f+b c^2 e f^2+2bcd e^2 f-11b d^2 e^3\right)x^5}{16e^3 f}+\frac{\left(5c^2 a f^3+2acde f^2-a d^2 e^2 f+b c^2 e f^2-2bcd e^2 f-5b d^2 e^3\right)x^3}{6e^2 f^2}+\frac{\left(11c^2 a f^3-2acde\right.}{}}{\left(f x^2+e\right)^3}$

```
[In] int((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] (1/16*(5*a*c^2*f^3+2*a*c*d*e*f^2+a*d^2*e^2*f+b*c^2*e*f^2+2*b*c*d*e^2*f-11*b
*d^2*e^3)/e^3/f*x^5+1/6*(5*a*c^2*f^3+2*a*c*d*e*f^2-a*d^2*e^2*f+b*c^2*e*f^2-
2*b*c*d*e^2*f-5*b*d^2*e^3)/e^2/f^2*x^3+1/16*(11*a*c^2*f^3-2*a*c*d*e*f^2-a*d
^2*e^2*f-b*c^2*e*f^2-2*b*c*d*e^2*f-5*b*d^2*e^3)/f^3/e*x)/(f*x^2+e)^3+1/16*(
5*a*c^2*f^3+2*a*c*d*e*f^2+a*d^2*e^2*f+b*c^2*e*f^2+2*b*c*d*e^2*f+5*b*d^2*e^3
)/e^3/f^3/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(224) = 448$.

Time = 0.27 (sec) , antiderivative size = 1024, normalized size of antiderivative = 4.27

$$\int \frac{(a+bx^2)\,(c+dx^2)^2}{(e+fx^2)^4}\,dx$$

$$= \left[-\frac{6\,(11\,bd^2e^4f^3 - 5\,ac^2ef^6 - (2\,bcd + ad^2)e^3f^4 - (bc^2 + 2\,acd)e^2f^5)x^5 + 16\,(5\,bd^2e^5f^2 - 5\,ac^2e^2f^5 + (2\,bc}{} \right.$$

$$-\frac{3\,(11\,bd^2e^4f^3 - 5\,ac^2ef^6 - (2\,bcd + ad^2)e^3f^4 - (bc^2 + 2\,acd)e^2f^5)x^5 + 8\,(5\,bd^2e^5f^2 - 5\,ac^2e^2f^5 + (2\,bcd}{}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="fricas")
```

```
[Out] [-1/96*(6*(11*b*d^2*e^4*f^3 - 5*a*c^2*e*f^6 - (2*b*c*d + a*d^2)*e^3*f^4 - (
b*c^2 + 2*a*c*d)*e^2*f^5)*x^5 + 16*(5*b*d^2*e^5*f^2 - 5*a*c^2*e^2*f^5 + (2*
b*c*d + a*d^2)*e^4*f^3 - (b*c^2 + 2*a*c*d)*e^3*f^4)*x^3 + 3*(5*b*d^2*e^6 +
```

```
5*a*c^2*e^3*f^3 + (2*b*c*d + a*d^2)*e^5*f + (b*c^2 + 2*a*c*d)*e^4*f^2 + (5*
b*d^2*e^3*f^3 + 5*a*c^2*f^6 + (2*b*c*d + a*d^2)*e^2*f^4 + (b*c^2 + 2*a*c*d)
*e*f^5)*x^6 + 3*(5*b*d^2*e^4*f^2 + 5*a*c^2*e*f^5 + (2*b*c*d + a*d^2)*e^3*f^
3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*x^4 + 3*(5*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 + (
2*b*c*d + a*d^2)*e^4*f^2 + (b*c^2 + 2*a*c*d)*e^3*f^3)*x^2)*sqrt(-e*f)*log((
f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 6*(5*b*d^2*e^6*f - 11*a*c^2*e^3*
f^4 + (2*b*c*d + a*d^2)*e^5*f^2 + (b*c^2 + 2*a*c*d)*e^4*f^3)*x)/(e^4*f^7*x^
6 + 3*e^5*f^6*x^4 + 3*e^6*f^5*x^2 + e^7*f^4), -1/48*(3*(11*b*d^2*e^4*f^3 -
5*a*c^2*e*f^6 - (2*b*c*d + a*d^2)*e^3*f^4 - (b*c^2 + 2*a*c*d)*e^2*f^5)*x^5
+ 8*(5*b*d^2*e^5*f^2 - 5*a*c^2*e^2*f^5 + (2*b*c*d + a*d^2)*e^4*f^3 - (b*c^2
 + 2*a*c*d)*e^3*f^4)*x^3 - 3*(5*b*d^2*e^6 + 5*a*c^2*e^3*f^3 + (2*b*c*d + a*
d^2)*e^5*f + (b*c^2 + 2*a*c*d)*e^4*f^2 + (5*b*d^2*e^3*f^3 + 5*a*c^2*f^6 + (
2*b*c*d + a*d^2)*e^2*f^4 + (b*c^2 + 2*a*c*d)*e*f^5)*x^6 + 3*(5*b*d^2*e^4*f^
2 + 5*a*c^2*e*f^5 + (2*b*c*d + a*d^2)*e^3*f^3 + (b*c^2 + 2*a*c*d)*e^2*f^4)*
x^4 + 3*(5*b*d^2*e^5*f + 5*a*c^2*e^2*f^4 + (2*b*c*d + a*d^2)*e^4*f^2 + (b*c
^2 + 2*a*c*d)*e^3*f^3)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + 3*(5*b*d^2*e^
6*f - 11*a*c^2*e^3*f^4 + (2*b*c*d + a*d^2)*e^5*f^2 + (b*c^2 + 2*a*c*d)*e^4*
f^3)*x)/(e^4*f^7*x^6 + 3*e^5*f^6*x^4 + 3*e^6*f^5*x^2 + e^7*f^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**2/(f*x**2+e)**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx$$
$$= \frac{(5\,bd^2e^3+2\,bcde^2f+ad^2e^2f+bc^2ef^2+2\,acdef^2+5\,ac^2f^3)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{16\,\sqrt{ef}e^3f^3} - \frac{33\,bd^2e^3f^2x^5-6\,bcde^2f^3x^5-3\,ad^2e^2f^3x^5-3\,bc^2ef^4x^5-6\,acdef^4x^5-15\,ac^2f^5x^5+40\,bd^2e^4fx^3+16\,bc}{}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^2/(f*x^2+e)^4,x, algorithm="giac")
```

```
[Out] 1/16*(5*b*d^2*e^3 + 2*b*c*d*e^2*f + a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f
^2 + 5*a*c^2*f^3)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e^3*f^3) - 1/48*(33*b*d^
2*e^3*f^2*x^5 - 6*b*c*d*e^2*f^3*x^5 - 3*a*d^2*e^2*f^3*x^5 - 3*b*c^2*e*f^4*x
^5 - 6*a*c*d*e*f^4*x^5 - 15*a*c^2*f^5*x^5 + 40*b*d^2*e^4*f*x^3 + 16*b*c*d*e
^3*f^2*x^3 + 8*a*d^2*e^3*f^2*x^3 - 8*b*c^2*e^2*f^3*x^3 - 16*a*c*d*e^2*f^3*x
^3 - 40*a*c^2*e*f^4*x^3 + 15*b*d^2*e^5*x + 6*b*c*d*e^4*f*x + 3*a*d^2*e^4*f*
x + 3*b*c^2*e^3*f^2*x + 6*a*c*d*e^3*f^2*x - 33*a*c^2*e^2*f^3*x)/((f*x^2 + e
)^3*e^3*f^3)
```

Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^2)(c+dx^2)^2}{(e+fx^2)^4}\,dx$$
$$= \frac{\frac{x^3\,(b\,c^2\,e\,f^2+5\,a\,c^2\,f^3-2\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2-5\,b\,d^2\,e^3-a\,d^2\,e^2\,f)}{6\,e^2\,f^2} - \frac{x\,(b\,c^2\,e\,f^2-11\,a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2+5\,b\,d^2\,e^3+a\,d^2\,e^2\,f)}{16\,e\,f^3} + \frac{x^5}{}}{e^3+3\,e^2\,f\,x^2+3\,e\,f^2\,x^4+f^3\,x^6}$$
$$+ \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,(b\,c^2\,e\,f^2+5\,a\,c^2\,f^3+2\,b\,c\,d\,e^2\,f+2\,a\,c\,d\,e\,f^2+5\,b\,d^2\,e^3+a\,d^2\,e^2\,f)}{16\,e^{7/2}\,f^{7/2}}$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^2)/(e + f*x^2)^4,x)
```

```
[Out] ((x^3*(5*a*c^2*f^3 - 5*b*d^2*e^3 - a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^
2 - 2*b*c*d*e^2*f))/(6*e^2*f^2) - (x*(5*b*d^2*e^3 - 11*a*c^2*f^3 + a*d^2*e^
2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 + 2*b*c*d*e^2*f))/(16*e*f^3) + (x^5*(5*a*
c^2*f^3 - 11*b*d^2*e^3 + a*d^2*e^2*f + b*c^2*e*f^2 + 2*a*c*d*e*f^2 + 2*b*c*
d*e^2*f))/(16*e^3*f))/(e^3 + f^3*x^6 + 3*e^2*f*x^2 + 3*e*f^2*x^4) + (atan((
f^(1/2)*x)/e^(1/2))*(5*a*c^2*f^3 + 5*b*d^2*e^3 + a*d^2*e^2*f + b*c^2*e*f^2
+ 2*a*c*d*e*f^2 + 2*b*c*d*e^2*f))/(16*e^(7/2)*f^(7/2))
```

3.16 $\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx$

Optimal result

Integrand size = 26, antiderivative size = 310

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx &= ac^3e^3x+\frac{1}{3}c^2e^2(bce+3a(de+cf))x^3\\&+\frac{3}{5}ce\left(bce(de+cf)+a\left(d^2e^2+3cdef+c^2f^2\right)\right)x^5\\&+\frac{1}{7}\left(3bce\left(d^2e^2+3cdef+c^2f^2\right)\right.\\&\qquad\left.+a\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^7\\&+\frac{1}{9}\left(3adf\left(d^2e^2+3cdef+c^2f^2\right)\right.\\&\qquad\left.+b\left(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\right)\right)x^9\\&+\frac{3}{11}df\left(adf(de+cf)+b\left(d^2e^2+3cdef+c^2f^2\right)\right)x^{11}\\&+\frac{1}{13}d^2f^2(adf+3b(de+cf))x^{13}+\frac{1}{15}bd^3f^3x^{15}\end{aligned}$$

```
[Out] a*c^3*e^3*x+1/3*c^2*e^2*(b*c*e+3*a*(c*f+d*e))*x^3+3/5*c*e*(b*c*e*(c*f+d*e)+
a*(c^2*f^2+3*c*d*e*f+d^2*e^2))*x^5+1/7*(3*b*c*e*(c^2*f^2+3*c*d*e*f+d^2*e^2)
+a*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^7+1/9*(3*a*d*f*(c^2*f^2
+3*c*d*e*f+d^2*e^2)+b*(c^3*f^3+9*c^2*d*e*f^2+9*c*d^2*e^2*f+d^3*e^3))*x^9+3/
11*d*f*(a*d*f*(c*f+d*e)+b*(c^2*f^2+3*c*d*e*f+d^2*e^2))*x^11+1/13*d^2*f^2*(a
*d*f+3*b*(c*f+d*e))*x^13+1/15*b*d^3*f^3*x^15
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.038, Rules used = {535}

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx = \frac{3}{11}dfx^{11}\big(adf(cf+de)+b\big(c^2f^2+3cdef+d^2e^2\big)\big) + \frac{3}{5}cex^5\big(a\big(c^2f^2+3cdef+d^2e^2\big)+bce(cf+de)\big) + \frac{1}{3}c^2e^2x^3(3a(cf+de)+bce) + \frac{1}{9}x^9\big(3adf\big(c^2f^2+3cdef+d^2e^2\big) + b\big(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\big)\big) + \frac{1}{7}x^7\big(a\big(c^3f^3+9c^2def^2+9cd^2e^2f+d^3e^3\big) + 3bce\big(c^2f^2+3cdef+d^2e^2\big)\big) + \frac{1}{13}d^2f^2x^{13}(adf+3b(cf+de)) + ac^3e^3x + \frac{1}{15}bd^3f^3x^{15}$$

```
[In] Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

```
[Out] a*c^3*e^3*x + (c^2*e^2*(b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (3*c*e*(b*c*e*(d*
e + c*f) + a*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/5 + ((3*b*c*e*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2) + a*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^
3))*x^7)/7 + ((3*a*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b*(d^3*e^3 + 9*c*d
^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^9)/9 + (3*d*f*(a*d*f*(d*e + c*f) + b
*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^11)/11 + (d^2*f^2*(a*d*f + 3*b*(d*e + c
*f))*x^13)/13 + (b*d^3*f^3*x^15)/15
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\text{integral} = \int \big(ac^3e^3+c^2e^2(bce+3a(de+cf))x^2+3ce\big(bce(de+cf)+a\big(d^2e^2+3cdef+c^2f^2\big)\big)x^4 + \big(3bce\big(d^2e^2+3cdef+c^2f^2\big)+a\big(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\big)\big)x^6 + \big(3adf\big(d^2e^2+3cdef+c^2f^2\big)+b\big(d^3e^3+9cd^2e^2f+9c^2def^2+c^3f^3\big)\big)x^8 + 3df\big(adf(de+cf)+b\big(d^2e^2+3cdef+c^2f^2\big)\big)x^{10}+d^2f^2(adf+3b(de+cf))x^{12} + bd^3f^3x^{14}\big)\,dx$$

$$= ac^3e^3x + \frac{1}{3}c^2e^2(bce + 3a(de + cf))x^3 + \frac{3}{5}ce\left(bce(de + cf) + a\left(d^2e^2 + 3cdef + c^2f^2\right)\right)x^5 + \frac{1}{7}\left(3bce\left(d^2e^2 + 3cdef + c^2f^2\right) + a\left(d^3e^3 + 9cd^2e^2f + 9c^2def^2 + c^3f^3\right)\right)x^7 + \frac{1}{9}\left(3adf\left(d^2e^2 + 3cdef + c^2f^2\right) + b\left(d^3e^3 + 9cd^2e^2f + 9c^2def^2 + c^3f^3\right)\right)x^9 + \frac{3}{11}df\left(adf(de + cf) + b\left(d^2e^2 + 3cdef + c^2f^2\right)\right)x^{11} + \frac{1}{13}d^2f^2(adf + 3b(de + cf))x^{13} + \frac{1}{15}bd^3f^3x^{15}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.00

$$\int \left(a + bx^2\right)\left(c + dx^2\right)^3\left(e + fx^2\right)^3 dx = ac^3e^3x + \frac{1}{3}c^2e^2(bce + 3a(de + cf))x^3 + \frac{3}{5}ce\left(bce(de + cf) + a\left(d^2e^2 + 3cdef + c^2f^2\right)\right)x^5 + \frac{1}{7}\left(3bce\left(d^2e^2 + 3cdef + c^2f^2\right) + a\left(d^3e^3 + 9cd^2e^2f + 9c^2def^2 + c^3f^3\right)\right)x^7 + \frac{1}{9}\left(3adf\left(d^2e^2 + 3cdef + c^2f^2\right) + b\left(d^3e^3 + 9cd^2e^2f + 9c^2def^2 + c^3f^3\right)\right)x^9 + \frac{3}{11}df\left(adf(de + cf) + b\left(d^2e^2 + 3cdef + c^2f^2\right)\right)x^{11} + \frac{1}{13}d^2f^2(adf + 3b(de + cf))x^{13} + \frac{1}{15}bd^3f^3x^{15}$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x]
```

```
[Out] a*c^3*e^3*x + (c^2*e^2*(b*c*e + 3*a*(d*e + c*f))*x^3)/3 + (3*c*e*(b*c*e*(d*
e + c*f) + a*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^5)/5 + ((3*b*c*e*(d^2*e^2 +
 3*c*d*e*f + c^2*f^2) + a*(d^3*e^3 + 9*c*d^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^
3))*x^7)/7 + ((3*a*d*f*(d^2*e^2 + 3*c*d*e*f + c^2*f^2) + b*(d^3*e^3 + 9*c*d
^2*e^2*f + 9*c^2*d*e*f^2 + c^3*f^3))*x^9)/9 + (3*d*f*(a*d*f*(d*e + c*f) + b
*(d^2*e^2 + 3*c*d*e*f + c^2*f^2))*x^11)/11 + (d^2*f^2*(a*d*f + 3*b*(d*e + c
*f))*x^13)/13 + (b*d^3*f^3*x^15)/15
```

Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.09

method	result
default	$\frac{b d^3 f^3 x^{15}}{15}+\frac{((a d^3+3bc d^2) f^3+3b d^3 e f^2) x^{13}}{13}+\frac{((3ac d^2+3b c^2 d) f^3+3(a d^3+3bc d^2) e f^2+3b d^3 e^2 f) x^{11}}{11}+$
norman	$a c^3 e^3 x+\left(c^3 a e^2 f+a c^2 d e^3+\frac{1}{3} b c^3 e^3\right) x^3+\left(\frac{3}{5} c^3 a e f^2+\frac{9}{5} a c^2 d e^2 f+\frac{3}{5} a c d^2 e^3+\frac{3}{5} b c^3 e^2 f+\right.$
gosper	$\frac{1}{7} x^7 c^3 a f^3+\frac{1}{7} x^7 a d^3 e^3+\frac{1}{9} x^9 b c^3 f^3+\frac{1}{9} x^9 b d^3 e^3+\frac{1}{13} x^{13} a d^3 f^3+\frac{1}{3} x^3 b c^3 e^3+x^9 b c^2 d e f^2+x^9 b$
risch	$\frac{1}{7} x^7 c^3 a f^3+\frac{1}{7} x^7 a d^3 e^3+\frac{1}{9} x^9 b c^3 f^3+\frac{1}{9} x^9 b d^3 e^3+\frac{1}{13} x^{13} a d^3 f^3+\frac{1}{3} x^3 b c^3 e^3+x^9 b c^2 d e f^2+x^9 b$
parallelrisch	$\frac{1}{7} x^7 c^3 a f^3+\frac{1}{7} x^7 a d^3 e^3+\frac{1}{9} x^9 b c^3 f^3+\frac{1}{9} x^9 b d^3 e^3+\frac{1}{13} x^{13} a d^3 f^3+\frac{1}{3} x^3 b c^3 e^3+x^9 b c^2 d e f^2+x^9 b$

```
[In] int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*b*d^3*f^3*x^15+1/13*((a*d^3+3*b*c*d^2)*f^3+3*b*d^3*e*f^2)*x^13+1/11*((
3*a*c*d^2+3*b*c^2*d)*f^3+3*(a*d^3+3*b*c*d^2)*e*f^2+3*b*d^3*e^2*f)*x^11+1/9*
((3*a*c^2*d+b*c^3)*f^3+3*(3*a*c*d^2+3*b*c^2*d)*e*f^2+3*(a*d^3+3*b*c*d^2)*e^
2*f+b*d^3*e^3)*x^9+1/7*(c^3*a*f^3+3*(3*a*c^2*d+b*c^3)*e*f^2+3*(3*a*c*d^2+3*
b*c^2*d)*e^2*f+(a*d^3+3*b*c*d^2)*e^3)*x^7+1/5*(3*c^3*a*e*f^2+3*(3*a*c^2*d+b
*c^3)*e^2*f+(3*a*c*d^2+3*b*c^2*d)*e^3)*x^5+1/3*(3*c^3*a*e^2*f+(3*a*c^2*d+b*
c^3)*e^3)*x^3+a*c^3*e^3*x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.05

$$\begin{aligned}
&\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^3\,dx \\
&= \frac{1}{15}\,bd^3f^3x^{15}+\frac{1}{13}\left(3\,bd^3ef^2+\left(3\,bcd^2+ad^3\right)f^3\right)x^{13} \\
&\quad+\frac{3}{11}\left(bd^3e^2f+\left(3\,bcd^2+ad^3\right)ef^2+\left(bc^2d+acd^2\right)f^3\right)x^{11} \\
&\quad+\frac{1}{9}\left(bd^3e^3+3\left(3\,bcd^2+ad^3\right)e^2f+9\left(bc^2d+acd^2\right)ef^2+\left(bc^3+3\,ac^2d\right)f^3\right)x^9+ac^3e^3x \\
&\quad+\frac{1}{7}\left(ac^3f^3+\left(3\,bcd^2+ad^3\right)e^3+9\left(bc^2d+acd^2\right)e^2f+3\left(bc^3+3\,ac^2d\right)ef^2\right)x^7 \\
&\quad+\frac{3}{5}\left(ac^3ef^2+\left(bc^2d+acd^2\right)e^3+\left(bc^3+3\,ac^2d\right)e^2f\right)x^5 \\
&\quad+\frac{1}{3}\left(3\,ac^3e^2f+\left(bc^3+3\,ac^2d\right)e^3\right)x^3
\end{aligned}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] 1/15*b*d^3*f^3*x^15 + 1/13*(3*b*d^3*e*f^2 + (3*b*c*d^2 + a*d^3)*f^3)*x^13 +
 3/11*(b*d^3*e^2*f + (3*b*c*d^2 + a*d^3)*e*f^2 + (b*c^2*d + a*c*d^2)*f^3)*x
```

```
^11 + 1/9*(b*d^3*e^3 + 3*(3*b*c*d^2 + a*d^3)*e^2*f + 9*(b*c^2*d + a*c*d^2)*
e*f^2 + (b*c^3 + 3*a*c^2*d)*f^3)*x^9 + a*c^3*e^3*x + 1/7*(a*c^3*f^3 + (3*b*
c*d^2 + a*d^3)*e^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f + 3*(b*c^3 + 3*a*c^2*d)*e*
f^2)*x^7 + 3/5*(a*c^3*e*f^2 + (b*c^2*d + a*c*d^2)*e^3 + (b*c^3 + 3*a*c^2*d)
*e^2*f)*x^5 + 1/3*(3*a*c^3*e^2*f + (b*c^3 + 3*a*c^2*d)*e^3)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.36

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx = ac^3e^3x+\frac{bd^3f^3x^{15}}{15}+x^{13}\left(\frac{ad^3f^3}{13}+\frac{3bcd^2f^3}{13}+\frac{3bd^3ef^2}{13}\right) + x^{11}\cdot\left(\frac{3acd^2f^3}{11}+\frac{3ad^3ef^2}{11}+\frac{3bc^2df^3}{11}+\frac{9bcd^2ef^2}{11}+\frac{3bd^3e^2f}{11}\right)+x^9\left(\frac{ac^2df^3}{3}+acd^2ef^2+\frac{ad^3e^2f}{3}+\frac{bc^3f^3}{9}+bc^2def^2+bcd^2e^2f+\frac{bd^3e^3}{9}\right) + x^7\left(\frac{ac^3f^3}{7}+\frac{9ac^2def^2}{7}+\frac{9acd^2e^2f}{7}+\frac{ad^3e^3}{7}+\frac{3bc^3ef^2}{7}+\frac{9bc^2de^2f}{7}+\frac{3bcd^2e^3}{7}\right) + x^5\cdot\left(\frac{3ac^3ef^2}{5}+\frac{9ac^2de^2f}{5}+\frac{3acd^2e^3}{5}+\frac{3bc^3e^2f}{5}+\frac{3bc^2de^3}{5}\right)+x^3\left(ac^3e^2f+ac^2de^3+\frac{bc^3e^3}{3}\right)$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e)**3,x)
```

```
[Out] a*c**3*e**3*x + b*d**3*f**3*x**15/15 + x**13*(a*d**3*f**3/13 + 3*b*c*d**2*f
**3/13 + 3*b*d**3*e*f**2/13) + x**11*(3*a*c*d**2*f**3/11 + 3*a*d**3*e*f**2/
11 + 3*b*c**2*d*f**3/11 + 9*b*c*d**2*e*f**2/11 + 3*b*d**3*e**2*f/11) + x**9
*(a*c**2*d*f**3/3 + a*c*d**2*e*f**2 + a*d**3*e**2*f/3 + b*c**3*f**3/9 + b*c
**2*d*e*f**2 + b*c*d**2*e**2*f + b*d**3*e**3/9) + x**7*(a*c**3*f**3/7 + 9*a
*c**2*d*e*f**2/7 + 9*a*c*d**2*e**2*f/7 + a*d**3*e**3/7 + 3*b*c**3*e*f**2/7
+ 9*b*c**2*d*e**2*f/7 + 3*b*c*d**2*e**3/7) + x**5*(3*a*c**3*e*f**2/5 + 9*a*
c**2*d*e**2*f/5 + 3*a*c*d**2*e**3/5 + 3*b*c**3*e**2*f/5 + 3*b*c**2*d*e**3/5
) + x**3*(a*c**3*e**2*f + a*c**2*d*e**3 + b*c**3*e**3/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.05

$$\begin{aligned}
&\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx \\
&= \frac{1}{15}bd^3f^3x^{15} + \frac{1}{13}\left(3\,bd^3ef^2 + \left(3\,bcd^2 + ad^3\right)f^3\right)x^{13} \\
&\quad + \frac{3}{11}\left(bd^3e^2f + \left(3\,bcd^2 + ad^3\right)ef^2 + \left(bc^2d + acd^2\right)f^3\right)x^{11} \\
&\quad + \frac{1}{9}\left(bd^3e^3 + 3\left(3\,bcd^2 + ad^3\right)e^2f + 9\left(bc^2d + acd^2\right)ef^2 + \left(bc^3 + 3\,ac^2d\right)f^3\right)x^9 + ac^3e^3x \\
&\quad + \frac{1}{7}\left(ac^3f^3 + \left(3\,bcd^2 + ad^3\right)e^3 + 9\left(bc^2d + acd^2\right)e^2f + 3\left(bc^3 + 3\,ac^2d\right)ef^2\right)x^7 \\
&\quad + \frac{3}{5}\left(ac^3ef^2 + \left(bc^2d + acd^2\right)e^3 + \left(bc^3 + 3\,ac^2d\right)e^2f\right)x^5 \\
&\quad + \frac{1}{3}\left(3\,ac^3e^2f + \left(bc^3 + 3\,ac^2d\right)e^3\right)x^3
\end{aligned}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b*d^3*f^3*x^15 + 1/13*(3*b*d^3*e*f^2 + (3*b*c*d^2 + a*d^3)*f^3)*x^13 +
 3/11*(b*d^3*e^2*f + (3*b*c*d^2 + a*d^3)*e*f^2 + (b*c^2*d + a*c*d^2)*f^3)*x
^11 + 1/9*(b*d^3*e^3 + 3*(3*b*c*d^2 + a*d^3)*e^2*f + 9*(b*c^2*d + a*c*d^2)*
e*f^2 + (b*c^3 + 3*a*c^2*d)*f^3)*x^9 + a*c^3*e^3*x + 1/7*(a*c^3*f^3 + (3*b*
c*d^2 + a*d^3)*e^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f + 3*(b*c^3 + 3*a*c^2*d)*e*
f^2)*x^7 + 3/5*(a*c^3*e*f^2 + (b*c^2*d + a*c*d^2)*e^3 + (b*c^3 + 3*a*c^2*d)
*e^2*f)*x^5 + 1/3*(3*a*c^3*e^2*f + (b*c^3 + 3*a*c^2*d)*e^3)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.32

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)^3\,dx = \frac{1}{15}bd^3f^3x^{15}+\frac{3}{13}bd^3ef^2x^{13}+\frac{3}{13}bcd^2f^3x^{13}+\frac{1}{13}ad^3f^3x^{13}+\frac{3}{11}bd^3e^2fx^{11}+\frac{9}{11}bcd^2ef^2x^{11}+\frac{3}{11}ad^3ef^2x^{11}+\frac{3}{11}bc^2df^3x^{11}+\frac{3}{11}acd^2f^3x^{11}+\frac{1}{9}bd^3e^3x^9+bcd^2e^2fx^9+\frac{1}{3}ad^3e^2fx^9+bc^2def^2x^9+acd^2ef^2x^9+\frac{1}{9}bc^3f^3x^9+\frac{1}{3}ac^2df^3x^9+\frac{3}{7}bcd^2e^3x^7+\frac{1}{7}ad^3e^3x^7+\frac{9}{7}bc^2de^2fx^7+\frac{9}{7}acd^2e^2fx^7+\frac{3}{7}bc^3ef^2x^7+\frac{9}{7}ac^2def^2x^7+\frac{1}{7}ac^3f^3x^7+\frac{3}{5}bc^2de^3x^5+\frac{3}{5}acd^2e^3x^5+\frac{3}{5}bc^3e^2fx^5+\frac{9}{5}ac^2de^2fx^5+\frac{3}{5}ac^3ef^2x^5+\frac{1}{3}bc^3e^3x^3+ac^2de^3x^3+ac^3e^2fx^3+ac^3e^3x$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/15*b*d^3*f^3*x^15 + 3/13*b*d^3*e*f^2*x^13 + 3/13*b*c*d^2*f^3*x^13 + 1/13*
a*d^3*f^3*x^13 + 3/11*b*d^3*e^2*f*x^11 + 9/11*b*c*d^2*e*f^2*x^11 + 3/11*a*d
^3*e*f^2*x^11 + 3/11*b*c^2*d*f^3*x^11 + 3/11*a*c*d^2*f^3*x^11 + 1/9*b*d^3*e
^3*x^9 + b*c*d^2*e^2*f*x^9 + 1/3*a*d^3*e^2*f*x^9 + b*c^2*d*e*f^2*x^9 + a*c*
d^2*e*f^2*x^9 + 1/9*b*c^3*f^3*x^9 + 1/3*a*c^2*d*f^3*x^9 + 3/7*b*c*d^2*e^3*x
^7 + 1/7*a*d^3*e^3*x^7 + 9/7*b*c^2*d*e^2*f*x^7 + 9/7*a*c*d^2*e^2*f*x^7 + 3/
7*b*c^3*e*f^2*x^7 + 9/7*a*c^2*d*e*f^2*x^7 + 1/7*a*c^3*f^3*x^7 + 3/5*b*c^2*d
*e^3*x^5 + 3/5*a*c*d^2*e^3*x^5 + 3/5*b*c^3*e^2*f*x^5 + 9/5*a*c^2*d*e^2*f*x^
5 + 3/5*a*c^3*e*f^2*x^5 + 1/3*b*c^3*e^3*x^3 + a*c^2*d*e^3*x^3 + a*c^3*e^2*f
*x^3 + a*c^3*e^3*x
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 335, normalized size of antiderivative = 1.08

$$\int (a+bx^2) (c+dx^2)^3 (e+fx^2)^3 \, dx = x^5 \left(\frac{3\,b\,c^3\,e^2\,f}{5}+\frac{3\,a\,c^3\,e\,f^2}{5}+\frac{3\,b\,c^2\,d\,e^3}{5}+\frac{9\,a\,c^2\,d\,e^2\,f}{5}+\frac{3\,a\,c\,d^2\,e^3}{5}\right)+x^{11}\left(\frac{3\,b\,c^2\,d\,f^3}{11}+\frac{9\,b\,c\,d^2\,e\,f^2}{11}+\frac{3\,a\,c\,d^2\,f^3}{11}+\frac{3\,b\,d^3\,e^2\,f}{11}+\frac{3\,a\,d^3\,e\,f^2}{11}\right)+x^7\left(\frac{3\,b\,c^3\,e\,f^2}{7}+\frac{a\,c^3\,f^3}{7}+\frac{9\,b\,c^2\,d\,e^2\,f}{7}+\frac{9\,a\,c^2\,d\,e\,f^2}{7}+\frac{3\,b\,c\,d^2\,e^3}{7}+\frac{9\,a\,c\,d^2\,e^2\,f}{7}+\frac{a\,d^3\,e^3}{7}\right)+x^9\left(\frac{b\,c^3\,f^3}{9}+b\,c^2\,d\,e\,f^2+\frac{a\,c^2\,d\,f^3}{3}+b\,c\,d^2\,e^2\,f+a\,c\,d^2\,e\,f^2+\frac{b\,d^3\,e^3}{9}+\frac{a\,d^3\,e^2\,f}{3}\right)+\frac{b\,d^3\,f^3\,x^{15}}{15}+\frac{c^2\,e^2\,x^3\,(3\,a\,c\,f+3\,a\,d\,e+b\,c\,e)}{3}+\frac{d^2\,f^2\,x^{13}\,(a\,d\,f+3\,b\,c\,f+3\,b\,d\,e)}{13}+a\,c^3\,e^3\,x$$

```
[In] int((a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^3,x)
```

```
[Out] x^5*((3*a*c*d^2*e^3)/5 + (3*b*c^2*d*e^3)/5 + (3*a*c^3*e*f^2)/5 + (3*b*c^3*e
^2*f)/5 + (9*a*c^2*d*e^2*f)/5) + x^11*((3*a*c*d^2*f^3)/11 + (3*b*c^2*d*f^3)
/11 + (3*a*d^3*e*f^2)/11 + (3*b*d^3*e^2*f)/11 + (9*b*c*d^2*e*f^2)/11) + x^7
*((a*c^3*f^3)/7 + (a*d^3*e^3)/7 + (3*b*c*d^2*e^3)/7 + (3*b*c^3*e*f^2)/7 + (
9*a*c*d^2*e^2*f)/7 + (9*a*c^2*d*e*f^2)/7 + (9*b*c^2*d*e^2*f)/7) + x^9*((b*c
^3*f^3)/9 + (b*d^3*e^3)/9 + (a*c^2*d*f^3)/3 + (a*d^3*e^2*f)/3 + a*c*d^2*e*f
^2 + b*c*d^2*e^2*f + b*c^2*d*e*f^2) + (b*d^3*f^3*x^15)/15 + (c^2*e^2*x^3*(3
*a*c*f + 3*a*d*e + b*c*e))/3 + (d^2*f^2*x^13*(a*d*f + 3*b*c*f + 3*b*d*e))/1
3 + a*c^3*e^3*x
```

3.17 $\int (a+bx^2)(c+dx^2)^3(e+fx^2)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 226

$$\begin{aligned}\int (a+bx^2)(c+dx^2)^3(e+fx^2)^2\,dx &= ac^3e^2x+\frac{1}{3}c^2e(bce+3ade+2acf)x^3\\ &\quad+\frac{1}{5}c\big(bce(3de+2cf)+a\big(3d^2e^2+6cdef+c^2f^2\big)\big)x^5\\ &\quad+\frac{1}{7}\big(bc\big(3d^2e^2+6cdef+c^2f^2\big)\\ &\qquad+ad\big(d^2e^2+6cdef+3c^2f^2\big)\big)x^7\\ &\quad+\frac{1}{9}d\big(adf(2de+3cf)+b\big(d^2e^2+6cdef+3c^2f^2\big)\big)x^9\\ &\quad+\frac{1}{11}d^2f(2bde+3bcf+adf)x^{11}+\frac{1}{13}bd^3f^2x^{13}\end{aligned}$$

```
[Out] a*c^3*e^2*x+1/3*c^2*e*(2*a*c*f+3*a*d*e+b*c*e)*x^3+1/5*c*(b*c*e*(2*c*f+3*d*e
)+a*(c^2*f^2+6*c*d*e*f+3*d^2*e^2))*x^5+1/7*(b*c*(c^2*f^2+6*c*d*e*f+3*d^2*e^
2)+a*d*(3*c^2*f^2+6*c*d*e*f+d^2*e^2))*x^7+1/9*d*(a*d*f*(3*c*f+2*d*e)+b*(3*c
^2*f^2+6*c*d*e*f+d^2*e^2))*x^9+1/11*d^2*f*(a*d*f+3*b*c*f+2*b*d*e)*x^11+1/13
*b*d^3*f^2*x^13
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used

$= \{535\}$

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx = \frac{1}{9}dx^9\left(adf(3cf+2de)+b\left(3c^2f^2+6cdef+d^2e^2\right)\right) + \frac{1}{7}x^7\left(ad\left(3c^2f^2+6cdef+d^2e^2\right) + bc\left(c^2f^2+6cdef+3d^2e^2\right)\right) + \frac{1}{5}cx^5\left(a\left(c^2f^2+6cdef+3d^2e^2\right)+bce(2cf+3de)\right) + \frac{1}{3}c^2ex^3(2acf+3ade+bce) + \frac{1}{11}d^2fx^{11}(adf+3bcf+2bde) + ac^3e^2x+\frac{1}{13}bd^3f^2x^{13}$$

```
[In] Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^2,x]
```

```
[Out] a*c^3*e^2*x + (c^2*e*(b*c*e + 3*a*d*e + 2*a*c*f)*x^3)/3 + (c*(b*c*e*(3*d*e
+ 2*c*f) + a*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^5)/5 + ((b*c*(3*d^2*e^2 +
 6*c*d*e*f + c^2*f^2) + a*d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + (d*
(a*d*f*(2*d*e + 3*c*f) + b*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^9)/9 + (d^2
*f*(2*b*d*e + 3*b*c*f + a*d*f)*x^11)/11 + (b*d^3*f^2*x^13)/13
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\text{integral} = \int \left(ac^3e^2+c^2e(bce+3ade+2acf)x^2+c\left(bce(3de+2cf)+a\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^4 + \left(bc\left(3d^2e^2+6cdef+c^2f^2\right)+ad\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^6 + d\left(adf(2de+3cf)+b\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^8+d^2f(2bde+3bcf+adf)x^{10} + bd^3f^2x^{12}\right)\,dx$$

$$= ac^3e^2x+\frac{1}{3}c^2e(bce+3ade+2acf)x^3+\frac{1}{5}c\left(bce(3de+2cf)+a\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^5 + \frac{1}{7}\left(bc\left(3d^2e^2+6cdef+c^2f^2\right)+ad\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^7 + \frac{1}{9}d\left(adf(2de+3cf)+b\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^9 + \frac{1}{11}d^2f(2bde+3bcf+adf)x^{11}+\frac{1}{13}bd^3f^2x^{13}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^2 dx = ac^3e^2x+\frac{1}{3}c^2e(bce+3ade+2acf)x^3 +\frac{1}{5}c\left(bce(3de+2cf)+a\left(3d^2e^2+6cdef+c^2f^2\right)\right)x^5 +\frac{1}{7}\left(bc\left(3d^2e^2+6cdef+c^2f^2\right) +ad\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^7 +\frac{1}{9}d\left(adf(2de+3cf)+b\left(d^2e^2+6cdef+3c^2f^2\right)\right)x^9 +\frac{1}{11}d^2f(2bde+3bcf+adf)x^{11}+\frac{1}{13}bd^3f^2x^{13}$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^2,x]
```

```
[Out] a*c^3*e^2*x + (c^2*e*(b*c*e + 3*a*d*e + 2*a*c*f)*x^3)/3 + (c*(b*c*e*(3*d*e
+ 2*c*f) + a*(3*d^2*e^2 + 6*c*d*e*f + c^2*f^2))*x^5)/5 + ((b*c*(3*d^2*e^2 +
 6*c*d*e*f + c^2*f^2) + a*d*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^7)/7 + (d*
(a*d*f*(2*d*e + 3*c*f) + b*(d^2*e^2 + 6*c*d*e*f + 3*c^2*f^2))*x^9)/9 + (d^2
*f*(2*b*d*e + 3*b*c*f + a*d*f)*x^11)/11 + (b*d^3*f^2*x^13)/13
```

Maple [A] (verified)

Time = 3.28 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.08

method	result
default	$\frac{b d^3 f^2 x^{13}}{13}+\frac{\left(\left(a d^3+3bc d^2\right) f^2+2b d^3 e f\right) x^{11}}{11}+\frac{\left(\left(3ac d^2+3b c^2 d\right) f^2+2\left(a d^3+3bc d^2\right) e f+b d^3 e^2\right) x^9}{9}+\frac{((3a c^2 d+c^3 b) f^2+2$
norman	$\frac{b d^3 f^2 x^{13}}{13}+\left(\frac{1}{11} a d^3 f^2+\frac{3}{11} bc d^2 f^2+\frac{2}{11} b d^3 e f\right) x^{11}+\left(\frac{1}{3} ac d^2 f^2+\frac{2}{9} a d^3 e f+\frac{1}{3} b c^2 d f^2+\frac{2}{3} bc d^2$
gosper	$\frac{1}{13} b d^3 f^2 x^{13}+\frac{1}{11} x^{11} a d^3 f^2+\frac{3}{11} x^{11} bc d^2 f^2+\frac{2}{11} x^{11} b d^3 e f+\frac{1}{3} x^9 ac d^2 f^2+\frac{2}{9} x^9 a d^3 e f+\frac{1}{3} x^9 b c^2$
risch	$\frac{1}{13} b d^3 f^2 x^{13}+\frac{1}{11} x^{11} a d^3 f^2+\frac{3}{11} x^{11} bc d^2 f^2+\frac{2}{11} x^{11} b d^3 e f+\frac{1}{3} x^9 ac d^2 f^2+\frac{2}{9} x^9 a d^3 e f+\frac{1}{3} x^9 b c^2$
parallelrisch	$\frac{1}{13} b d^3 f^2 x^{13}+\frac{1}{11} x^{11} a d^3 f^2+\frac{3}{11} x^{11} bc d^2 f^2+\frac{2}{11} x^{11} b d^3 e f+\frac{1}{3} x^9 ac d^2 f^2+\frac{2}{9} x^9 a d^3 e f+\frac{1}{3} x^9 b c^2$

```
[In] int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*b*d^3*f^2*x^13+1/11*((a*d^3+3*b*c*d^2)*f^2+2*b*d^3*e*f)*x^11+1/9*((3*a
*c*d^2+3*b*c^2*d)*f^2+2*(a*d^3+3*b*c*d^2)*e*f+b*d^3*e^2)*x^9+1/7*((3*a*c^2*
d+b*c^3)*f^2+2*(3*a*c*d^2+3*b*c^2*d)*e*f+(a*d^3+3*b*c*d^2)*e^2)*x^7+1/5*(c^
3*a*f^2+2*(3*a*c^2*d+b*c^3)*e*f+(3*a*c*d^2+3*b*c^2*d)*e^2)*x^5+1/3*(2*c^3*a
*e*f+(3*a*c^2*d+b*c^3)*e^2)*x^3+a*c^3*e^2*x
```

Fricas [A] (verification not implemented)

none

Time = 0.33 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx$$
$$=\frac{1}{13}bd^3f^2x^{13}+\frac{1}{11}\left(2\,bd^3ef+\left(3\,bcd^2+ad^3\right)f^2\right)x^{11}+\frac{1}{9}\left(bd^3e^2+2\left(3\,bcd^2+ad^3\right)ef+3\left(bc^2d+acd^2\right)f^2\right)x^9+\frac{1}{7}\left(\left(3\,bcd^2+ad^3\right)e^2+6\left(bc^2d+acd^2\right)ef+\left(bc^3+3\,ac^2d\right)f^2\right)x^7+ac^3e^2x+\frac{1}{5}\left(ac^3f^2+3\left(bc^2d+acd^2\right)e^2+2\left(bc^3+3\,ac^2d\right)ef\right)x^5+\frac{1}{3}\left(2\,ac^3ef+\left(bc^3+3\,ac^2d\right)e^2\right)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] 1/13*b*d^3*f^2*x^13 + 1/11*(2*b*d^3*e*f + (3*b*c*d^2 + a*d^3)*f^2)*x^11 + 1
/9*(b*d^3*e^2 + 2*(3*b*c*d^2 + a*d^3)*e*f + 3*(b*c^2*d + a*c*d^2)*f^2)*x^9
+ 1/7*((3*b*c*d^2 + a*d^3)*e^2 + 6*(b*c^2*d + a*c*d^2)*e*f + (b*c^3 + 3*a*c
^2*d)*f^2)*x^7 + a*c^3*e^2*x + 1/5*(a*c^3*f^2 + 3*(b*c^2*d + a*c*d^2)*e^2 +
 2*(b*c^3 + 3*a*c^2*d)*e*f)*x^5 + 1/3*(2*a*c^3*e*f + (b*c^3 + 3*a*c^2*d)*e^
2)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.35

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx=ac^3e^2x+\frac{bd^3f^2x^{13}}{13}+x^{11}\left(\frac{ad^3f^2}{11}+\frac{3bcd^2f^2}{11}+\frac{2bd^3ef}{11}\right)+x^9\left(\frac{acd^2f^2}{3}+\frac{2ad^3ef}{9}+\frac{bc^2df^2}{3}+\frac{2bcd^2ef}{3}+\frac{bd^3e^2}{9}\right)+x^7\cdot\left(\frac{3ac^2df^2}{7}+\frac{6acd^2ef}{7}+\frac{ad^3e^2}{7}+\frac{bc^3f^2}{7}+\frac{6bc^2def}{7}+\frac{3bcd^2e^2}{7}\right)+x^5\left(\frac{ac^3f^2}{5}+\frac{6ac^2def}{5}+\frac{3acd^2e^2}{5}+\frac{2bc^3ef}{5}+\frac{3bc^2de^2}{5}\right)+x^3\cdot\left(\frac{2ac^3ef}{3}+ac^2de^2+\frac{bc^3e^2}{3}\right)$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e)**2,x)
```

```
[Out] a*c**3*e**2*x + b*d**3*f**2*x**13/13 + x**11*(a*d**3*f**2/11 + 3*b*c*d**2*f
**2/11 + 2*b*d**3*e*f/11) + x**9*(a*c*d**2*f**2/3 + 2*a*d**3*e*f/9 + b*c**2
*d*f**2/3 + 2*b*c*d**2*e*f/3 + b*d**3*e**2/9) + x**7*(3*a*c**2*d*f**2/7 + 6
*a*c*d**2*e*f/7 + a*d**3*e**2/7 + b*c**3*f**2/7 + 6*b*c**2*d*e*f/7 + 3*b*c*
d**2*e**2/7) + x**5*(a*c**3*f**2/5 + 6*a*c**2*d*e*f/5 + 3*a*c*d**2*e**2/5 +
 2*b*c**3*e*f/5 + 3*b*c**2*d*e**2/5) + x**3*(2*a*c**3*e*f/3 + a*c**2*d*e**2
 + b*c**3*e**2/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

$$\begin{aligned}
&\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx \\
&= \frac{1}{13}bd^3f^2x^{13} + \frac{1}{11}\left(2\,bd^3ef + \left(3\,bcd^2+ad^3\right)f^2\right)x^{11} \\
&\quad + \frac{1}{9}\left(bd^3e^2 + 2\left(3\,bcd^2+ad^3\right)ef + 3\left(bc^2d+acd^2\right)f^2\right)x^9 \\
&\quad + \frac{1}{7}\left(\left(3\,bcd^2+ad^3\right)e^2 + 6\left(bc^2d+acd^2\right)ef + \left(bc^3+3\,ac^2d\right)f^2\right)x^7 \\
&\quad + ac^3e^2x + \frac{1}{5}\left(ac^3f^2 + 3\left(bc^2d+acd^2\right)e^2 + 2\left(bc^3+3\,ac^2d\right)ef\right)x^5 \\
&\quad + \frac{1}{3}\left(2\,ac^3ef + \left(bc^3+3\,ac^2d\right)e^2\right)x^3
\end{aligned}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] 1/13*b*d^3*f^2*x^13 + 1/11*(2*b*d^3*e*f + (3*b*c*d^2 + a*d^3)*f^2)*x^11 + 1
/9*(b*d^3*e^2 + 2*(3*b*c*d^2 + a*d^3)*e*f + 3*(b*c^2*d + a*c*d^2)*f^2)*x^9
+ 1/7*((3*b*c*d^2 + a*d^3)*e^2 + 6*(b*c^2*d + a*c*d^2)*e*f + (b*c^3 + 3*a*c
^2*d)*f^2)*x^7 + a*c^3*e^2*x + 1/5*(a*c^3*f^2 + 3*(b*c^2*d + a*c*d^2)*e^2 +
 2*(b*c^3 + 3*a*c^2*d)*e*f)*x^5 + 1/3*(2*a*c^3*e*f + (b*c^3 + 3*a*c^2*d)*e^
2)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.28

$$\int (a+bx^2)\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx = \frac{1}{13}bd^3f^2x^{13}+\frac{2}{11}bd^3efx^{11}+\frac{3}{11}bcd^2f^2x^{11}+\frac{1}{11}ad^3f^2x^{11}+\frac{1}{9}bd^3e^2x^9+\frac{2}{3}bcd^2efx^9+\frac{2}{9}ad^3efx^9+\frac{1}{3}bc^2df^2x^9+\frac{1}{3}acd^2f^2x^9+\frac{3}{7}bcd^2e^2x^7+\frac{1}{7}ad^3e^2x^7+\frac{6}{7}bc^2defx^7+\frac{6}{7}acd^2efx^7+\frac{1}{7}bc^3f^2x^7+\frac{3}{7}ac^2df^2x^7+\frac{3}{5}bc^2de^2x^5+\frac{3}{5}acd^2e^2x^5+\frac{2}{5}bc^3efx^5+\frac{6}{5}ac^2defx^5+\frac{1}{5}ac^3f^2x^5+\frac{1}{3}bc^3e^2x^3+ac^2de^2x^3+\frac{2}{3}ac^3efx^3+ac^3e^2x$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] 1/13*b*d^3*f^2*x^13 + 2/11*b*d^3*e*f*x^11 + 3/11*b*c*d^2*f^2*x^11 + 1/11*a*
d^3*f^2*x^11 + 1/9*b*d^3*e^2*x^9 + 2/3*b*c*d^2*e*f*x^9 + 2/9*a*d^3*e*f*x^9
+ 1/3*b*c^2*d*f^2*x^9 + 1/3*a*c*d^2*f^2*x^9 + 3/7*b*c*d^2*e^2*x^7 + 1/7*a*d
^3*e^2*x^7 + 6/7*b*c^2*d*e*f*x^7 + 6/7*a*c*d^2*e*f*x^7 + 1/7*b*c^3*f^2*x^7
+ 3/7*a*c^2*d*f^2*x^7 + 3/5*b*c^2*d*e^2*x^5 + 3/5*a*c*d^2*e^2*x^5 + 2/5*b*c
^3*e*f*x^5 + 6/5*a*c^2*d*e*f*x^5 + 1/5*a*c^3*f^2*x^5 + 1/3*b*c^3*e^2*x^3 +
a*c^2*d*e^2*x^3 + 2/3*a*c^3*e*f*x^3 + a*c^3*e^2*x
```

Mupad [B] (verification not implemented)

Time = 5.18 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.03

$$\int (a+bx^2)\left(c+dx^2\right)^3\left(e+fx^2\right)^2\,dx = x^5\left(\frac{2\,b\,c^3\,e\,f}{5}+\frac{a\,c^3\,f^2}{5}+\frac{3\,b\,c^2\,d\,e^2}{5}+\frac{6\,a\,c^2\,d\,e\,f}{5}+\frac{3\,a\,c\,d^2\,e^2}{5}\right)+x^9\left(\frac{b\,c^2\,d\,f^2}{3}+\frac{2\,b\,c\,d^2\,e\,f}{3}+\frac{a\,c\,d^2\,f^2}{3}+\frac{b\,d^3\,e^2}{9}+\frac{2\,a\,d^3\,e\,f}{9}\right)+x^7\left(\frac{b\,c^3\,f^2}{7}+\frac{6\,b\,c^2\,d\,e\,f}{7}+\frac{3\,a\,c^2\,d\,f^2}{7}+\frac{3\,b\,c\,d^2\,e^2}{7}+\frac{6\,a\,c\,d^2\,e\,f}{7}+\frac{a\,d^3\,e^2}{7}\right)+\frac{b\,d^3\,f^2\,x^{13}}{13}+\frac{c^2\,e\,x^3\,\left(2\,a\,c\,f+3\,a\,d\,e+b\,c\,e\right)}{3}+\frac{d^2\,f\,x^{11}\,\left(a\,d\,f+3\,b\,c\,f+2\,b\,d\,e\right)}{11}+a\,c^3\,e^2\,x$$

```
[In] int((a + b*x^2)*(c + d*x^2)^3*(e + f*x^2)^2,x)
```

```
[Out] x^5*((a*c^3*f^2)/5 + (2*b*c^3*e*f)/5 + (3*a*c*d^2*e^2)/5 + (3*b*c^2*d*e^2)/
5 + (6*a*c^2*d*e*f)/5) + x^9*((b*d^3*e^2)/9 + (2*a*d^3*e*f)/9 + (a*c*d^2*f^
2)/3 + (b*c^2*d*f^2)/3 + (2*b*c*d^2*e*f)/3) + x^7*((a*d^3*e^2)/7 + (b*c^3*f
^2)/7 + (3*a*c^2*d*f^2)/7 + (3*b*c*d^2*e^2)/7 + (6*a*c*d^2*e*f)/7 + (6*b*c^
2*d*e*f)/7) + (b*d^3*f^2*x^13)/13 + (c^2*e*x^3*(2*a*c*f + 3*a*d*e + b*c*e))
/3 + (d^2*f*x^11*(a*d*f + 3*b*c*f + 2*b*d*e))/11 + a*c^3*e^2*x
```

3.18 $\int (a+bx^2)(c+dx^2)^3(e+fx^2)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)\,dx = ac^3ex+\frac{1}{3}c^2(bce+3ade+acf)x^3 +\frac{1}{5}c(3ad(de+cf)+bc(3de+cf))x^5 +\frac{1}{7}d(3bc(de+cf)+ad(de+3cf))x^7 +\frac{1}{9}d^2(bde+3bcf+adf)x^9+\frac{1}{11}bd^3fx^{11}$$

```
[Out] a*c^3*e*x+1/3*c^2*(a*c*f+3*a*d*e+b*c*e)*x^3+1/5*c*(3*a*d*(c*f+d*e)+b*c*(c*f
+3*d*e))*x^5+1/7*d*(3*b*c*(c*f+d*e)+a*d*(3*c*f+d*e))*x^7+1/9*d^2*(a*d*f+3*b
*c*f+b*d*e)*x^9+1/11*b*d^3*f*x^11
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {535}

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)\,dx = \frac{1}{3}c^2x^3(acf+3ade+bce)+\frac{1}{9}d^2x^9(adf+3bcf+bde) +\frac{1}{7}dx^7(ad(3cf+de)+3bc(cf+de)) +\frac{1}{5}cx^5(3ad(cf+de)+bc(cf+3de)) +ac^3ex+\frac{1}{11}bd^3fx^{11}$$

```
[In] Int[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2),x]
```

```
[Out] a*c^3*e*x + (c^2*(b*c*e + 3*a*d*e + a*c*f)*x^3)/3 + (c*(3*a*d*(d*e + c*f) +
 b*c*(3*d*e + c*f))*x^5)/5 + (d*(3*b*c*(d*e + c*f) + a*d*(d*e + 3*c*f))*x^7
)/7 + (d^2*(b*d*e + 3*b*c*f + a*d*f)*x^9)/9 + (b*d^3*f*x^11)/11
```

Rule 535

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c +
d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && IGtQ[p
, 0] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \int \left(ac^3e + c^2(bce + 3ade + acf)x^2 + c(3ad(de + cf) + bc(3de + cf))x^4 \right.\\ &\qquad \left. + d(3bc(de + cf) + ad(de + 3cf))x^6 + d^2(bde + 3bcf + adf)x^8 + bd^3fx^{10}\right)\,dx\\ &= ac^3ex + \frac{1}{3}c^2(bce + 3ade + acf)x^3 + \frac{1}{5}c(3ad(de + cf) + bc(3de + cf))x^5\\ &\quad + \frac{1}{7}d(3bc(de + cf) + ad(de + 3cf))x^7 + \frac{1}{9}d^2(bde + 3bcf + adf)x^9 + \frac{1}{11}bd^3fx^{11}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \left(a + bx^2\right)\left(c + dx^2\right)^3\left(e + fx^2\right)\,dx = ac^3ex &+ \frac{1}{3}c^2(bce + 3ade + acf)x^3\\ &+ \frac{1}{5}c(3ad(de + cf) + bc(3de + cf))x^5\\ &+ \frac{1}{7}d(3bc(de + cf) + ad(de + 3cf))x^7\\ &+ \frac{1}{9}d^2(bde + 3bcf + adf)x^9 + \frac{1}{11}bd^3fx^{11}\end{aligned}$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^3*(e + f*x^2),x]
```

```
[Out] a*c^3*e*x + (c^2*(b*c*e + 3*a*d*e + a*c*f)*x^3)/3 + (c*(3*a*d*(d*e + c*f) +
 b*c*(3*d*e + c*f))*x^5)/5 + (d*(3*b*c*(d*e + c*f) + a*d*(d*e + 3*c*f))*x^7
)/7 + (d^2*(b*d*e + 3*b*c*f + a*d*f)*x^9)/9 + (b*d^3*f*x^11)/11
```

Maple [A] (verified)

Time = 3.33 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.11

method	result
norman	$\frac{b\,d^{3} f\,x^{11}}{11}+\left(\frac{1}{9} a\,d^{3} f+\frac{1}{3} b c\,d^{2} f+\frac{1}{9} b\,d^{3} e\right) x^{9}+\left(\frac{3}{7} a c\,d^{2} f+\frac{1}{7} a\,d^{3} e+\frac{3}{7} b\,c^{2} f d+\frac{3}{7} b c e\,d^{2}\right) x^{7}+\left(\frac{3}{5} a\right.$
default	$\frac{b\,d^{3} f\,x^{11}}{11}+\frac{\left(\left(a\,d^{3}+3 b c\,d^{2}\right) f+b\,d^{3} e\right) x^{9}}{9}+\frac{\left(\left(3 a c\,d^{2}+3 b\,c^{2} d\right) f+\left(a\,d^{3}+3 b c\,d^{2}\right) e\right) x^{7}}{7}+\frac{\left(\left(3 a\,c^{2} d+c^{3} b\right) f+\left(3 a c\,d^{2}+3 b\,c^{2} d\right) e\right) x^{5}}{5}$
gosper	$\frac{1}{11} b\,d^{3} f\,x^{11}+\frac{1}{9} x^{9} a\,d^{3} f+\frac{1}{3} x^{9} b c\,d^{2} f+\frac{1}{9} x^{9} b\,d^{3} e+\frac{3}{7} x^{7} a c\,d^{2} f+\frac{1}{7} x^{7} a\,d^{3} e+\frac{3}{7} x^{7} b\,c^{2} f d+\frac{3}{7} x^{7} b c e$
risch	$\frac{1}{11} b\,d^{3} f\,x^{11}+\frac{1}{9} x^{9} a\,d^{3} f+\frac{1}{3} x^{9} b c\,d^{2} f+\frac{1}{9} x^{9} b\,d^{3} e+\frac{3}{7} x^{7} a c\,d^{2} f+\frac{1}{7} x^{7} a\,d^{3} e+\frac{3}{7} x^{7} b\,c^{2} f d+\frac{3}{7} x^{7} b c e$
parallelrisch	$\frac{1}{11} b\,d^{3} f\,x^{11}+\frac{1}{9} x^{9} a\,d^{3} f+\frac{1}{3} x^{9} b c\,d^{2} f+\frac{1}{9} x^{9} b\,d^{3} e+\frac{3}{7} x^{7} a c\,d^{2} f+\frac{1}{7} x^{7} a\,d^{3} e+\frac{3}{7} x^{7} b\,c^{2} f d+\frac{3}{7} x^{7} b c e$

```
[In] int((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*b*d^3*f*x^11+(1/9*a*d^3*f+1/3*b*c*d^2*f+1/9*b*d^3*e)*x^9+(3/7*a*c*d^2*
f+1/7*a*d^3*e+3/7*b*c^2*f*d+3/7*b*c*e*d^2)*x^7+(3/5*a*c^2*d*f+3/5*a*c*d^2*e
+1/5*b*c^3*f+3/5*b*c^2*d*e)*x^5+(1/3*c^3*a*f+a*c^2*d*e+1/3*b*c^3*e)*x^3+a*c
^3*e*x
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.12

$$\int (a+bx^2)(c+dx^2)^3(e+fx^2)\,dx = \frac{1}{11}bd^3fx^{11}+\frac{1}{9}\left(bd^3e+\left(3\,bcd^2+ad^3\right)f\right)x^9 + \frac{1}{7}\left(\left(3\,bcd^2+ad^3\right)e+3\left(bc^2d+acd^2\right)f\right)x^7+ac^3ex + \frac{1}{5}\left(3\left(bc^2d+acd^2\right)e+\left(bc^3+3\,ac^2d\right)f\right)x^5 + \frac{1}{3}\left(ac^3f+\left(bc^3+3\,ac^2d\right)e\right)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="fricas")
```

```
[Out] 1/11*b*d^3*f*x^11 + 1/9*(b*d^3*e + (3*b*c*d^2 + a*d^3)*f)*x^9 + 1/7*((3*b*c
*d^2 + a*d^3)*e + 3*(b*c^2*d + a*c*d^2)*f)*x^7 + a*c^3*e*x + 1/5*(3*(b*c^2*
d + a*c*d^2)*e + (b*c^3 + 3*a*c^2*d)*f)*x^5 + 1/3*(a*c^3*f + (b*c^3 + 3*a*c
^2*d)*e)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.33

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)\,dx = ac^3ex + \frac{bd^3fx^{11}}{11} + x^9\left(\frac{ad^3f}{9} + \frac{bcd^2f}{3} + \frac{bd^3e}{9}\right) + x^7\cdot\left(\frac{3acd^2f}{7} + \frac{ad^3e}{7} + \frac{3bc^2df}{7} + \frac{3bcd^2e}{7}\right) + x^5\cdot\left(\frac{3ac^2df}{5} + \frac{3acd^2e}{5} + \frac{bc^3f}{5} + \frac{3bc^2de}{5}\right) + x^3\left(\frac{ac^3f}{3} + ac^2de + \frac{bc^3e}{3}\right)$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3*(f*x**2+e),x)
```

```
[Out] a*c**3*e*x + b*d**3*f*x**11/11 + x**9*(a*d**3*f/9 + b*c*d**2*f/3 + b*d**3*e
/9) + x**7*(3*a*c*d**2*f/7 + a*d**3*e/7 + 3*b*c**2*d*f/7 + 3*b*c*d**2*e/7)
+ x**5*(3*a*c**2*d*f/5 + 3*a*c*d**2*e/5 + b*c**3*f/5 + 3*b*c**2*d*e/5) + x*
*3*(a*c**3*f/3 + a*c**2*d*e + b*c**3*e/3)
```

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.12

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)\,dx = \frac{1}{11}bd^3fx^{11} + \frac{1}{9}\left(bd^3e + \left(3\,bcd^2 + ad^3\right)f\right)x^9 + \frac{1}{7}\left(\left(3\,bcd^2 + ad^3\right)e + 3\left(bc^2d + acd^2\right)f\right)x^7 + ac^3ex + \frac{1}{5}\left(3\left(bc^2d + acd^2\right)e + \left(bc^3 + 3\,ac^2d\right)f\right)x^5 + \frac{1}{3}\left(ac^3f + \left(bc^3 + 3\,ac^2d\right)e\right)x^3$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="maxima")
```

```
[Out] 1/11*b*d^3*f*x^11 + 1/9*(b*d^3*e + (3*b*c*d^2 + a*d^3)*f)*x^9 + 1/7*((3*b*c
*d^2 + a*d^3)*e + 3*(b*c^2*d + a*c*d^2)*f)*x^7 + a*c^3*e*x + 1/5*(3*(b*c^2*
d + a*c*d^2)*e + (b*c^3 + 3*a*c^2*d)*f)*x^5 + 1/3*(a*c^3*f + (b*c^3 + 3*a*c
^2*d)*e)*x^3
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.27

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)\,dx = \frac{1}{11}bd^3fx^{11}+\frac{1}{9}bd^3ex^9+\frac{1}{3}bcd^2fx^9+\frac{1}{9}ad^3fx^9+\frac{3}{7}bcd^2ex^7+\frac{1}{7}ad^3ex^7+\frac{3}{7}bc^2dfx^7+\frac{3}{7}acd^2fx^7+\frac{3}{5}bc^2dex^5+\frac{3}{5}acd^2ex^5+\frac{1}{5}bc^3fx^5+\frac{3}{5}ac^2dfx^5+\frac{1}{3}bc^3ex^3+ac^2dex^3+\frac{1}{3}ac^3fx^3+ac^3ex$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3*(f*x^2+e),x, algorithm="giac")
```

```
[Out] 1/11*b*d^3*f*x^11 + 1/9*b*d^3*e*x^9 + 1/3*b*c*d^2*f*x^9 + 1/9*a*d^3*f*x^9 +
 3/7*b*c*d^2*e*x^7 + 1/7*a*d^3*e*x^7 + 3/7*b*c^2*d*f*x^7 + 3/7*a*c*d^2*f*x^
7 + 3/5*b*c^2*d*e*x^5 + 3/5*a*c*d^2*e*x^5 + 1/5*b*c^3*f*x^5 + 3/5*a*c^2*d*f
*x^5 + 1/3*b*c^3*e*x^3 + a*c^2*d*e*x^3 + 1/3*a*c^3*f*x^3 + a*c^3*e*x
```

Mupad [B] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^3\left(e+fx^2\right)\,dx = x^5\left(\frac{b\,c^3\,f}{5}+\frac{3\,a\,c\,d^2\,e}{5}+\frac{3\,a\,c^2\,d\,f}{5}+\frac{3\,b\,c^2\,d\,e}{5}\right)+x^7\left(\frac{a\,d^3\,e}{7}+\frac{3\,a\,c\,d^2\,f}{7}+\frac{3\,b\,c\,d^2\,e}{7}+\frac{3\,b\,c^2\,d\,f}{7}\right)+x^3\left(\frac{a\,c^3\,f}{3}+\frac{b\,c^3\,e}{3}+a\,c^2\,d\,e\right)+x^9\left(\frac{a\,d^3\,f}{9}+\frac{b\,d^3\,e}{9}+\frac{b\,c\,d^2\,f}{3}\right)+a\,c^3\,e\,x+\frac{b\,d^3\,f\,x^{11}}{11}$$

```
[In] int((a + b*x^2)*(c + d*x^2)^3*(e + f*x^2),x)
```

```
[Out] x^5*((b*c^3*f)/5 + (3*a*c*d^2*e)/5 + (3*a*c^2*d*f)/5 + (3*b*c^2*d*e)/5) + x
^7*((a*d^3*e)/7 + (3*a*c*d^2*f)/7 + (3*b*c*d^2*e)/7 + (3*b*c^2*d*f)/7) + x^
3*((a*c^3*f)/3 + (b*c^3*e)/3 + a*c^2*d*e) + x^9*((a*d^3*f)/9 + (b*d^3*e)/9
+ (b*c*d^2*f)/3) + a*c^3*e*x + (b*d^3*f*x^11)/11
```

3.19 $\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 227

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2} dx = \frac{(7adf(15d^2e^2-40cdef+33c^2f^2)-b(105d^3e^3-280cd^2e^2f+231c^2def^2-48c^3f^3))x}{105f^4} - \frac{(7adf(5de-9cf)-b(35d^2e^2-63cdef+24c^2f^2))x(c+dx^2)}{105f^3} - \frac{(7bde-6bcf-7adf)x(c+dx^2)^2}{35f^2} + \frac{bx(c+dx^2)^3}{7f} + \frac{(be-af)(de-cf)^3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{9/2}}$$

```
[Out] 1/105*(7*a*d*f*(33*c^2*f^2-40*c*d*e*f+15*d^2*e^2)-b*(-48*c^3*f^3+231*c^2*d*
e*f^2-280*c*d^2*e^2*f+105*d^3*e^3))*x/f^4-1/105*(7*a*d*f*(-9*c*f+5*d*e)-b*(
24*c^2*f^2-63*c*d*e*f+35*d^2*e^2))*x*(d*x^2+c)/f^3-1/35*(-7*a*d*f-6*b*c*f+7
*b*d*e)*x*(d*x^2+c)^2/f^2+1/7*b*x*(d*x^2+c)^3/f+(-a*f+b*e)*(-c*f+d*e)^3*arc
tan(x*f^(1/2)/e^(1/2))/f^(9/2)/e^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used

$= \{542, 396, 211\}$

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx$$
$$= \frac{(be-af)\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^3}{\sqrt{e}f^{9/2}}$$
$$- \frac{x(c+dx^2)\left(7adf(5de-9cf)-b(24c^2f^2-63cdef+35d^2e^2)\right)}{105f^3}$$
$$+ \frac{x\left(7adf(33c^2f^2-40cdef+15d^2e^2)-b(-48c^3f^3+231c^2def^2-280cd^2e^2f+105d^3e^3)\right)}{105f^4}$$
$$- \frac{x(c+dx^2)^2(-7adf-6bcf+7bde)}{35f^2} + \frac{bx(c+dx^2)^3}{7f}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x]
```

```
[Out] ((7*a*d*f*(15*d^2*e^2 - 40*c*d*e*f + 33*c^2*f^2) - b*(105*d^3*e^3 - 280*c*d
^2*e^2*f + 231*c^2*d*e*f^2 - 48*c^3*f^3))*x)/(105*f^4) - ((7*a*d*f*(5*d*e -
 9*c*f) - b*(35*d^2*e^2 - 63*c*d*e*f + 24*c^2*f^2))*x*(c + d*x^2))/(105*f^3
) - ((7*b*d*e - 6*b*c*f - 7*a*d*f)*x*(c + d*x^2)^2)/(35*f^2) + (b*x*(c + d*
x^2)^3)/(7*f) + ((b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sq
rt[e]*f^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx(c+dx^2)^3}{7f} + \frac{\int \frac{(c+dx^2)^2(-c(be-7af)+(-7bde+6bcf+7adf)x^2)}{e+fx^2}\,dx}{7f} \\
&= -\frac{(7bde-6bcf-7adf)x(c+dx^2)^2}{35f^2} + \frac{bx(c+dx^2)^3}{7f} \\
&\quad + \frac{\int \frac{(c+dx^2)(c(be(7de-11cf)-7af(de-5cf))+(-7adf(5de-9cf)+b(35d^2e^2-63cdef+24c^2f^2))x^2)}{e+fx^2}\,dx}{35f^2} \\
&= -\frac{(7adf(5de-9cf)-b(35d^2e^2-63cdef+24c^2f^2))\,x(c+dx^2)}{105f^3} \\
&\quad - \frac{(7bde-6bcf-7adf)x(c+dx^2)^2}{35f^2} + \frac{bx(c+dx^2)^3}{7f} \\
&\quad + \frac{\int \frac{c(7af(5d^2e^2-12cdef+15c^2f^2)-be(35d^2e^2-84cdef+57c^2f^2))+(7adf(15d^2e^2-40cdef+33c^2f^2)-b(105d^3e^3-280cd^2e^2f+231}{e+fx^2}}{105f^3} \\
&= \frac{(7adf(15d^2e^2-40cdef+33c^2f^2)-b(105d^3e^3-280cd^2e^2f+231c^2def^2-48c^3f^3))\,x}{105f^4} \\
&\quad - \frac{(7adf(5de-9cf)-b(35d^2e^2-63cdef+24c^2f^2))\,x(c+dx^2)}{105f^3} \\
&\quad - \frac{(7bde-6bcf-7adf)x(c+dx^2)^2}{35f^2} \\
&\quad + \frac{bx(c+dx^2)^3}{7f} + \frac{((be-af)(de-cf)^3)\int \frac{1}{e+fx^2}\,dx}{f^4} \\
&= \frac{(7adf(15d^2e^2-40cdef+33c^2f^2)-b(105d^3e^3-280cd^2e^2f+231c^2def^2-48c^3f^3))\,x}{105f^4} \\
&\quad - \frac{(7adf(5de-9cf)-b(35d^2e^2-63cdef+24c^2f^2))\,x(c+dx^2)}{105f^3} \\
&\quad - \frac{(7bde-6bcf-7adf)x(c+dx^2)^2}{35f^2} + \frac{bx(c+dx^2)^3}{7f} \\
&\quad + \frac{(be-af)(de-cf)^3\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx = \frac{(-b(de-cf)^3+adf(d^2e^2-3cdef+3c^2f^2))\,x}{f^4} + \frac{d(adf(-de+3cf)+b(d^2e^2-3cdef+3c^2f^2))\,x^3}{3f^3} + \frac{d^2(-bde+3bcf+adf)x^5}{5f^2} + \frac{bd^3x^7}{7f} + \frac{(be-af)(de-cf)^3\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{\sqrt{e}f^{9/2}}$$

[In] Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x]

[Out] ((-(b*(d*e - c*f)^3) + a*d*f*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x)/f^4 + (d *(a*d*f*(-(d*e) + 3*c*f) + b*(d^2*e^2 - 3*c*d*e*f + 3*c^2*f^2))*x^3)/(3*f^3) + (d^2*(-(b*d*e) + 3*b*c*f + a*d*f)*x^5)/(5*f^2) + (b*d^3*x^7)/(7*f) + ((b*e - a*f)*(d*e - c*f)^3*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(Sqrt[e]*f^(9/2))

Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.32

method	result
default	$\frac{\frac{1}{7}b\,d^3x^7f^3+\frac{1}{5}a\,d^3f^3x^5+\frac{3}{5}bc\,d^2f^3x^5-\frac{1}{5}b\,d^3e\,f^2x^5+ac\,d^2f^3x^3-\frac{1}{3}a\,d^3e\,f^2x^3+b\,c^2d\,f^3x^3-bc\,d^2e\,f^2x^3+\frac{1}{3}b\,d^3e^2f\,x^3+3a\,c^2d\,f^3x-3ac\,d}{f^4}$
risch	$\frac{b\,d^3x^7}{7f}+\frac{a\,d^3x^5}{5f}+\frac{b\,c^3x}{f}-\frac{\ln(fx+\sqrt{-ef})\,a\,c^3}{2\sqrt{-ef}}+\frac{ac\,d^2x^3}{f}-\frac{a\,d^3e\,x^3}{3f^2}+\frac{b\,c^2d\,x^3}{f}+\frac{b\,d^3e^2x^3}{3f^3}-\frac{3\ln(-fx+\sqrt{-ef})\,a\,c^2de}{2f\sqrt{-ef}}+$

[In] int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x,method=_RETURNVERBOSE)

[Out] 1/f^4*(1/7*b*d^3*x^7*f^3+1/5*a*d^3*f^3*x^5+3/5*b*c*d^2*f^3*x^5-1/5*b*d^3*e* f^2*x^5+a*c*d^2*f^3*x^3-1/3*a*d^3*e*f^2*x^3+b*c^2*d*f^3*x^3-b*c*d^2*e*f^2*x ^3+1/3*b*d^3*e^2*f*x^3+3*a*c^2*d*f^3*x-3*a*c*d^2*e*f^2*x+a*d^3*e^2*f*x+b*c^ 3*f^3*x-3*b*c^2*d*e*f^2*x+3*b*c*d^2*e^2*f*x-b*d^3*e^3*x)+(a*c^3*f^4-3*a*c^2 *d*e*f^3+3*a*c*d^2*e^2*f^2-a*d^3*e^3*f-b*c^3*e*f^3+3*b*c^2*d*e^2*f^2-3*b*c* d^2*e^3*f+b*d^3*e^4)/f^4/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.58

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx$$
$$= \left[\frac{30\,bd^3ef^4x^7 - 42\,(bd^3e^2f^3 - (3\,bcd^2 + ad^3)ef^4)x^5 + 70\,(bd^3e^3f^2 - (3\,bcd^2 + ad^3)e^2f^3 + 3\,(bc^2d + acd^2)e\right.$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="fricas")
```

```
[Out] [1/210*(30*b*d^3*e*f^4*x^7 - 42*(b*d^3*e^2*f^3 - (3*b*c*d^2 + a*d^3)*e*f^4)
*x^5 + 70*(b*d^3*e^3*f^2 - (3*b*c*d^2 + a*d^3)*e^2*f^3 + 3*(b*c^2*d + a*c*d
^2)*e*f^4)*x^3 - 105*(b*d^3*e^4 + a*c^3*f^4 - (3*b*c*d^2 + a*d^3)*e^3*f + 3
*(b*c^2*d + a*c*d^2)*e^2*f^2 - (b*c^3 + 3*a*c^2*d)*e*f^3)*sqrt(-e*f)*log((f
*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) - 210*(b*d^3*e^4*f - (3*b*c*d^2 + a
*d^3)*e^3*f^2 + 3*(b*c^2*d + a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c^2*d)*e*f^4)*
x)/(e*f^5), 1/105*(15*b*d^3*e*f^4*x^7 - 21*(b*d^3*e^2*f^3 - (3*b*c*d^2 + a*
d^3)*e*f^4)*x^5 + 35*(b*d^3*e^3*f^2 - (3*b*c*d^2 + a*d^3)*e^2*f^3 + 3*(b*c^
2*d + a*c*d^2)*e*f^4)*x^3 + 105*(b*d^3*e^4 + a*c^3*f^4 - (3*b*c*d^2 + a*d^3
)*e^3*f + 3*(b*c^2*d + a*c*d^2)*e^2*f^2 - (b*c^3 + 3*a*c^2*d)*e*f^3)*sqrt(e
*f)*arctan(sqrt(e*f)*x/e) - 105*(b*d^3*e^4*f - (3*b*c*d^2 + a*d^3)*e^3*f^2
+ 3*(b*c^2*d + a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c^2*d)*e*f^4)*x)/(e*f^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(228) = 456$.

Time = 0.85 (sec) , antiderivative size = 508, normalized size of antiderivative = 2.24

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx$$
$$= \frac{bd^3x^7}{7f} + x^5\left(\frac{ad^3}{5f} + \frac{3bcd^2}{5f} - \frac{bd^3e}{5f^2}\right) + x^3\left(\frac{acd^2}{f} - \frac{ad^3e}{3f^2} + \frac{bc^2d}{f} - \frac{bcd^2e}{f^2} + \frac{bd^3e^2}{3f^3}\right)$$
$$+ x\left(\frac{3ac^2d}{f} - \frac{3acd^2e}{f^2} + \frac{ad^3e^2}{f^3} + \frac{bc^3}{f} - \frac{3bc^2de}{f^2} + \frac{3bcd^2e^2}{f^3} - \frac{bd^3e^3}{f^4}\right)$$
$$- \frac{\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3\log\left(-\frac{ef^4\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3}{ac^3f^4-3ac^2def^3+3acd^2e^2f^2-ad^3e^3f-bc^3ef^3+3bc^2de^2f^2-3bcd^2e^3f+bd^3e^4}+x\right)}{2}$$
$$+ \frac{\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3\log\left(\frac{ef^4\sqrt{-\frac{1}{ef^9}}(af-be)(cf-de)^3}{ac^3f^4-3ac^2def^3+3acd^2e^2f^2-ad^3e^3f-bc^3ef^3+3bc^2de^2f^2-3bcd^2e^3f+bd^3e^4}+x\right)}{2}$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e),x)
```

```
[Out] b*d**3*x**7/(7*f) + x**5*(a*d**3/(5*f) + 3*b*c*d**2/(5*f) - b*d**3*e/(5*f**
2)) + x**3*(a*c*d**2/f - a*d**3*e/(3*f**2) + b*c**2*d/f - b*c*d**2*e/f**2 +
 b*d**3*e**2/(3*f**3)) + x*(3*a*c**2*d/f - 3*a*c*d**2*e/f**2 + a*d**3*e**2/
f**3 + b*c**3/f - 3*b*c**2*d*e/f**2 + 3*b*c*d**2*e**2/f**3 - b*d**3*e**3/f*
*4) - sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3*log(-e*f**4*sqrt(-1/(e*f
**9))*(a*f - b*e)*(c*f - d*e)**3/(a*c**3*f**4 - 3*a*c**2*d*e*f**3 + 3*a*c*d
**2*e**2*f**2 - a*d**3*e**3*f - b*c**3*e*f**3 + 3*b*c**2*d*e**2*f**2 - 3*b*
c*d**2*e**3*f + b*d**3*e**4) + x)/2 + sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f -
d*e)**3*log(e*f**4*sqrt(-1/(e*f**9))*(a*f - b*e)*(c*f - d*e)**3/(a*c**3*f**
4 - 3*a*c**2*d*e*f**3 + 3*a*c*d**2*e**2*f**2 - a*d**3*e**3*f - b*c**3*e*f**
3 + 3*b*c**2*d*e**2*f**2 - 3*b*c*d**2*e**3*f + b*d**3*e**4) + x)/2
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx$$
$$= \frac{(bd^3e^4 - 3\,bcd^2e^3f - ad^3e^3f + 3\,bc^2de^2f^2 + 3\,acd^2e^2f^2 - bc^3ef^3 - 3\,ac^2def^3 + ac^3f^4)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{\sqrt{ef}\,f^4}$$
$$+ \frac{15\,bd^3f^6x^7 - 21\,bd^3ef^5x^5 + 63\,bcd^2f^6x^5 + 21\,ad^3f^6x^5 + 35\,bd^3e^2f^4x^3 - 105\,bcd^2ef^5x^3 - 35\,ad^3ef^5x^3 + \cdots}{\cdots}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e),x, algorithm="giac")
```

```
[Out] (b*d^3*e^4 - 3*b*c*d^2*e^3*f - a*d^3*e^3*f + 3*b*c^2*d*e^2*f^2 + 3*a*c*d^2*
e^2*f^2 - b*c^3*e*f^3 - 3*a*c^2*d*e*f^3 + a*c^3*f^4)*arctan(f*x/sqrt(e*f))/
```

```
(sqrt(e*f)*f^4) + 1/105*(15*b*d^3*f^6*x^7 - 21*b*d^3*e*f^5*x^5 + 63*b*c*d^2
*f^6*x^5 + 21*a*d^3*f^6*x^5 + 35*b*d^3*e^2*f^4*x^3 - 105*b*c*d^2*e*f^5*x^3
- 35*a*d^3*e*f^5*x^3 + 105*b*c^2*d*f^6*x^3 + 105*a*c*d^2*f^6*x^3 - 105*b*d^
3*e^3*f^3*x + 315*b*c*d^2*e^2*f^4*x + 105*a*d^3*e^2*f^4*x - 315*b*c^2*d*e*f
^5*x - 315*a*c*d^2*e*f^5*x + 105*b*c^3*f^6*x + 315*a*c^2*d*f^6*x)/f^7
```

Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.37

$$\int \frac{(a+bx^2)(c+dx^2)^3}{e+fx^2}\,dx = x\left(\frac{b\,c^3+3\,a\,d\,c^2}{f}+\frac{e\left(\frac{e\left(\frac{a\,d^3+3\,b\,c\,d^2}{f}-\frac{b\,d^3\,e}{f^2}\right)}{f}-\frac{3\,c\,d\,(a\,d+b\,c)}{f}\right)}{f}\right) + x^5\left(\frac{a\,d^3+3\,b\,c\,d^2}{5\,f}-\frac{b\,d^3\,e}{5\,f^2}\right)-x^3\left(\frac{e\left(\frac{a\,d^3+3\,b\,c\,d^2}{f}-\frac{b\,d^3\,e}{f^2}\right)}{3\,f}-\frac{c\,d\,(a\,d+b\,c)}{f}\right)+\frac{b\,d^3\,x^7}{7\,f} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(a\,f-b\,e)\,(c\,f-d\,e)^3}{\sqrt{e}\,(-b\,c^3\,e\,f^3+a\,c^3\,f^4+3\,b\,c^2\,d\,e^2\,f^2-3\,a\,c^2\,d\,e\,f^3-3\,b\,c\,d^2\,e^3\,f+3\,a\,c\,d^2\,e^2\,f^2+b\,d^3\,e^4-a\,d^3\,e^3\,f)}\right)\,(a\,f-b\,e)\,(c\,f-d\,e)^3}{\sqrt{e}\,f^{9/2}}$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2),x)
```

```
[Out] x*((b*c^3 + 3*a*c^2*d)/f + (e*((e*((a*d^3 + 3*b*c*d^2)/f - (b*d^3*e)/f^2))/
f - (3*c*d*(a*d + b*c))/f))/f) + x^5*((a*d^3 + 3*b*c*d^2)/(5*f) - (b*d^3*e)
/(5*f^2)) - x^3*((e*((a*d^3 + 3*b*c*d^2)/f - (b*d^3*e)/f^2))/(3*f) - (c*d*(
a*d + b*c))/f) + (b*d^3*x^7)/(7*f) + (atan((f^(1/2)*x*(a*f - b*e)*(c*f - d*
e)^3)/(e^(1/2)*(a*c^3*f^4 + b*d^3*e^4 - a*d^3*e^3*f - b*c^3*e*f^3 - 3*a*c^2
*d*e*f^3 - 3*b*c*d^2*e^3*f + 3*a*c*d^2*e^2*f^2 + 3*b*c^2*d*e^2*f^2)))*(a*f
- b*e)*(c*f - d*e)^3)/(e^(1/2)*f^(9/2))
```

3.20 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2} dx$

Optimal result

Integrand size = 26, antiderivative size = 242

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2} dx = -\frac{d(5af(15d^2e^2-22cdef+3c^2f^2)-be(105d^2e^2-190cdef+81c^2f^2))x}{30ef^4} - \frac{d(be(35de-33cf)-5af(5de-3cf))x(c+dx^2)}{30ef^3} + \frac{d(7be-5af)x(c+dx^2)^2}{10ef^2} - \frac{(be-af)x(c+dx^2)^3}{2ef(e+fx^2)} - \frac{(de-cf)^2(be(7de-cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{9/2}}$$

```
[Out] -1/30*d*(5*a*f*(3*c^2*f^2-22*c*d*e*f+15*d^2*e^2)-b*e*(81*c^2*f^2-190*c*d*e*
f+105*d^2*e^2))*x/e/f^4-1/30*d*(b*e*(-33*c*f+35*d*e)-5*a*f*(-3*c*f+5*d*e))*
x*(d*x^2+c)/e/f^3+1/10*d*(-5*a*f+7*b*e)*x*(d*x^2+c)^2/e/f^2-1/2*(-a*f+b*e)*
x*(d*x^2+c)^3/e/f/(f*x^2+e)-1/2*(-c*f+d*e)^2*(b*e*(-c*f+7*d*e)-a*f*(c*f+5*d
*e))*arctan(x*f^(1/2)/e^(1/2))/e^(3/2)/f^(9/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used

$= \{540, 542, 396, 211\}$

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx$$
$$= -\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)^2(be(7de-cf)-af(cf+5de))}{2e^{3/2}f^{9/2}}$$
$$-\frac{dx(5af(3c^2f^2-22cdef+15d^2e^2)-be(81c^2f^2-190cdef+105d^2e^2))}{30ef^4}$$
$$-\frac{dx(c+dx^2)(be(35de-33cf)-5af(5de-3cf))}{30ef^3}$$
$$+\frac{dx(c+dx^2)^2(7be-5af)}{10ef^2}-\frac{x(c+dx^2)^3(be-af)}{2ef(e+fx^2)}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

```
[Out] -1/30*(d*(5*a*f*(15*d^2*e^2 - 22*c*d*e*f + 3*c^2*f^2) - b*e*(105*d^2*e^2 -
190*c*d*e*f + 81*c^2*f^2))*x)/(e*f^4) - (d*(b*e*(35*d*e - 33*c*f) - 5*a*f*(
5*d*e - 3*c*f))*x*(c + d*x^2))/(30*e*f^3) + (d*(7*b*e - 5*a*f)*x*(c + d*x^2
)^2)/(10*e*f^2) - ((b*e - a*f)*x*(c + d*x^2)^3)/(2*e*f*(e + f*x^2)) - ((d*e
 - c*f)^2*(b*e*(7*d*e - c*f) - a*f*(5*d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e
]])/(2*e^(3/2)*f^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(be-af)x\left(c+dx^2\right)^3}{2ef\left(e+fx^2\right)}-\frac{\int \frac{\left(c+dx^2\right)^2\left(-c(be+af)-d(7be-5af)x^2\right)}{e+fx^2}\,dx}{2ef} \\
&= \frac{d(7be-5af)x\left(c+dx^2\right)^2}{10ef^2}-\frac{(be-af)x\left(c+dx^2\right)^3}{2ef\left(e+fx^2\right)} \\
&\quad -\frac{\int \frac{\left(c+dx^2\right)\left(c(be(7de-5cf)-5af(de+cf))+d(be(35de-33cf)-5af(5de-3cf))x^2\right)}{e+fx^2}\,dx}{10ef^2} \\
&= -\frac{d(be(35de-33cf)-5af(5de-3cf))x\left(c+dx^2\right)}{30ef^3} \\
&\quad +\frac{d(7be-5af)x\left(c+dx^2\right)^2}{10ef^2}-\frac{(be-af)x\left(c+dx^2\right)^3}{2ef\left(e+fx^2\right)} \\
&\quad -\frac{\int \frac{c\left(5af\left(5d^2e^2-6cdef-3c^2f^2\right)-be\left(35d^2e^2-54cdef+15c^2f^2\right)\right)+d\left(5af\left(15d^2e^2-22cdef+3c^2f^2\right)-be\left(105d^2e^2-190cdef+81c^2f^2\right)\right)x}{e+fx^2}}{30ef^3} \\
&= -\frac{d\left(5af\left(15d^2e^2-22cdef+3c^2f^2\right)-be\left(105d^2e^2-190cdef+81c^2f^2\right)\right)x}{30ef^4} \\
&\quad -\frac{d(be(35de-33cf)-5af(5de-3cf))x\left(c+dx^2\right)}{30ef^3}+\frac{d(7be-5af)x\left(c+dx^2\right)^2}{10ef^2} \\
&\quad -\frac{(be-af)x\left(c+dx^2\right)^3}{2ef\left(e+fx^2\right)}-\frac{\left((de-cf)^2(be(7de-cf)-af(5de+cf))\right)\int \frac{1}{e+fx^2}\,dx}{2ef^4} \\
&= -\frac{d\left(5af\left(15d^2e^2-22cdef+3c^2f^2\right)-be\left(105d^2e^2-190cdef+81c^2f^2\right)\right)x}{30ef^4} \\
&\quad -\frac{d(be(35de-33cf)-5af(5de-3cf))x\left(c+dx^2\right)}{30ef^3}+\frac{d(7be-5af)x\left(c+dx^2\right)^2}{10ef^2} \\
&\quad -\frac{(be-af)x\left(c+dx^2\right)^3}{2ef\left(e+fx^2\right)}-\frac{(de-cf)^2(be(7de-cf)-af(5de+cf))\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{9/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx = \frac{d(3b(de-cf)^2+adf(-2de+3cf))\,x}{f^4} + \frac{d^2(-2bde+3bcf+adf)x^3}{3f^3} + \frac{bd^3x^5}{5f^2} + \frac{(be-af)(de-cf)^3x}{2ef^4(e+fx^2)} - \frac{(de-cf)^2(be(7de-cf)-af(5de+cf))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{2e^{3/2}f^{9/2}}$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x]
```

```
[Out] (d*(3*b*(d*e - c*f)^2 + a*d*f*(-2*d*e + 3*c*f))*x)/f^4 + (d^2*(-2*b*d*e + 3
*b*c*f + a*d*f)*x^3)/(3*f^3) + (b*d^3*x^5)/(5*f^2) + ((b*e - a*f)*(d*e - c*
f)^3*x)/(2*e*f^4*(e + f*x^2)) - ((d*e - c*f)^2*(b*e*(7*d*e - c*f) - a*f*(5*
d*e + c*f))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(2*e^(3/2)*f^(9/2))
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.27

method	result
default	$\frac{d\left(\frac{1}{5}b\,d^2x^5f^2+\frac{1}{3}a\,d^2f^2x^3+bcd\,f^2x^3-\frac{2}{3}b\,d^2ef\,x^3+3acd\,f^2x-2a\,d^2efx+3b\,c^2f^2x-6bcdefx+3b\,d^2e^2x\right)}{f^4}+\frac{\left(a\,c^3f^4-3a\,c^2de\,f^3+3ac\,d^2\right.}{}$
risch	$\frac{3d^2acx}{f^2}-\frac{2d^3aex}{f^3}+\frac{3db\,c^2x}{f^2}+\frac{3d^3b\,e^2x}{f^4}-\frac{\ln(fx+\sqrt{-ef})a\,c^3}{4\sqrt{-ef}\,e}+\frac{9e\ln(fx+\sqrt{-ef})ac\,d^2}{4f^2\sqrt{-ef}}+\frac{9e\ln(fx+\sqrt{-ef})b\,c^2d}{4f^2\sqrt{-ef}}-\frac{15e^2}{}$

```
[In] int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d/f^4*(1/5*b*d^2*x^5*f^2+1/3*a*d^2*f^2*x^3+b*c*d*f^2*x^3-2/3*b*d^2*e*f*x^3+
3*a*c*d*f^2*x-2*a*d^2*e*f*x+3*b*c^2*f^2*x-6*b*c*d*e*f*x+3*b*d^2*e^2*x)+1/f^
4*(1/2*(a*c^3*f^4-3*a*c^2*d*e*f^3+3*a*c*d^2*e^2*f^2-a*d^3*e^3*f-b*c^3*e*f^3
+3*b*c^2*d*e^2*f^2-3*b*c*d^2*e^3*f+b*d^3*e^4)/e*x/(f*x^2+e)+1/2*(a*c^3*f^4+
3*a*c^2*d*e*f^3-9*a*c*d^2*e^2*f^2+5*a*d^3*e^3*f+b*c^3*e*f^3-9*b*c^2*d*e^2*f
^2+15*b*c*d^2*e^3*f-7*b*d^3*e^4)/e/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 834, normalized size of antiderivative = 3.45

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx$$

$$= \left[\frac{12\,bd^3e^2f^4x^7 - 4\,(7\,bd^3e^3f^3 - 5\,(3\,bcd^2 + ad^3)e^2f^4)x^5 + 20\,(7\,bd^3e^4f^2 - 5\,(3\,bcd^2 + ad^3)e^3f^3 + 9\,(bc^2d + \right.$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(12*b*d^3*e^2*f^4*x^7 - 4*(7*b*d^3*e^3*f^3 - 5*(3*b*c*d^2 + a*d^3)*e^
2*f^4)*x^5 + 20*(7*b*d^3*e^4*f^2 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c^2
*d + a*c*d^2)*e^2*f^4)*x^3 + 15*(7*b*d^3*e^5 - a*c^3*e*f^4 - 5*(3*b*c*d^2 +
 a*d^3)*e^4*f + 9*(b*c^2*d + a*c*d^2)*e^3*f^2 - (b*c^3 + 3*a*c^2*d)*e^2*f^3
 + (7*b*d^3*e^4*f - a*c^3*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^2 + 9*(b*c^2*d
+ a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c^2*d)*e*f^4)*x^2)*sqrt(-e*f)*log((f*x^2
- 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 30*(7*b*d^3*e^5*f + a*c^3*e*f^5 - 5*(3
*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^2*d + a*c*d^2)*e^3*f^3 - (b*c^3 + 3*a*c^
2*d)*e^2*f^4)*x)/(e^2*f^6*x^2 + e^3*f^5), 1/30*(6*b*d^3*e^2*f^4*x^7 - 2*(7*
b*d^3*e^3*f^3 - 5*(3*b*c*d^2 + a*d^3)*e^2*f^4)*x^5 + 10*(7*b*d^3*e^4*f^2 -
5*(3*b*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f^4)*x^3 - 15*(7*
b*d^3*e^5 - a*c^3*e*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^4*f + 9*(b*c^2*d + a*c*d^
2)*e^3*f^2 - (b*c^3 + 3*a*c^2*d)*e^2*f^3 + (7*b*d^3*e^4*f - a*c^3*f^5 - 5*(
3*b*c*d^2 + a*d^3)*e^3*f^2 + 9*(b*c^2*d + a*c*d^2)*e^2*f^3 - (b*c^3 + 3*a*c
^2*d)*e*f^4)*x^2)*sqrt(e*f)*arctan(sqrt(e*f)*x/e) + 15*(7*b*d^3*e^5*f + a*c
^3*e*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^2*d + a*c*d^2)*e^3*f^3 -
(b*c^3 + 3*a*c^2*d)*e^2*f^4)*x)/(e^2*f^6*x^2 + e^3*f^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(231) = 462$.

Time = 2.22 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.73

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^2}\,dx$$
$$= \frac{bd^3x^5}{5f^2} + x^3\left(\frac{ad^3}{3f^2} + \frac{bcd^2}{f^2} - \frac{2bd^3e}{3f^3}\right) + x\left(\frac{3acd^2}{f^2} - \frac{2ad^3e}{f^3} + \frac{3bc^2d}{f^2} - \frac{6bcd^2e}{f^3} + \frac{3bd^3e^2}{f^4}\right)$$
$$+ \frac{x(ac^3f^4 - 3ac^2def^3 + 3acd^2e^2f^2 - ad^3e^3f - bc^3ef^3 + 3bc^2de^2f^2 - 3bcd^2e^3f + bd^3e^4)}{2e^2f^4 + 2ef^5x^2}$$
$$- \frac{\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2 + 5adef + bcef - 7bde^2)\log\left(-\frac{e^2f^4\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2+5adef+bcef-7}{ac^3f^4+3ac^2def^3-9acd^2e^2f^2+5ad^3e^3f+bc^3ef^3-9bc^2de^2f^2}\right.}{4}$$
$$+ \frac{\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2 + 5adef + bcef - 7bde^2)\log\left(\frac{e^2f^4\sqrt{-\frac{1}{e^3f^9}}(cf-de)^2(acf^2+5adef+bcef-7bd}{ac^3f^4+3ac^2def^3-9acd^2e^2f^2+5ad^3e^3f+bc^3ef^3-9bc^2de^2f^2+1}\right.}{4}$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**2,x)
```

```
[Out] b*d**3*x**5/(5*f**2) + x**3*(a*d**3/(3*f**2) + b*c*d**2/f**2 - 2*b*d**3*e/(
3*f**3)) + x*(3*a*c*d**2/f**2 - 2*a*d**3*e/f**3 + 3*b*c**2*d/f**2 - 6*b*c*d
**2*e/f**3 + 3*b*d**3*e**2/f**4) + x*(a*c**3*f**4 - 3*a*c**2*d*e*f**3 + 3*a
*c*d**2*e**2*f**2 - a*d**3*e**3*f - b*c**3*e*f**3 + 3*b*c**2*d*e**2*f**2 -
3*b*c*d**2*e**3*f + b*d**3*e**4)/(2*e**2*f**4 + 2*e*f**5*x**2) - sqrt(-1/(e
**3*f**9))*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e**2)*log
(-e**2*f**4*sqrt(-1/(e**3*f**9))*(c*f - d*e)**2*(a*c*f**2 + 5*a*d*e*f + b*c
*e*f - 7*b*d*e**2)/(a*c**3*f**4 + 3*a*c**2*d*e*f**3 - 9*a*c*d**2*e**2*f**2
+ 5*a*d**3*e**3*f + b*c**3*e*f**3 - 9*b*c**2*d*e**2*f**2 + 15*b*c*d**2*e**3
*f - 7*b*d**3*e**4) + x)/4 + sqrt(-1/(e**3*f**9))*(c*f - d*e)**2*(a*c*f**2
+ 5*a*d*e*f + b*c*e*f - 7*b*d*e**2)*log(e**2*f**4*sqrt(-1/(e**3*f**9))*(c*f
 - d*e)**2*(a*c*f**2 + 5*a*d*e*f + b*c*e*f - 7*b*d*e**2)/(a*c**3*f**4 + 3*a
*c**2*d*e*f**3 - 9*a*c*d**2*e**2*f**2 + 5*a*d**3*e**3*f + b*c**3*e*f**3 - 9
*b*c**2*d*e**2*f**2 + 15*b*c*d**2*e**3*f - 7*b*d**3*e**4) + x)/4
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)\,(c+dx^2)^3}{(e+fx^2)^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.38

$$\int \frac{(a+bx^2)\,(c+dx^2)^3}{(e+fx^2)^2}\,dx =$$

$$-\frac{(7\,bd^3e^4 - 15\,bcd^2e^3f - 5\,ad^3e^3f + 9\,bc^2de^2f^2 + 9\,acd^2e^2f^2 - bc^3ef^3 - 3\,ac^2def^3 - ac^3f^4)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{2\,\sqrt{ef}ef^4}$$
$$+\frac{bd^3e^4x - 3\,bcd^2e^3fx - ad^3e^3fx + 3\,bc^2de^2f^2x + 3\,acd^2e^2f^2x - bc^3ef^3x - 3\,ac^2def^3x + ac^3f^4x}{2\,(fx^2+e)ef^4}$$
$$+\frac{3\,bd^3f^8x^5 - 10\,bd^3ef^7x^3 + 15\,bcd^2f^8x^3 + 5\,ad^3f^8x^3 + 45\,bd^3e^2f^6x - 90\,bcd^2ef^7x - 30\,ad^3ef^7x + 45\,bc^2d}{15\,f^{10}}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*b*d^3*e^4 - 15*b*c*d^2*e^3*f - 5*a*d^3*e^3*f + 9*b*c^2*d*e^2*f^2 +
9*a*c*d^2*e^2*f^2 - b*c^3*e*f^3 - 3*a*c^2*d*e*f^3 - a*c^3*f^4)*arctan(f*x/s
qrt(e*f))/(sqrt(e*f)*e*f^4) + 1/2*(b*d^3*e^4*x - 3*b*c*d^2*e^3*f*x - a*d^3*
e^3*f*x + 3*b*c^2*d*e^2*f^2*x + 3*a*c*d^2*e^2*f^2*x - b*c^3*e*f^3*x - 3*a*c
^2*d*e*f^3*x + a*c^3*f^4*x)/((f*x^2 + e)*e*f^4) + 1/15*(3*b*d^3*f^8*x^5 - 1
0*b*d^3*e*f^7*x^3 + 15*b*c*d^2*f^8*x^3 + 5*a*d^3*f^8*x^3 + 45*b*d^3*e^2*f^6
*x - 90*b*c*d^2*e*f^7*x - 30*a*d^3*e*f^7*x + 45*b*c^2*d*f^8*x + 45*a*c*d^2*
f^8*x)/f^10
```

Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.61

$$\int \frac{(a+bx^2)\,(c+dx^2)^3}{(e+fx^2)^2}\,dx = x^3\left(\frac{a\,d^3+3\,b\,c\,d^2}{3\,f^2}-\frac{2\,b\,d^3\,e}{3\,f^3}\right)$$
$$-x\left(\frac{2\,e\left(\frac{a\,d^3+3\,b\,c\,d^2}{f^2}-\frac{2\,b\,d^3\,e}{f^3}\right)}{f}+\frac{b\,d^3\,e^2}{f^4}-\frac{3\,c\,d\,(a\,d+b\,c)}{f^2}\right)+\frac{b\,d^3\,x^5}{5\,f^2}$$
$$+\frac{x\,(-b\,c^3\,e\,f^3+a\,c^3\,f^4+3\,b\,c^2\,d\,e^2\,f^2-3\,a\,c^2\,d\,e\,f^3-3\,b\,c\,d^2\,e^3\,f+3\,a\,c\,d^2\,e^2\,f^2+b\,d^3\,e^4-a\,d^3\,e^3\,f)}{2\,e\,(f^5\,x^2+e\,f^4)}$$
$$+\frac{\operatorname{atan}\left(\frac{\sqrt{f}\,x\,(c\,f-d\,e)^2\,(a\,c\,f^2-7\,b\,d\,e^2+5\,a\,d\,e\,f+b\,c\,e\,f)}{\sqrt{e}\,(b\,c^3\,e\,f^3+a\,c^3\,f^4-9\,b\,c^2\,d\,e^2\,f^2+3\,a\,c^2\,d\,e\,f^3+15\,b\,c\,d^2\,e^3\,f-9\,a\,c\,d^2\,e^2\,f^2-7\,b\,d^3\,e^4+5\,a\,d^3\,e^3\,f)}\right)\,(c\,f-d\,e)^2\,(a\,c\,f^2-7}{2\,e^{3/2}\,f^{9/2}}$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^2,x)
```

```
[Out] x^3*((a*d^3 + 3*b*c*d^2)/(3*f^2) - (2*b*d^3*e)/(3*f^3)) - x*((2*e*((a*d^3 +
 3*b*c*d^2)/f^2 - (2*b*d^3*e)/f^3))/f + (b*d^3*e^2)/f^4 - (3*c*d*(a*d + b*c
))/f^2) + (b*d^3*x^5)/(5*f^2) + (x*(a*c^3*f^4 + b*d^3*e^4 - a*d^3*e^3*f - b
*c^3*e*f^3 - 3*a*c^2*d*e*f^3 - 3*b*c*d^2*e^3*f + 3*a*c*d^2*e^2*f^2 + 3*b*c^
2*d*e^2*f^2))/(2*e*(e*f^4 + f^5*x^2)) + (atan((f^(1/2)*x*(c*f - d*e)^2*(a*c
*f^2 - 7*b*d*e^2 + 5*a*d*e*f + b*c*e*f))/(e^(1/2)*(a*c^3*f^4 - 7*b*d^3*e^4
+ 5*a*d^3*e^3*f + b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 + 15*b*c*d^2*e^3*f - 9*a*c*
d^2*e^2*f^2 - 9*b*c^2*d*e^2*f^2)))*(c*f - d*e)^2*(a*c*f^2 - 7*b*d*e^2 + 5*a
*d*e*f + b*c*e*f))/(2*e^(3/2)*f^(9/2))
```

3.21 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3} dx$

Optimal result

Integrand size = 26, antiderivative size = 291

$$
\begin{aligned}
\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3} dx
&= \frac{d(3af(15d^2e^2-4cdef-3c^2f^2)-be(105d^2e^2-100cdef+3c^2f^2))x}{24e^2f^4} \\
&+ \frac{d(be(35de-3cf)-3af(5de+3cf))x(c+dx^2)}{24e^2f^3} \\
&- \frac{(be-af)x(c+dx^2)^3}{4ef(e+fx^2)^2} - \frac{(be(7de-cf)-3af(de+cf))x(c+dx^2)^2}{8e^2f^2(e+fx^2)} \\
&+ \frac{(de-cf)(be(35d^2e^2-10cdef-c^2f^2)-3af(5d^2e^2+2cdef+c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{9/2}}
\end{aligned}
$$

```
[Out] 1/24*d*(3*a*f*(-3*c^2*f^2-4*c*d*e*f+15*d^2*e^2)-b*e*(3*c^2*f^2-100*c*d*e*f+
105*d^2*e^2))*x/e^2/f^4+1/24*d*(b*e*(-3*c*f+35*d*e)-3*a*f*(3*c*f+5*d*e))*x*
(d*x^2+c)/e^2/f^3-1/4*(-a*f+b*e)*x*(d*x^2+c)^3/e/f/(f*x^2+e)^2-1/8*(b*e*(-c
*f+7*d*e)-3*a*f*(c*f+d*e))*x*(d*x^2+c)^2/e^2/f^2/(f*x^2+e)+1/8*(-c*f+d*e)*(
b*e*(-c^2*f^2-10*c*d*e*f+35*d^2*e^2)-3*a*f*(c^2*f^2+2*c*d*e*f+5*d^2*e^2))*a
rctan(x*f^(1/2)/e^(1/2))/e^(5/2)/f^(9/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {540, 542, 396, 211}

$$\begin{aligned}&\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx\\&=\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)(de-cf)(be(-c^2f^2-10cdef+35d^2e^2)-3af(c^2f^2+2cdef+5d^2e^2))}{8e^{5/2}f^{9/2}}\\&+\frac{dx(3af(-3c^2f^2-4cdef+15d^2e^2)-be(3c^2f^2-100cdef+105d^2e^2))}{24e^2f^4}\\&+\frac{dx(c+dx^2)(be(35de-3cf)-3af(3cf+5de))}{24e^2f^3}\\&-\frac{x(c+dx^2)^2(be(7de-cf)-3af(cf+de))}{8e^2f^2(e+fx^2)}-\frac{x(c+dx^2)^3(be-af)}{4ef(e+fx^2)^2}\end{aligned}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

```
[Out] (d*(3*a*f*(15*d^2*e^2 - 4*c*d*e*f - 3*c^2*f^2) - b*e*(105*d^2*e^2 - 100*c*d
*e*f + 3*c^2*f^2))*x)/(24*e^2*f^4) + (d*(b*e*(35*d*e - 3*c*f) - 3*a*f*(5*d*
e + 3*c*f))*x*(c + d*x^2))/(24*e^2*f^3) - ((b*e - a*f)*x*(c + d*x^2)^3)/(4*
e*f*(e + f*x^2)^2) - ((b*e*(7*d*e - c*f) - 3*a*f*(d*e + c*f))*x*(c + d*x^2)
^2)/(8*e^2*f^2*(e + f*x^2)) + ((d*e - c*f)*(b*e*(35*d^2*e^2 - 10*c*d*e*f -
c^2*f^2) - 3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f^2))*ArcTan[(Sqrt[f]*x)/Sqrt
[e]])/(8*e^(5/2)*f^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
```

```
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\text{integral} = -\frac{(be-af)x(c+dx^2)^3}{4ef(e+fx^2)^2} - \frac{\int \frac{(c+dx^2)^2(-c(be+3af)-d(7be-3af)x^2)}{(e+fx^2)^2}\,dx}{4ef}$$

$$= -\frac{(be-af)x(c+dx^2)^3}{4ef(e+fx^2)^2} - \frac{(be(7de-cf)-3af(de+cf))x(c+dx^2)^2}{8e^2f^2(e+fx^2)} + \frac{\int \frac{(c+dx^2)(-c(3af(de-cf)-be(7de+cf))+d(be(35de-3cf)-3af(5de+3cf))x^2)}{e+fx^2}\,dx}{8e^2f^2}$$

$$= \frac{d(be(35de-3cf)-3af(5de+3cf))x(c+dx^2)}{24e^2f^3} - \frac{(be-af)x(c+dx^2)^3}{4ef(e+fx^2)^2} - \frac{(be(7de-cf)-3af(de+cf))x(c+dx^2)^2}{8e^2f^2(e+fx^2)} + \frac{\int \frac{-c(be(35d^2e^2-24cdef-3c^2f^2)-3af(5d^2e^2+3c^2f^2))+d(3af(15d^2e^2-4cdef-3c^2f^2)-be(105d^2e^2-100cdef+3c^2f^2))x^2}{e+fx^2}\,dx}{24e^2f^3}$$

$$= \frac{d(3af(15d^2e^2-4cdef-3c^2f^2)-be(105d^2e^2-100cdef+3c^2f^2))x}{24e^2f^4} + \frac{d(be(35de-3cf)-3af(5de+3cf))x(c+dx^2)}{24e^2f^3} - \frac{(be-af)x(c+dx^2)^3}{4ef(e+fx^2)^2} - \frac{(be(7de-cf)-3af(de+cf))x(c+dx^2)^2}{8e^2f^2(e+fx^2)} + \frac{((de-cf)(be(35d^2e^2-10cdef-c^2f^2)-3af(5d^2e^2+2cdef+c^2f^2)))\int \frac{1}{e+fx^2}\,dx}{8e^2f^4}$$

$$
\begin{aligned}
&= \frac{d(3af(15d^2e^2-4cdef-3c^2f^2)-be(105d^2e^2-100cdef+3c^2f^2))\,x}{24e^2f^4} \\
&\quad + \frac{d(be(35de-3cf)-3af(5de+3cf))x(c+dx^2)}{24e^2f^3} \\
&\quad - \frac{(be-af)x(c+dx^2)^3}{4ef\,(e+fx^2)^2} - \frac{(be(7de-cf)-3af(de+cf))x(c+dx^2)^2}{8e^2f^2\,(e+fx^2)} \\
&\quad + \frac{(de-cf)\,(be(35d^2e^2-10cdef-c^2f^2)-3af(5d^2e^2+2cdef+c^2f^2))\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{9/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.75

$$
\begin{aligned}
&\int \frac{(a+bx^2)\,(c+dx^2)^3}{(e+fx^2)^3}\,dx \\
&= \frac{d^2(-3bde+3bcf+adf)x}{f^4} + \frac{bd^3x^3}{3f^3} + \frac{(be-af)(de-cf)^3x}{4ef^4\,(e+fx^2)^2} \\
&\quad - \frac{(de-cf)^2(be(13de-cf)-3af(3de+cf))x}{8e^2f^4\,(e+fx^2)} \\
&\quad + \frac{(de-cf)\,(be(35d^2e^2-10cdef-c^2f^2)-3af(5d^2e^2+2cdef+c^2f^2))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{8e^{5/2}f^{9/2}}
\end{aligned}
$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x]
```

```
[Out] (d^2*(-3*b*d*e + 3*b*c*f + a*d*f)*x)/f^4 + (b*d^3*x^3)/(3*f^3) + ((b*e - a*
f)*(d*e - c*f)^3*x)/(4*e*f^4*(e + f*x^2)^2) - ((d*e - c*f)^2*(b*e*(13*d*e -
 c*f) - 3*a*f*(3*d*e + c*f))*x)/(8*e^2*f^4*(e + f*x^2)) + ((d*e - c*f)*(b*e
*(35*d^2*e^2 - 10*c*d*e*f - c^2*f^2) - 3*a*f*(5*d^2*e^2 + 2*c*d*e*f + c^2*f
^2))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(8*e^(5/2)*f^(9/2))
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.18

method	result
default	$\frac{d^2\left(\frac{1}{3}bdf\,x^3+adfx+3bcfx-3bdex\right)}{f^4}+\frac{\frac{f\left(3a\,c^3f^4+3a\,c^2de\,f^3-15ac\,d^2e^2f^2+9a\,d^3e^3f+b\,c^3e\,f^3-15b\,c^2d\,e^2f^2+27bc\,d^2e^3f-13b\,d^3e^4\right)x^3}{8e^2}+\frac{(5}{}}{\left(f\,x^2+e\right)^2}$
risch	$\frac{d^3b\,x^3}{3f^3}+\frac{d^3ax}{f^3}+\frac{3d^2bcx}{f^3}-\frac{3d^3bex}{f^4}+\frac{\frac{f\left(3a\,c^3f^4+3a\,c^2de\,f^3-15ac\,d^2e^2f^2+9a\,d^3e^3f+b\,c^3e\,f^3-15b\,c^2d\,e^2f^2+27bc\,d^2e^3f-13b\,d^3e^4\right)x^3}{8e^2}}{f^4(f\,x^2}$

```
[In] int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x,method=_RETURNVERBOSE)
```

```
[Out] d^2/f^4*(1/3*b*d*f*x^3+a*d*f*x+3*b*c*f*x-3*b*d*e*x)+1/f^4*((1/8*f*(3*a*c^3*
f^4+3*a*c^2*d*e*f^3-15*a*c*d^2*e^2*f^2+9*a*d^3*e^3*f+b*c^3*e*f^3-15*b*c^2*d
*e^2*f^2+27*b*c*d^2*e^3*f-13*b*d^3*e^4)/e^2*x^3+1/8*(5*a*c^3*f^4-3*a*c^2*d*
e*f^3-9*a*c*d^2*e^2*f^2+7*a*d^3*e^3*f-b*c^3*e*f^3-9*b*c^2*d*e^2*f^2+21*b*c*
d^2*e^3*f-11*b*d^3*e^4)/e*x)/(f*x^2+e)^2+1/8*(3*a*c^3*f^4+3*a*c^2*d*e*f^3+9
*a*c*d^2*e^2*f^2-15*a*d^3*e^3*f+b*c^3*e*f^3+9*b*c^2*d*e^2*f^2-45*b*c*d^2*e^
3*f+35*b*d^3*e^4)/e^2/(e*f)^(1/2)*arctan(f*x/(e*f)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 1102, normalized size of antiderivative = 3.79

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="fricas")
```

```
[Out] [1/48*(16*b*d^3*e^3*f^4*x^7 - 16*(7*b*d^3*e^4*f^3 - 3*(3*b*c*d^2 + a*d^3)*e
^3*f^4)*x^5 - 2*(175*b*d^3*e^5*f^2 - 9*a*c^3*e*f^6 - 75*(3*b*c*d^2 + a*d^3)
*e^4*f^3 + 45*(b*c^2*d + a*c*d^2)*e^3*f^4 - 3*(b*c^3 + 3*a*c^2*d)*e^2*f^5)*
x^3 - 3*(35*b*d^3*e^6 + 3*a*c^3*e^2*f^4 - 15*(3*b*c*d^2 + a*d^3)*e^5*f + 9*
(b*c^2*d + a*c*d^2)*e^4*f^2 + (b*c^3 + 3*a*c^2*d)*e^3*f^3 + (35*b*d^3*e^4*f
^2 + 3*a*c^3*f^6 - 15*(3*b*c*d^2 + a*d^3)*e^3*f^3 + 9*(b*c^2*d + a*c*d^2)*e
^2*f^4 + (b*c^3 + 3*a*c^2*d)*e*f^5)*x^4 + 2*(35*b*d^3*e^5*f + 3*a*c^3*e*f^5
 - 15*(3*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^2*d + a*c*d^2)*e^3*f^3 + (b*c^3
+ 3*a*c^2*d)*e^2*f^4)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x
^2 + e)) - 6*(35*b*d^3*e^6*f - 5*a*c^3*e^2*f^5 - 15*(3*b*c*d^2 + a*d^3)*e^5
*f^2 + 9*(b*c^2*d + a*c*d^2)*e^4*f^3 + (b*c^3 + 3*a*c^2*d)*e^3*f^4)*x)/(e^3
*f^7*x^4 + 2*e^4*f^6*x^2 + e^5*f^5), 1/24*(8*b*d^3*e^3*f^4*x^7 - 8*(7*b*d^3
*e^4*f^3 - 3*(3*b*c*d^2 + a*d^3)*e^3*f^4)*x^5 - (175*b*d^3*e^5*f^2 - 9*a*c^
3*e*f^6 - 75*(3*b*c*d^2 + a*d^3)*e^4*f^3 + 45*(b*c^2*d + a*c*d^2)*e^3*f^4 -
 3*(b*c^3 + 3*a*c^2*d)*e^2*f^5)*x^3 + 3*(35*b*d^3*e^6 + 3*a*c^3*e^2*f^4 - 1
5*(3*b*c*d^2 + a*d^3)*e^5*f + 9*(b*c^2*d + a*c*d^2)*e^4*f^2 + (b*c^3 + 3*a*
c^2*d)*e^3*f^3 + (35*b*d^3*e^4*f^2 + 3*a*c^3*f^6 - 15*(3*b*c*d^2 + a*d^3)*e
^3*f^3 + 9*(b*c^2*d + a*c*d^2)*e^2*f^4 + (b*c^3 + 3*a*c^2*d)*e*f^5)*x^4 + 2
*(35*b*d^3*e^5*f + 3*a*c^3*e*f^5 - 15*(3*b*c*d^2 + a*d^3)*e^4*f^2 + 9*(b*c^
2*d + a*c*d^2)*e^3*f^3 + (b*c^3 + 3*a*c^2*d)*e^2*f^4)*x^2)*sqrt(e*f)*arctan
(sqrt(e*f)*x/e) - 3*(35*b*d^3*e^6*f - 5*a*c^3*e^2*f^5 - 15*(3*b*c*d^2 + a*d
^3)*e^5*f^2 + 9*(b*c^2*d + a*c*d^2)*e^4*f^3 + (b*c^3 + 3*a*c^2*d)*e^3*f^4)*
x)/(e^3*f^7*x^4 + 2*e^4*f^6*x^2 + e^5*f^5)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. 2(291) = 582.

Time = 58.36 (sec) , antiderivative size = 865, normalized size of antiderivative = 2.97

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx = \frac{bd^3x^3}{3f^3} + x\left(\frac{ad^3}{f^3} + \frac{3bcd^2}{f^3} - \frac{3bd^3e}{f^4}\right)$$
$$- \frac{\sqrt{-\frac{1}{e^5f^9}}(cf-de)(3ac^2f^3+6acdef^2+15ad^2e^2f+bc^2ef^2+10bcde^2f-35bd^2e^3)\log\left(-\frac{e^3f^4\sqrt{-\frac{1}{e^5f^9}}(cf\ldots}{3ac^3f^4+3ac^2def^3\ldots}\right.}{16}$$
$$+ \frac{\sqrt{-\frac{1}{e^5f^9}}(cf-de)(3ac^2f^3+6acdef^2+15ad^2e^2f+bc^2ef^2+10bcde^2f-35bd^2e^3)\log\left(\frac{e^3f^4\sqrt{-\frac{1}{e^5f^9}}(cf-\ldots}{3ac^3f^4+3ac^2def^3+9\ldots}\right.}{16}$$
$$+ \frac{x^3\cdot(3ac^3f^5+3ac^2def^4-15acd^2e^2f^3+9ad^3e^3f^2+bc^3ef^4-15bc^2de^2f^3+27bcd^2e^3f^2-13bd^3e^4f)+\ldots}{8e^4f^4+16e^3f^5x^2\ldots}$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**3,x)
```

```
[Out] b*d**3*x**3/(3*f**3) + x*(a*d**3/f**3 + 3*b*c*d**2/f**3 - 3*b*d**3*e/f**4)
- sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 15*a*d
**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)*log(-e**3*f*
*4*sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 15*a*
d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)/(3*a*c**3*f
**4 + 3*a*c**2*d*e*f**3 + 9*a*c*d**2*e**2*f**2 - 15*a*d**3*e**3*f + b*c**3*
e*f**3 + 9*b*c**2*d*e**2*f**2 - 45*b*c*d**2*e**3*f + 35*b*d**3*e**4) + x)/1
6 + sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 15*a
*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)*log(e**3*f
**4*sqrt(-1/(e**5*f**9))*(c*f - d*e)*(3*a*c**2*f**3 + 6*a*c*d*e*f**2 + 15*a
*d**2*e**2*f + b*c**2*e*f**2 + 10*b*c*d*e**2*f - 35*b*d**2*e**3)/(3*a*c**3*
f**4 + 3*a*c**2*d*e*f**3 + 9*a*c*d**2*e**2*f**2 - 15*a*d**3*e**3*f + b*c**3
*e*f**3 + 9*b*c**2*d*e**2*f**2 - 45*b*c*d**2*e**3*f + 35*b*d**3*e**4) + x)/
16 + (x**3*(3*a*c**3*f**5 + 3*a*c**2*d*e*f**4 - 15*a*c*d**2*e**2*f**3 + 9*a
*d**3*e**3*f**2 + b*c**3*e*f**4 - 15*b*c**2*d*e**2*f**3 + 27*b*c*d**2*e**3*
f**2 - 13*b*d**3*e**4*f) + x*(5*a*c**3*e*f**4 - 3*a*c**2*d*e**2*f**3 - 9*a*
c*d**2*e**3*f**2 + 7*a*d**3*e**4*f - b*c**3*e**2*f**3 - 9*b*c**2*d*e**3*f**
2 + 21*b*c*d**2*e**4*f - 11*b*d**3*e**5))/(8*e**4*f**4 + 16*e**3*f**5*x**2
+ 8*e**2*f**6*x**4)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx$$

$$= \frac{(35\,bd^3e^4 - 45\,bcd^2e^3f - 15\,ad^3e^3f + 9\,bc^2de^2f^2 + 9\,acd^2e^2f^2 + bc^3ef^3 + 3\,ac^2def^3 + 3\,ac^3f^4)\arctan\left(\frac{fx}{\sqrt{ef}}\right)}{8\sqrt{ef}e^2f^4}$$

$$- \frac{13\,bd^3e^4fx^3 - 27\,bcd^2e^3f^2x^3 - 9\,ad^3e^3f^2x^3 + 15\,bc^2de^2f^3x^3 + 15\,acd^2e^2f^3x^3 - bc^3ef^4x^3 - 3\,ac^2def^4x^3 - \cdots}{8\cdots}$$

$$+ \frac{bd^3f^6x^3 - 9\,bd^3ef^5x + 9\,bcd^2f^6x + 3\,ad^3f^6x}{3\,f^9}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*b*d^3*e^4 - 45*b*c*d^2*e^3*f - 15*a*d^3*e^3*f + 9*b*c^2*d*e^2*f^2 +
 9*a*c*d^2*e^2*f^2 + b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 + 3*a*c^3*f^4)*arctan(f*
x/sqrt(e*f))/(sqrt(e*f)*e^2*f^4) - 1/8*(13*b*d^3*e^4*f*x^3 - 27*b*c*d^2*e^3
*f^2*x^3 - 9*a*d^3*e^3*f^2*x^3 + 15*b*c^2*d*e^2*f^3*x^3 + 15*a*c*d^2*e^2*f^
3*x^3 - b*c^3*e*f^4*x^3 - 3*a*c^2*d*e*f^4*x^3 - 3*a*c^3*f^5*x^3 + 11*b*d^3*
e^5*x - 21*b*c*d^2*e^4*f*x - 7*a*d^3*e^4*f*x + 9*b*c^2*d*e^3*f^2*x + 9*a*c*
d^2*e^3*f^2*x + b*c^3*e^2*f^3*x + 3*a*c^2*d*e^2*f^3*x - 5*a*c^3*e*f^4*x)/((
f*x^2 + e)^2*e^2*f^4) + 1/3*(b*d^3*f^6*x^3 - 9*b*d^3*e*f^5*x + 9*b*c*d^2*f^
6*x + 3*a*d^3*f^6*x)/f^9
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 495, normalized size of antiderivative = 1.70

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^3}\,dx$$
$$= \frac{\frac{x^3\,(b\,c^3\,e\,f^4+3\,a\,c^3\,f^5-15\,b\,c^2\,d\,e^2\,f^3+3\,a\,c^2\,d\,e\,f^4+27\,b\,c\,d^2\,e^3\,f^2-15\,a\,c\,d^2\,e^2\,f^3-13\,b\,d^3\,e^4\,f+9\,a\,d^3\,e^3\,f^2)}{8\,e^2} - \frac{x\,(b\,c^3\,e\,f^3-5\,a\,c^3\,f^4+9\,b\,c^2\,d\,e^2 \cdots}{\cdots}}{e^2\,f^4+2\,e\,f^5\,x^2+f^6\,x^4}$$
$$+ x\left(\frac{a\,d^3+3\,b\,c\,d^2}{f^3}-\frac{3\,b\,d^3\,e}{f^4}\right)+\frac{b\,d^3\,x^3}{3\,f^3}$$
$$+ \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x\,(c\,f-d\,e)\,(b\,c^2\,e\,f^2+3\,a\,c^2\,f^3+10\,b\,c\,d\,e^2\,f+6\,a\,c\,d\,e\,f^2-35\,b\,d^2\,e^3+15\,a\,d^2\,e^2\,f)}{\sqrt{e}\,(b\,c^3\,e\,f^3+3\,a\,c^3\,f^4+9\,b\,c^2\,d\,e^2\,f^2+3\,a\,c^2\,d\,e\,f^3-45\,b\,c\,d^2\,e^3\,f+9\,a\,c\,d^2\,e^2\,f^2+35\,b\,d^3\,e^4-15\,a\,d^3\,e^3\,f)}\right)\,(c\,f-d\,e)\,(b\,c^2\,e\,f^2 \cdots}{8\,e^{5/2}\,f^{9/2}}$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^3,x)
```

```
[Out] ((x^3*(3*a*c^3*f^5 + 9*a*d^3*e^3*f^2 + b*c^3*e*f^4 - 13*b*d^3*e^4*f + 3*a*c
^2*d*e*f^4 - 15*a*c*d^2*e^2*f^3 + 27*b*c*d^2*e^3*f^2 - 15*b*c^2*d*e^2*f^3))
/(8*e^2) - (x*(11*b*d^3*e^4 - 5*a*c^3*f^4 - 7*a*d^3*e^3*f + b*c^3*e*f^3 + 3
*a*c^2*d*e*f^3 - 21*b*c*d^2*e^3*f + 9*a*c*d^2*e^2*f^2 + 9*b*c^2*d*e^2*f^2))
/(8*e))/(e^2*f^4 + f^6*x^4 + 2*e*f^5*x^2) + x*((a*d^3 + 3*b*c*d^2)/f^3 - (3
*b*d^3*e)/f^4) + (b*d^3*x^3)/(3*f^3) + (atan((f^(1/2)*x*(c*f - d*e)*(3*a*c^
2*f^3 - 35*b*d^2*e^3 + 15*a*d^2*e^2*f + b*c^2*e*f^2 + 6*a*c*d*e*f^2 + 10*b*
c*d*e^2*f))/(e^(1/2)*(3*a*c^3*f^4 + 35*b*d^3*e^4 - 15*a*d^3*e^3*f + b*c^3*e
*f^3 + 3*a*c^2*d*e*f^3 - 45*b*c*d^2*e^3*f + 9*a*c*d^2*e^2*f^2 + 9*b*c^2*d*e
^2*f^2)))*(c*f - d*e)*(3*a*c^2*f^3 - 35*b*d^2*e^3 + 15*a*d^2*e^2*f + b*c^2*
e*f^2 + 6*a*c*d*e*f^2 + 10*b*c*d*e^2*f))/(8*e^(5/2)*f^(9/2))
```

3.22 $\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4} dx$

Optimal result

Integrand size = 26, antiderivative size = 348

$$\begin{aligned}
&\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4} dx \\
&= \frac{d(be(105d^2e^2-10cdef-3c^2f^2)-af(15d^2e^2+14cdef+15c^2f^2))x}{48e^3f^4} \\
&\quad - \frac{(be-af)x(c+dx^2)^3}{6ef(e+fx^2)^3} - \frac{(be(7de-cf)-af(de+5cf))x(c+dx^2)^2}{24e^2f^2(e+fx^2)^2} \\
&\quad - \frac{(be(35d^2e^2-8cdef-3c^2f^2)-af(5d^2e^2+4cdef+15c^2f^2))x(c+dx^2)}{48e^3f^3(e+fx^2)} \\
&\quad - \frac{(be(35d^3e^3-15cd^2e^2f-3c^2def^2-c^3f^3)-af(5d^3e^3+3cd^2e^2f+3c^2def^2+5c^3f^3))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{9/2}}
\end{aligned}$$

```
[Out] 1/48*d*(b*e*(-3*c^2*f^2-10*c*d*e*f+105*d^2*e^2)-a*f*(15*c^2*f^2+14*c*d*e*f+
15*d^2*e^2))*x/e^3/f^4-1/6*(-a*f+b*e)*x*(d*x^2+c)^3/e/f/(f*x^2+e)^3-1/24*(b
*e*(-c*f+7*d*e)-a*f*(5*c*f+d*e))*x*(d*x^2+c)^2/e^2/f^2/(f*x^2+e)^2-1/48*(b*
e*(-3*c^2*f^2-8*c*d*e*f+35*d^2*e^2)-a*f*(15*c^2*f^2+4*c*d*e*f+5*d^2*e^2))*x
*(d*x^2+c)/e^3/f^3/(f*x^2+e)-1/16*(b*e*(-c^3*f^3-3*c^2*d*e*f^2-15*c*d^2*e^2
*f+35*d^3*e^3)-a*f*(5*c^3*f^3+3*c^2*d*e*f^2+3*c*d^2*e^2*f+5*d^3*e^3))*arcta
n(x*f^(1/2)/e^(1/2))/e^(7/2)/f^(9/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {540, 396, 211}

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx =$$
$$-\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\left(be(-c^3f^3-3c^2def^2-15cd^2e^2f+35d^3e^3)-af(5c^3f^3+3c^2def^2+3cd^2e^2f+5d^3e^3)\right)}{16e^{7/2}f^{9/2}}$$
$$+\frac{dx(be(-3c^2f^2-10cdef+105d^2e^2)-af(15c^2f^2+14cdef+15d^2e^2))}{48e^3f^4}$$
$$-\frac{x(c+dx^2)\left(be(-3c^2f^2-8cdef+35d^2e^2)-af(15c^2f^2+4cdef+5d^2e^2)\right)}{48e^3f^3(e+fx^2)}$$
$$-\frac{x(c+dx^2)^2(be(7de-cf)-af(5cf+de))}{24e^2f^2(e+fx^2)^2}-\frac{x(c+dx^2)^3(be-af)}{6ef(e+fx^2)^3}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

```
[Out] (d*(b*e*(105*d^2*e^2 - 10*c*d*e*f - 3*c^2*f^2) - a*f*(15*d^2*e^2 + 14*c*d*e
*f + 15*c^2*f^2))*x)/(48*e^3*f^4) - ((b*e - a*f)*x*(c + d*x^2)^3)/(6*e*f*(e
 + f*x^2)^3) - ((b*e*(7*d*e - c*f) - a*f*(d*e + 5*c*f))*x*(c + d*x^2)^2)/(2
4*e^2*f^2*(e + f*x^2)^2) - ((b*e*(35*d^2*e^2 - 8*c*d*e*f - 3*c^2*f^2) - a*f
*(5*d^2*e^2 + 4*c*d*e*f + 15*c^2*f^2))*x*(c + d*x^2))/(48*e^3*f^3*(e + f*x^
2)) - ((b*e*(35*d^3*e^3 - 15*c*d^2*e^2*f - 3*c^2*d*e*f^2 - c^3*f^3) - a*f*(
5*d^3*e^3 + 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f]*x)/
Sqrt[e]])/(16*e^(7/2)*f^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
```

```
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\text{integral} = -\frac{(be-af)x\left(c+dx^2\right)^3}{6ef\left(e+fx^2\right)^3} - \frac{\int \frac{\left(c+dx^2\right)^2\left(-c(be+5af)-d(7be-af)x^2\right)}{\left(e+fx^2\right)^3}\,dx}{6ef}$$

$$= -\frac{(be-af)x\left(c+dx^2\right)^3}{6ef\left(e+fx^2\right)^3} - \frac{(be(7de-cf)-af(de+5cf))x\left(c+dx^2\right)^2}{24e^2f^2\left(e+fx^2\right)^2} + \frac{\int \frac{\left(c+dx^2\right)\left(c(de(7be-af)+3cf(be+5af))+d(be(35de-cf)-5af(de+cf))x^2\right)}{\left(e+fx^2\right)^2}\,dx}{24e^2f^2}$$

$$= -\frac{(be-af)x\left(c+dx^2\right)^3}{6ef\left(e+fx^2\right)^3} - \frac{(be(7de-cf)-af(de+5cf))x\left(c+dx^2\right)^2}{24e^2f^2\left(e+fx^2\right)^2} - \frac{\left(be\left(35d^2e^2-8cdef-3c^2f^2\right)-af\left(5d^2e^2+4cdef+15c^2f^2\right)\right)x\left(c+dx^2\right)}{48e^3f^3\left(e+fx^2\right)} - \frac{\int \frac{c\left(af\left(5d^2e^2+6cdef-15c^2f^2\right)-be\left(35d^2e^2+6cdef+3c^2f^2\right)\right)-d\left(be\left(105d^2e^2-10cdef-3c^2f^2\right)-af\left(15d^2e^2+14cdef+15c^2f^2\right)\right)x^2}{e+fx^2}\,dx}{48e^3f^3}$$

$$= \frac{d\left(be\left(105d^2e^2-10cdef-3c^2f^2\right)-af\left(15d^2e^2+14cdef+15c^2f^2\right)\right)x}{48e^3f^4} - \frac{(be-af)x\left(c+dx^2\right)^3}{6ef\left(e+fx^2\right)^3} - \frac{(be(7de-cf)-af(de+5cf))x\left(c+dx^2\right)^2}{24e^2f^2\left(e+fx^2\right)^2} - \frac{\left(be\left(35d^2e^2-8cdef-3c^2f^2\right)-af\left(5d^2e^2+4cdef+15c^2f^2\right)\right)x\left(c+dx^2\right)}{48e^3f^3\left(e+fx^2\right)} - \frac{\left(be\left(35d^3e^3-15cd^2e^2f-3c^2def^2-c^3f^3\right)-af\left(5d^3e^3+3cd^2e^2f+3c^2def^2+5c^3f^3\right)\right)\int \frac{1}{e+fx^2}\,dx}{16e^3f^4}$$

$$= \frac{d\left(be\left(105d^2e^2-10cdef-3c^2f^2\right)-af\left(15d^2e^2+14cdef+15c^2f^2\right)\right)x}{48e^3f^4} - \frac{(be-af)x\left(c+dx^2\right)^3}{6ef\left(e+fx^2\right)^3} - \frac{(be(7de-cf)-af(de+5cf))x\left(c+dx^2\right)^2}{24e^2f^2\left(e+fx^2\right)^2} - \frac{\left(be\left(35d^2e^2-8cdef-3c^2f^2\right)-af\left(5d^2e^2+4cdef+15c^2f^2\right)\right)x\left(c+dx^2\right)}{48e^3f^3\left(e+fx^2\right)} - \frac{\left(be\left(35d^3e^3-15cd^2e^2f-3c^2def^2-c^3f^3\right)-af\left(5d^3e^3+3cd^2e^2f+3c^2def^2+5c^3f^3\right)\right)\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{9/2}}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 295, normalized size of antiderivative = 0.85

$$
\begin{aligned}
&\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx \\
&= \frac{bd^3x}{f^4} + \frac{(be-af)(de-cf)^3x}{6ef^4(e+fx^2)^3} - \frac{(de-cf)^2(be(19de-cf)-af(13de+5cf))x}{24e^2f^4(e+fx^2)^2} \\
&\quad + \frac{(de-cf)\left(be(29d^2e^2-4cdef-c^2f^2)-af(11d^2e^2+8cdef+5c^2f^2)\right)x}{16e^3f^4(e+fx^2)} \\
&\quad - \frac{(be(35d^3e^3-15cd^2e^2f-3c^2def^2-c^3f^3)-af(5d^3e^3+3cd^2e^2f+3c^2def^2+5c^3f^3))\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)}{16e^{7/2}f^{9/2}}
\end{aligned}
$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x]
```

```
[Out] (b*d^3*x)/f^4 + ((b*e - a*f)*(d*e - c*f)^3*x)/(6*e*f^4*(e + f*x^2)^3) - ((d
*e - c*f)^2*(b*e*(19*d*e - c*f) - a*f*(13*d*e + 5*c*f))*x)/(24*e^2*f^4*(e +
 f*x^2)^2) + ((d*e - c*f)*(b*e*(29*d^2*e^2 - 4*c*d*e*f - c^2*f^2) - a*f*(11
*d^2*e^2 + 8*c*d*e*f + 5*c^2*f^2))*x)/(16*e^3*f^4*(e + f*x^2)) - ((b*e*(35*
d^3*e^3 - 15*c*d^2*e^2*f - 3*c^2*d*e*f^2 - c^3*f^3) - a*f*(5*d^3*e^3 + 3*c*
d^2*e^2*f + 3*c^2*d*e*f^2 + 5*c^3*f^3))*ArcTan[(Sqrt[f]*x)/Sqrt[e]])/(16*e^
(7/2)*f^(9/2))
```

Maple [A] (verified)

Time = 3.37 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.20

method	result
default	$\frac{b\,d^3x}{f^4}+\frac{\frac{f^2\left(5a\,c^3f^4+3a\,c^2de\,f^3+3ac\,d^2e^2f^2-11a\,d^3e^3f+b\,c^3e\,f^3+3b\,c^2d\,e^2f^2-33bc\,d^2e^3f+29b\,d^3e^4\right)x^5}{16e^3}+\frac{f\left(5a\,c^3f^4+3a\,c^2de\,f^3-3ac\,d^2e^2f^2\right.}{}}{}$
risch	$\frac{b\,d^3x}{f^4}+\frac{\frac{f^2\left(5a\,c^3f^4+3a\,c^2de\,f^3+3ac\,d^2e^2f^2-11a\,d^3e^3f+b\,c^3e\,f^3+3b\,c^2d\,e^2f^2-33bc\,d^2e^3f+29b\,d^3e^4\right)x^5}{16e^3}+\frac{f\left(5a\,c^3f^4+3a\,c^2de\,f^3-3ac\,d^2e^2f^2\right.}{}}{}$

```
[In] int((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x,method=_RETURNVERBOSE)
```

```
[Out] b*d^3/f^4*x+1/f^4*((1/16*f^2*(5*a*c^3*f^4+3*a*c^2*d*e*f^3+3*a*c*d^2*e^2*f^2
-11*a*d^3*e^3*f+b*c^3*e*f^3+3*b*c^2*d*e^2*f^2-33*b*c*d^2*e^3*f+29*b*d^3*e^4
)/e^3*x^5+1/6*f*(5*a*c^3*f^4+3*a*c^2*d*e*f^3-3*a*c*d^2*e^2*f^2-5*a*d^3*e^3*
f+b*c^3*e*f^3-3*b*c^2*d*e^2*f^2-15*b*c*d^2*e^3*f+17*b*d^3*e^4)/e^2*x^3+1/16
*(11*a*c^3*f^4-3*a*c^2*d*e*f^3-3*a*c*d^2*e^2*f^2-5*a*d^3*e^3*f-b*c^3*e*f^3-
3*b*c^2*d*e^2*f^2-15*b*c*d^2*e^3*f+19*b*d^3*e^4)/e*x)/(f*x^2+e)^3+1/16*(5*a
*c^3*f^4+3*a*c^2*d*e*f^3+3*a*c*d^2*e^2*f^2+5*a*d^3*e^3*f+b*c^3*e*f^3+3*b*c^
```

```
2*d*e^2*f^2+15*b*c*d^2*e^3*f-35*b*d^3*e^4)/e^3/(e*f)^(1/2)*arctan(f*x/(e*f)
^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 701 vs. 2(330) = 660.

Time = 0.29 (sec) , antiderivative size = 1422, normalized size of antiderivative = 4.09

$$\int \frac{(a+bx^2)\,(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="fricas")
```

```
[Out] [1/96*(96*b*d^3*e^4*f^4*x^7 + 6*(77*b*d^3*e^5*f^3 + 5*a*c^3*e*f^7 - 11*(3*b
*c*d^2 + a*d^3)*e^4*f^4 + 3*(b*c^2*d + a*c*d^2)*e^3*f^5 + (b*c^3 + 3*a*c^2*
d)*e^2*f^6)*x^5 + 16*(35*b*d^3*e^6*f^2 + 5*a*c^3*e^2*f^6 - 5*(3*b*c*d^2 + a
*d^3)*e^5*f^3 - 3*(b*c^2*d + a*c*d^2)*e^4*f^4 + (b*c^3 + 3*a*c^2*d)*e^3*f^5
)*x^3 + 3*(35*b*d^3*e^7 - 5*a*c^3*e^3*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^6*f - 3
*(b*c^2*d + a*c*d^2)*e^5*f^2 - (b*c^3 + 3*a*c^2*d)*e^4*f^3 + (35*b*d^3*e^4*
f^3 - 5*a*c^3*f^7 - 5*(3*b*c*d^2 + a*d^3)*e^3*f^4 - 3*(b*c^2*d + a*c*d^2)*e
^2*f^5 - (b*c^3 + 3*a*c^2*d)*e*f^6)*x^6 + 3*(35*b*d^3*e^5*f^2 - 5*a*c^3*e*f
^6 - 5*(3*b*c*d^2 + a*d^3)*e^4*f^3 - 3*(b*c^2*d + a*c*d^2)*e^3*f^4 - (b*c^3
 + 3*a*c^2*d)*e^2*f^5)*x^4 + 3*(35*b*d^3*e^6*f - 5*a*c^3*e^2*f^5 - 5*(3*b*c
*d^2 + a*d^3)*e^5*f^2 - 3*(b*c^2*d + a*c*d^2)*e^4*f^3 - (b*c^3 + 3*a*c^2*d)
*e^3*f^4)*x^2)*sqrt(-e*f)*log((f*x^2 - 2*sqrt(-e*f)*x - e)/(f*x^2 + e)) + 6
*(35*b*d^3*e^7*f + 11*a*c^3*e^3*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^6*f^2 - 3*(b*
c^2*d + a*c*d^2)*e^5*f^3 - (b*c^3 + 3*a*c^2*d)*e^4*f^4)*x)/(e^4*f^8*x^6 + 3
*e^5*f^7*x^4 + 3*e^6*f^6*x^2 + e^7*f^5), 1/48*(48*b*d^3*e^4*f^4*x^7 + 3*(77
*b*d^3*e^5*f^3 + 5*a*c^3*e*f^7 - 11*(3*b*c*d^2 + a*d^3)*e^4*f^4 + 3*(b*c^2*
d + a*c*d^2)*e^3*f^5 + (b*c^3 + 3*a*c^2*d)*e^2*f^6)*x^5 + 8*(35*b*d^3*e^6*f
^2 + 5*a*c^3*e^2*f^6 - 5*(3*b*c*d^2 + a*d^3)*e^5*f^3 - 3*(b*c^2*d + a*c*d^2
)*e^4*f^4 + (b*c^3 + 3*a*c^2*d)*e^3*f^5)*x^3 - 3*(35*b*d^3*e^7 - 5*a*c^3*e^
3*f^4 - 5*(3*b*c*d^2 + a*d^3)*e^6*f - 3*(b*c^2*d + a*c*d^2)*e^5*f^2 - (b*c^
3 + 3*a*c^2*d)*e^4*f^3 + (35*b*d^3*e^4*f^3 - 5*a*c^3*f^7 - 5*(3*b*c*d^2 + a
*d^3)*e^3*f^4 - 3*(b*c^2*d + a*c*d^2)*e^2*f^5 - (b*c^3 + 3*a*c^2*d)*e*f^6)*
x^6 + 3*(35*b*d^3*e^5*f^2 - 5*a*c^3*e*f^6 - 5*(3*b*c*d^2 + a*d^3)*e^4*f^3 -
 3*(b*c^2*d + a*c*d^2)*e^3*f^4 - (b*c^3 + 3*a*c^2*d)*e^2*f^5)*x^4 + 3*(35*b
*d^3*e^6*f - 5*a*c^3*e^2*f^5 - 5*(3*b*c*d^2 + a*d^3)*e^5*f^2 - 3*(b*c^2*d +
 a*c*d^2)*e^4*f^3 - (b*c^3 + 3*a*c^2*d)*e^3*f^4)*x^2)*sqrt(e*f)*arctan(sqrt
(e*f)*x/e) + 3*(35*b*d^3*e^7*f + 11*a*c^3*e^3*f^5 - 5*(3*b*c*d^2 + a*d^3)*e
^6*f^2 - 3*(b*c^2*d + a*c*d^2)*e^5*f^3 - (b*c^3 + 3*a*c^2*d)*e^4*f^4)*x)/(e
^4*f^8*x^6 + 3*e^5*f^7*x^4 + 3*e^6*f^6*x^2 + e^7*f^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**3/(f*x**2+e)**4,x)
```

```
[Out] Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.36

$$\int \frac{(a+bx^2)(c+dx^2)^3}{(e+fx^2)^4}\,dx = \frac{bd^3x}{f^4} - \frac{(35\,bd^3e^4 - 15\,bcd^2e^3f - 5\,ad^3e^3f - 3\,bc^2de^2f^2 - 3\,acd^2e^2f^2 - bc^3ef^3 - 3\,ac^2def^3 - 5\,ac^3f^4)\arctan\left(\right.}{16\sqrt{ef}e^3f^4} + \frac{87\,bd^3e^4f^2x^5 - 99\,bcd^2e^3f^3x^5 - 33\,ad^3e^3f^3x^5 + 9\,bc^2de^2f^4x^5 + 9\,acd^2e^2f^4x^5 + 3\,bc^3ef^5x^5 + 9\,ac^2def^5x}{}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^3/(f*x^2+e)^4,x, algorithm="giac")
```

```
[Out] b*d^3*x/f^4 - 1/16*(35*b*d^3*e^4 - 15*b*c*d^2*e^3*f - 5*a*d^3*e^3*f - 3*b*c
^2*d*e^2*f^2 - 3*a*c*d^2*e^2*f^2 - b*c^3*e*f^3 - 3*a*c^2*d*e*f^3 - 5*a*c^3*
f^4)*arctan(f*x/sqrt(e*f))/(sqrt(e*f)*e^3*f^4) + 1/48*(87*b*d^3*e^4*f^2*x^5
 - 99*b*c*d^2*e^3*f^3*x^5 - 33*a*d^3*e^3*f^3*x^5 + 9*b*c^2*d*e^2*f^4*x^5 +
9*a*c*d^2*e^2*f^4*x^5 + 3*b*c^3*e*f^5*x^5 + 9*a*c^2*d*e*f^5*x^5 + 15*a*c^3*
```

```
f^6*x^5 + 136*b*d^3*e^5*f*x^3 - 120*b*c*d^2*e^4*f^2*x^3 - 40*a*d^3*e^4*f^2*
x^3 - 24*b*c^2*d*e^3*f^3*x^3 - 24*a*c*d^2*e^3*f^3*x^3 + 8*b*c^3*e^2*f^4*x^3
 + 24*a*c^2*d*e^2*f^4*x^3 + 40*a*c^3*e*f^5*x^3 + 57*b*d^3*e^6*x - 45*b*c*d^
2*e^5*f*x - 15*a*d^3*e^5*f*x - 9*b*c^2*d*e^4*f^2*x - 9*a*c*d^2*e^4*f^2*x -
3*b*c^3*e^3*f^3*x - 9*a*c^2*d*e^3*f^3*x + 33*a*c^3*e^2*f^4*x)/((f*x^2 + e)^
3*e^3*f^4)
```

Mupad [B] (verification not implemented)

Time = 5.66 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx^2)\,(c+dx^2)^3}{(e+fx^2)^4}\,dx$$
$$= \frac{\frac{x^3\,(b\,c^3\,e\,f^4+5\,a\,c^3\,f^5-3\,b\,c^2\,d\,e^2\,f^3+3\,a\,c^2\,d\,e\,f^4-15\,b\,c\,d^2\,e^3\,f^2-3\,a\,c\,d^2\,e^2\,f^3+17\,b\,d^3\,e^4\,f-5\,a\,d^3\,e^3\,f^2)}{6\,e^2}+\frac{x^5\,(b\,c^3\,e\,f^5+5\,a\,c^3\,f^6+3\,b\,c^2\,d\,e^2\,f^4}{}}{e^3\,f^4-}$$
$$+\frac{b\,d^3\,x}{f^4}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{e}}\right)\,(b\,c^3\,e\,f^3+5\,a\,c^3\,f^4+3\,b\,c^2\,d\,e^2\,f^2+3\,a\,c^2\,d\,e\,f^3+15\,b\,c\,d^2\,e^3\,f+3\,a\,c\,d^2\,e^2\,f^2-35\,b\,d^3\,e^4+}{16\,e^{7/2}\,f^{9/2}}$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^3)/(e + f*x^2)^4,x)
```

```
[Out] ((x^3*(5*a*c^3*f^5 - 5*a*d^3*e^3*f^2 + b*c^3*e*f^4 + 17*b*d^3*e^4*f + 3*a*c
^2*d*e*f^4 - 3*a*c*d^2*e^2*f^3 - 15*b*c*d^2*e^3*f^2 - 3*b*c^2*d*e^2*f^3))/(
6*e^2) + (x^5*(5*a*c^3*f^6 - 11*a*d^3*e^3*f^3 + 29*b*d^3*e^4*f^2 + b*c^3*e*
f^5 + 3*a*c^2*d*e*f^5 + 3*a*c*d^2*e^2*f^4 - 33*b*c*d^2*e^3*f^3 + 3*b*c^2*d*
e^2*f^4))/(16*e^3) - (x*(5*a*d^3*e^3*f - 19*b*d^3*e^4 - 11*a*c^3*f^4 + b*c^
3*e*f^3 + 3*a*c^2*d*e*f^3 + 15*b*c*d^2*e^3*f + 3*a*c*d^2*e^2*f^2 + 3*b*c^2*
d*e^2*f^2))/(16*e))/(e^3*f^4 + f^7*x^6 + 3*e*f^6*x^4 + 3*e^2*f^5*x^2) + (b*
d^3*x)/f^4 + (atan((f^(1/2)*x)/e^(1/2))*(5*a*c^3*f^4 - 35*b*d^3*e^4 + 5*a*d
^3*e^3*f + b*c^3*e*f^3 + 3*a*c^2*d*e*f^3 + 15*b*c*d^2*e^3*f + 3*a*c*d^2*e^2
*f^2 + 3*b*c^2*d*e^2*f^2))/(16*e^(7/2)*f^(9/2))
```

3.23 $\int (a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 544

$$
\begin{aligned}
&\int (a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}\,dx = \\
&-\frac{(7adf(2d^2e^2-7cdef-3c^2f^2)-b(8d^3e^3-19cd^2e^2f+9c^2def^2-6c^3f^3))\,x\sqrt{c+dx^2}}{105d^2f^2\sqrt{e+fx^2}} \\
&+\frac{(7adf(de+3cf)-b(4d^2e^2-6cdef+6c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105df^2} \\
&+\frac{(bde-2bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35df}+\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7d} \\
&+\frac{\sqrt{e}(7adf(2d^2e^2-7cdef-3c^2f^2)-b(8d^3e^3-19cd^2e^2f+9c^2def^2-6c^3f^3))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1\right.}{105d^2f^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&-\frac{e^{3/2}(7adf(de-9cf)-b(4d^2e^2-9cdef-3c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{105df^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

```
[Out] -1/105*(7*a*d*f*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2)-b*(-6*c^3*f^3+9*c^2*d*e*f^
2-19*c*d^2*e^2*f+8*d^3*e^3))*x*(d*x^2+c)^(1/2)/d^2/f^2/(f*x^2+e)^(1/2)-1/10
5*e^(3/2)*(7*a*d*f*(-9*c*f+d*e)-b*(-3*c^2*f^2-9*c*d*e*f+4*d^2*e^2))*(1/(1+f
*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1
/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/d/f^(5/2)/(e*(d*x^2+c)/c/(f*x^2+e))^
(1/2)/(f*x^2+e)^(1/2)+1/105*(7*a*d*f*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2)-b*(-6
*c^3*f^3+9*c^2*d*e*f^2-19*c*d^2*e^2*f+8*d^3*e^3))*(1/(1+f*x^2/e))^(1/2)*(1+
f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1
/2))*e^(1/2)*(d*x^2+c)^(1/2)/d^2/f^(5/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f
*x^2+e)^(1/2)+1/35*(7*a*d*f-2*b*c*f+b*d*e)*x*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2
```

```
)/d/f+1/7*b*x*(d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/d+1/105*(7*a*d*f*(3*c*f+d*e)-
b*(6*c^2*f^2-6*c*d*e*f+4*d^2*e^2))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/f^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {542, 545, 429, 506, 422}

$$\int (a+bx^2)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx =$$

$$-\frac{e^{3/2}\sqrt{c+dx^2}(7adf(de-9cf)-b(-3c^2f^2-9cdef+4d^2e^2))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{105df^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$$+\frac{\sqrt{e}\sqrt{c+dx^2}(7adf(-3c^2f^2-7cdef+2d^2e^2)-b(-6c^3f^3+9c^2def^2-19cd^2e^2f+8d^3e^3))\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\right)}{105d^2f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$$+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(7adf(3cf+de)-b(6c^2f^2-6cdef+4d^2e^2))}{105df^2}$$

$$-\frac{x\sqrt{c+dx^2}(7adf(-3c^2f^2-7cdef+2d^2e^2)-b(-6c^3f^3+9c^2def^2-19cd^2e^2f+8d^3e^3))}{105d^2f^2\sqrt{e+fx^2}}$$

$$+\frac{x(c+dx^2)^{3/2}\sqrt{e+fx^2}(7adf-2bcf+bde)}{35df}+\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7d}$$

```
[In] Int[(a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2],x]
```

```
[Out] -1/105*((7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) - b*(8*d^3*e^3 - 19*c*
d^2*e^2*f + 9*c^2*d*e*f^2 - 6*c^3*f^3))*x*Sqrt[c + d*x^2])/(d^2*f^2*Sqrt[e
+ f*x^2]) + ((7*a*d*f*(d*e + 3*c*f) - b*(4*d^2*e^2 - 6*c*d*e*f + 6*c^2*f^2)
)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(105*d*f^2) + ((b*d*e - 2*b*c*f + 7*a*
d*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(35*d*f) + (b*x*(c + d*x^2)^(5/2)
*Sqrt[e + f*x^2])/(7*d) + (Sqrt[e]*(7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*
f^2) - b*(8*d^3*e^3 - 19*c*d^2*e^2*f + 9*c^2*d*e*f^2 - 6*c^3*f^3))*Sqrt[c +
 d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d^2*f
^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (e^(3/2)*(7
*a*d*f*(d*e - 9*c*f) - b*(4*d^2*e^2 - 9*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^
2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d*f^(5/2)*
Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7d} + \frac{\int \frac{(c+dx^2)^{3/2}(-((bc-7ad)e)+(bde-2bcf+7adf)x^2)}{\sqrt{e+fx^2}}\,dx}{7d} \\
&= \frac{(bde-2bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35df} + \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7d} \\
&\quad + \frac{\int \frac{\sqrt{c+dx^2}(-ce(bde+3bcf-28adf)+(7adf(de+3cf)-b(4d^2e^2-6cdef+6c^2f^2))x^2)}{\sqrt{e+fx^2}}\,dx}{35df}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{(7adf(de+3cf)-b(4d^2e^2-6cdef+6c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105df^2} \\
&\quad + \frac{(bde-2bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35df} + \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7d} \\
&\quad + \frac{\int \frac{-ce(7adf(de-9cf)-b(4d^2e^2-9cdef-3c^2f^2))+(-7adf(2d^2e^2-7cdef-3c^2f^2)+b(8d^3e^3-19cd^2e^2f+9c^2def^2-6c^3f^3))x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{105df^2} \\
&= \frac{(7adf(de+3cf)-b(4d^2e^2-6cdef+6c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105df^2} \\
&\quad + \frac{(bde-2bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35df} + \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7d} \\
&\quad - \frac{(ce(7adf(de-9cf)-b(4d^2e^2-9cdef-3c^2f^2)))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{105df^2} \\
&\quad - \frac{(7adf(2d^2e^2-7cdef-3c^2f^2)-b(8d^3e^3-19cd^2e^2f+9c^2def^2-6c^3f^3))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{105df^2} \\
&= -\frac{(7adf(2d^2e^2-7cdef-3c^2f^2)-b(8d^3e^3-19cd^2e^2f+9c^2def^2-6c^3f^3))\,x\sqrt{c+dx^2}}{105d^2f^2\sqrt{e+fx^2}} \\
&\quad + \frac{(7adf(de+3cf)-b(4d^2e^2-6cdef+6c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105df^2} \\
&\quad + \frac{(bde-2bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35df} + \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7d} \\
&\quad - \frac{e^{3/2}(7adf(de-9cf)-b(4d^2e^2-9cdef-3c^2f^2))\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105df^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{(e(7adf(2d^2e^2-7cdef-3c^2f^2)-b(8d^3e^3-19cd^2e^2f+9c^2def^2-6c^3f^3)))\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{105d^2f^2} \\
&= -\frac{(7adf(2d^2e^2-7cdef-3c^2f^2)-b(8d^3e^3-19cd^2e^2f+9c^2def^2-6c^3f^3))\,x\sqrt{c+dx^2}}{105d^2f^2\sqrt{e+fx^2}} \\
&\quad + \frac{(7adf(de+3cf)-b(4d^2e^2-6cdef+6c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105df^2} \\
&\quad + \frac{(bde-2bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35df} + \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7d} \\
&\quad + \frac{\sqrt{e}(7adf(2d^2e^2-7cdef-3c^2f^2)-b(8d^3e^3-19cd^2e^2f+9c^2def^2-6c^3f^3))\sqrt{c+dx^2}\,E\Big(\tan^{-1}}{105d^2f^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{e^{3/2}(7adf(de-9cf)-b(4d^2e^2-9cdef-3c^2f^2))\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105df^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.13 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.69

$$\int (a+bx^2)\,(c+dx^2)^{3/2}\sqrt{e+fx^2}\,dx = \frac{\sqrt{\frac{d}{c}}fx(c+dx^2)\,(e+fx^2)\,(7adf(6cf+d(e+3fx^2))+b(3c^2f^2+3cdf(3e+8fx^2)\cdots}{\cdots}$$

```
[In] Integrate[(a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(7*a*d*f*(6*c*f + d*(e + 3*f*x^2)) +
 b*(3*c^2*f^2 + 3*c*d*f*(3*e + 8*f*x^2) + d^2*(-4*e^2 + 3*e*f*x^2 + 15*f^2*
x^4))) + I*e*(7*a*d*f*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) + b*(-8*d^3*e^3 +
 19*c*d^2*e^2*f - 9*c^2*d*e*f^2 + 6*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
(f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) + c
*f)*(-14*a*d*f*(d*e - 3*c*f) + b*(8*d^2*e^2 - 15*c*d*e*f + 3*c^2*f^2))*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)
/(d*e)])/(105*d*Sqrt[d/c]*f^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 5.01 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.28

method	result
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{bd\,x^5\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{7}+\frac{\left(a\,d^2f+2bcfd+b\,d^2e-\frac{bd(6cf+6de)}{7}\right)x^3\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{5df}+\frac{\left(2acdf+ae\,d^2+c^2b\cdots\right.}{\cdots}\right.}{\cdots}$
risch	$\frac{x\left(15b\,x^4d^2f^2+21a\,d^2f^2x^2+24bcd\,f^2x^2+3b\,d^2ef\,x^2+42acd\,f^2+7a\,d^2ef+3b\,c^2f^2+9bcdef-4b\,d^2e^2\right)\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{105d\,f^2}+\frac{\left(-\frac{3b\,c^3\cdots}{\cdots}\right.}{\cdots}$
default	Expression too large to display

```
[In] int((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/7*b*d*x^5*(d
*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/5*(a*d^2*f+2*b*c*f*d+b*d^2*e-1/7*b*d*(6
*c*f+6*d*e))/d/f*x^3*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/3*(2*a*c*d*f+a*e
*d^2+c^2*b*f+9/7*b*c*d*e-1/5*(a*d^2*f+2*b*c*f*d+b*d^2*e-1/7*b*d*(6*c*f+6*d*
e))/d/f*(4*c*f+4*d*e))/d/f*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(c^2*a*e-1
/3*(2*a*c*d*f+a*e*d^2+c^2*b*f+9/7*b*c*d*e-1/5*(a*d^2*f+2*b*c*f*d+b*d^2*e-1/
7*b*d*(6*c*f+6*d*e))/d/f*(4*c*f+4*d*e))/d/f*c*e)/(-d/c)^(1/2)*(1+d*x^2/c)^(
1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(c^2*a*f+2*a*c*d*e+b*c^2*e-3/5*(a*d^2*f+
2*b*c*f*d+b*d^2*e-1/7*b*d*(6*c*f+6*d*e))/d/f*c*e-1/3*(2*a*c*d*f+a*e*d^2+c^2
*b*f+9/7*b*c*d*e-1/5*(a*d^2*f+2*b*c*f*d+b*d^2*e-1/7*b*d*(6*c*f+6*d*e))/d/f*
(4*c*f+4*d*e))/d/f*(2*c*f+2*d*e))*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2
/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(
-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))
))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.85

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx =$$

$$-\frac{(8\,bd^3e^4-(19\,bcd^2+14\,ad^3)e^3f+(9\,bc^2d+49\,acd^2)e^2f^2-3\,(2\,bc^3-7\,ac^2d)ef^3)\sqrt{df}x\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-}}{x}\right.}{}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/105*((8*b*d^3*e^4 - (19*b*c*d^2 + 14*a*d^3)*e^3*f + (9*b*c^2*d + 49*a*c*
d^2)*e^2*f^2 - 3*(2*b*c^3 - 7*a*c^2*d)*e*f^3)*sqrt(d*f)*x*sqrt(-e/f)*ellipt
ic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (8*b*d^3*e^4 - (19*b*c*d^2 + 14*a*d
^3)*e^3*f + (9*b*c^2*d + (49*a + 4*b)*c*d^2)*e^2*f^2 - (6*b*c^3 - 3*(7*a -
3*b)*c^2*d + 7*a*c*d^2)*e*f^3 - 3*(b*c^3 - 21*a*c^2*d)*f^4)*sqrt(d*f)*x*sqr
t(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (15*b*d^3*f^4*x^6 + 8
*b*d^3*e^3*f - (19*b*c*d^2 + 14*a*d^3)*e^2*f^2 + (9*b*c^2*d + 49*a*c*d^2)*e
*f^3 - 3*(2*b*c^3 - 7*a*c^2*d)*f^4 + 3*(b*d^3*e*f^3 + (8*b*c*d^2 + 7*a*d^3)
*f^4)*x^4 - (4*b*d^3*e^2*f^2 - (9*b*c*d^2 + 7*a*d^3)*e*f^3 - 3*(b*c^2*d + 1
4*a*c*d^2)*f^4)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d^2*f^4*x)
```

Sympy [F]

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx=\int \left(a+bx^2\right)\left(c+dx^2\right)^{\frac{3}{2}}\sqrt{e+fx^2}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(3/2)*(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)*(c + d*x**2)**(3/2)*sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx=\int \left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{fx^2+e}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx=\int \left(bx^2+a\right)\left(dx^2+c\right)^{\frac{3}{2}}\sqrt{fx^2+e}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+bx^2\right)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}\,dx=\int \left(b\,x^2+a\right)\,\left(d\,x^2+c\right)^{3/2}\,\sqrt{f\,x^2+e}\,dx$$

```
[In] int((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2),x)
[Out] int((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2), x)
```

3.24 $\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 381

$$\begin{aligned}
&\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx \\
&= \frac{(5adf(de+cf)-2b(d^2e^2-cdef+c^2f^2))\,x\sqrt{c+dx^2}}{15d^2f\sqrt{e+fx^2}} \\
&\quad+\frac{(bde-2bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15df}+\frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5d} \\
&\quad-\frac{\sqrt{e}(5adf(de+cf)-2b(d^2e^2-cdef+c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15d^2f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad-\frac{e^{3/2}(bde+bcf-10adf)\sqrt{c+dx^2}\,\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15df^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

```
[Out] 1/15*(5*a*d*f*(c*f+d*e)-2*b*(c^2*f^2-c*d*e*f+d^2*e^2))*x*(d*x^2+c)^(1/2)/d^
2/f/(f*x^2+e)^(1/2)-1/15*e^(3/2)*(-10*a*d*f+b*c*f+b*d*e)*(1/(1+f*x^2/e))^(1
/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/
c/f)^(1/2))*(d*x^2+c)^(1/2)/d/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^
2+e)^(1/2)-1/15*(5*a*d*f*(c*f+d*e)-2*b*(c^2*f^2-c*d*e*f+d^2*e^2))*(1/(1+f*x
^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2
),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/d^2/f^(3/2)/(e*(d*x^2+c)/c/(f*
x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/5*b*x*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/d+1/1
5*(5*a*d*f-2*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {542, 545, 429, 506, 422}

$$\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$
$$= -\frac{\sqrt{e}\sqrt{c+dx^2}(5adf(cf+de)-2b(c^2f^2-cdef+d^2e^2))\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15d^2f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{e^{3/2}\sqrt{c+dx^2}(-10adf+bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15df^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{x\sqrt{c+dx^2}(5adf(cf+de)-2b(c^2f^2-cdef+d^2e^2))}{15d^2f\sqrt{e+fx^2}}$$
$$+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-2bcf+bde)}{15df}+\frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5d}$$

```
[In] Int[(a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

```
[Out] ((5*a*d*f*(d*e + c*f) - 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*x*Sqrt[c + d*x^2
])/(15*d^2*f*Sqrt[e + f*x^2]) + ((b*d*e - 2*b*c*f + 5*a*d*f)*x*Sqrt[c + d*x
^2]*Sqrt[e + f*x^2])/(15*d*f) + (b*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*
d) - (Sqrt[e]*(5*a*d*f*(d*e + c*f) - 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*Sqr
t[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*d
^2*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (e^(3/2
)*(b*d*e + b*c*f - 10*a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(15*d*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5d} + \frac{\int \frac{\sqrt{c+dx^2}\left(-((bc-5ad)e)+(bde-2bcf+5adf)x^2\right)}{\sqrt{e+fx^2}}\,dx}{5d} \\
&= \frac{(bde-2bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15df} + \frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5d} \\
&\quad + \frac{\int \frac{-ce(bde+bcf-10adf)+\left(5adf(de+cf)-2b\left(d^2e^2-cdef+c^2f^2\right)\right)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15df} \\
&= \frac{(bde-2bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15df} + \frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5d} \\
&\quad - \frac{(ce(bde+bcf-10adf))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15df} \\
&\quad + \frac{(5adf(de+cf)-2b(d^2e^2-cdef+c^2f^2))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15df}
\end{aligned}$$

$$= \frac{(5adf(de+cf)-2b(d^2e^2-cdef+c^2f^2))\,x\sqrt{c+dx^2}}{15d^2f\sqrt{e+fx^2}} + \frac{(bde-2bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15df} + \frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5d} - \frac{e^{3/2}(bde+bcf-10adf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15df^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{(e(5adf(de+cf)-2b(d^2e^2-cdef+c^2f^2)))\int\frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{15d^2f}$$

$$= \frac{(5adf(de+cf)-2b(d^2e^2-cdef+c^2f^2))\,x\sqrt{c+dx^2}}{15d^2f\sqrt{e+fx^2}} + \frac{(bde-2bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15df} + \frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5d} - \frac{\sqrt{e}(5adf(de+cf)-2b(d^2e^2-cdef+c^2f^2))\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15d^2f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{e^{3/2}(bde+bcf-10adf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15df^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.60 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.70

$$\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

$$= \frac{\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)(bcf+5adf+bd(e+3fx^2))+ie(-5adf(de+cf)+2b(d^2e^2-cdef+c^2f^2))\ \cdots}{\cdots}$$

```
[In] Integrate[(a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(b*c*f + 5*a*d*f + b*d*(e + 3*f*x^2)
) + I*e*(-5*a*d*f*(d*e + c*f) + 2*b*(d^2*e^2 - c*d*e*f + c^2*f^2))*Sqrt[1 +
 (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*
e)] - I*e*(-(d*e) + c*f)*(-2*b*d*e + b*c*f + 5*a*d*f)*Sqrt[1 + (d*x^2)/c]*S
qrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(15*d*Sq
rt[d/c]*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 431, normalized size of antiderivative = 1.13

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(\frac{b x^3\sqrt{df x^4+cf x^2+de x^2+ce}}{5}+\frac{\left(adf+bcf+bde-\frac{b(4cf+4de)}{5}\right)x\sqrt{df x^4+cf x^2+de x^2+ce}}{3df}+\frac{\left(ace-\frac{\left(adf+bcf+bde-\frac{b(4cf+}{5}\right.}{3df}\right.}{\sqrt{-}}\right.}{}$
risch	$\frac{x(3bdf x^2+5adf+bcf+bde)\sqrt{d x^2+c}\sqrt{f x^2+e}}{15df}+\frac{\left(-\frac{b c^2 ef\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\sqrt{df x^4+cf x^2+de x^2+ce}}-\frac{bcd e^2\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}F\left(x\right.}{\sqrt{-\frac{d}{c}}\sqrt{df x^4+cf x^2}}\right.}{}$
default	$\frac{\sqrt{d x^2+c}\sqrt{f x^2+e}\left(3\sqrt{-\frac{d}{c}}b d^2 f^3 x^7+5\sqrt{-\frac{d}{c}}a d^2 f^3 x^5+4\sqrt{-\frac{d}{c}}bcd f^3 x^5+4\sqrt{-\frac{d}{c}}b d^2 e f^2 x^5+5\sqrt{-\frac{d}{c}}acd f^3 x^3+5\sqrt{-\frac{d}{c}}a d^2 e f^2 x^3+\right.}{}$

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/5*b*x^3*(d*f
*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/3*(a*d*f+b*c*f+b*d*e-1/5*b*(4*c*f+4*d*e))
/d/f*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(a*c*e-1/3*(a*d*f+b*c*f+b*d*e-1/
5*b*(4*c*f+4*d*e))/d/f*c*e)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2
)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e
)/e/d)^(1/2))-(a*c*f+a*d*e+2/5*b*c*e-1/3*(a*d*f+b*c*f+b*d*e-1/5*b*(4*c*f+4*
d*e))/d/f*(2*c*f+2*d*e))*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)
/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d
*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.80

$$\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

$$=\frac{(2\,bd^2e^3-(2\,bcd+5\,ad^2)e^2f+(2\,bc^2-5\,acd)ef^2)\sqrt{df}\,x\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right)\mid\frac{cf}{de})-(2\,bd^2e^3-(2\,bcd+}{}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*((2*b*d^2*e^3 - (2*b*c*d + 5*a*d^2)*e^2*f + (2*b*c^2 - 5*a*c*d)*e*f^2)
*sqrt(d*f)*x*sqrt(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (2*b*
```

```
d^2*e^3 - (2*b*c*d + 5*a*d^2)*e^2*f + (2*b*c^2 - (5*a - b)*c*d)*e*f^2 + (b*
c^2 - 10*a*c*d)*f^3)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x)
, c*f/(d*e)) + (3*b*d^2*f^3*x^4 - 2*b*d^2*e^2*f + (2*b*c*d + 5*a*d^2)*e*f^2
 - (2*b*c^2 - 5*a*c*d)*f^3 + (b*d^2*e*f^2 + (b*c*d + 5*a*d^2)*f^3)*x^2)*sqr
t(d*x^2 + c)*sqrt(f*x^2 + e))/(d^2*f^3*x)
```

Sympy [F]

$$\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)*(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int (bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int (bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}\,dx = \int (b\,x^2+a)\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}\,dx$$

```
[In] int((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2),x)
[Out] int((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2), x)
```

3.25 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 283

$$\begin{aligned}\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx &= \frac{(bde-2bcf+3adf)x\sqrt{c+dx^2}}{3d^2\sqrt{e+fx^2}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d}\\ &\quad-\frac{\sqrt{e}(bde-2bcf+3adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3d^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\\ &\quad-\frac{(bc-3ad)e^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3cd\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

```
[Out] 1/3*(3*a*d*f-2*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)/d^2/(f*x^2+e)^(1/2)-1/3*(-3*a
*d+b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)
/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/c/d/f^(1/2)/(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(3*a*d*f-2*b*c*f+b*d*e)*
(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^
2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/d^2/f^(1/2)/(e*(d*x^2
+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*b*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1
/2)/d
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {542, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = -\frac{\sqrt{e}\sqrt{c+dx^2}(3adf-2bcf+bde)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3d^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{e^{3/2}\sqrt{c+dx^2}(bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3cd\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{x\sqrt{c+dx^2}(3adf-2bcf+bde)}{3d^2\sqrt{e+fx^2}} + \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d}$$

```
[In] Int[((a + b*x^2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] ((b*d*e - 2*b*c*f + 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*d^2*Sqrt[e + f*x^2]) + (
b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*d) - (Sqrt[e]*(b*d*e - 2*b*c*f + 3*
a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(3*d^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
- ((b*c - 3*a*d)*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(3*c*d*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]
*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d} + \frac{\int \frac{-((bc-3ad)e)+(bde-2bcf+3adf)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3d} \\
&= \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d} - \frac{((bc-3ad)e)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3d} \\
&\quad + \frac{(bde-2bcf+3adf)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3d} \\
&= \frac{(bde-2bcf+3adf)x\sqrt{c+dx^2}}{3d^2\sqrt{e+fx^2}} + \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d} \\
&\quad - \frac{(bc-3ad)e^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{(e(bde-2bcf+3adf))\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{3d^2} \\
&= \frac{(bde-2bcf+3adf)x\sqrt{c+dx^2}}{3d^2\sqrt{e+fx^2}} + \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3d} \\
&\quad - \frac{\sqrt{e}(bde-2bcf+3adf)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3d^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{(bc-3ad)e^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.42 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$
$$= \frac{b\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)+ie(-bde+2bcf-3adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ibe(-}{3d\sqrt{\frac{d}{c}}f\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[In] Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/Sqrt[c + d*x^2],x]
```

```
[Out] (b*Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2) + I*e*(-(b*d*e) + 2*b*c*f - 3*a*d*
f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x]
, (c*f)/(d*e)] - I*b*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/
e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*d*Sqrt[d/c]*f*Sqrt[c
+ d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 5.03 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.10

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{bx\sqrt{dfx^4+cfx^2+dex^2+ce}}{3d}+\frac{\left(ae-\frac{ceb}{3d}\right)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+ce}}-\frac{\left(af+be-\frac{b(2cf+2de)}{3d}\right)e\sqrt{1+}}{}\right.}{\sqrt{dx^2+c}\sqrt{fx^2+e}}$
risch	$\frac{bx\sqrt{dx^2+c}\sqrt{fx^2+e}}{3d}+\frac{\left(\frac{3ade\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+ce}}-\frac{bce\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+ce}}-\frac{(3adf-}{}\right.}{3d\sqrt{dx^2+c}}$
default	$\frac{\sqrt{fx^2+e}\sqrt{dx^2+c}\left(\sqrt{-\frac{d}{c}}bdf^2x^5+\sqrt{-\frac{d}{c}}bcf^2x^3+\sqrt{-\frac{d}{c}}bdefx^3+\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)bcef-\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}\right.}{}$

```
[In] int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/3*b/d*x*(d*f
*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(a*e-1/3*c/d*e*b)/(-d/c)^(1/2)*(1+d*x^2/c)^
(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d
/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(a*f+b*e-1/3*b/d*(2*c*f+2*d*e))*e/(-d/c
)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(
1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/
c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(bde^2-(2\,bc-3\,ad)ef)\sqrt{df}x\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right)\mid\frac{cf}{de})-(bde^2-(2\,bc-3\,ad)ef-(bc-3\,ad)f^2)\sqrt{d}}{3\,d^2f^2x}$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*((b*d*e^2 - (2*b*c - 3*a*d)*e*f)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_e(arc
sin(sqrt(-e/f)/x), c*f/(d*e)) - (b*d*e^2 - (2*b*c - 3*a*d)*e*f - (b*c - 3*a
*d)*f^2)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*e))
 - (b*d*f^2*x^2 + b*d*e*f - (2*b*c - 3*a*d)*f^2)*sqrt(d*x^2 + c)*sqrt(f*x^2
 + e))/(d^2*f^2*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx$$

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(e + f*x**2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a + bx^2)\sqrt{e + fx^2}}{\sqrt{c + dx^2}}\,dx = \int \frac{(bx^2 + a)\sqrt{fx^2 + e}}{\sqrt{dx^2 + c}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + bx^2)\sqrt{e + fx^2}}{\sqrt{c + dx^2}}\,dx = \int \frac{(b\,x^2 + a)\,\sqrt{f\,x^2 + e}}{\sqrt{d\,x^2 + c}}\,dx$$

```
[In] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(1/2),x)
[Out] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(1/2), x)
```

3.26 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 271

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \frac{(2bc-ad)fx\sqrt{c+dx^2}}{cd^2\sqrt{e+fx^2}} - \frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}} - \frac{(2bc-ad)\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{cd\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] (-a*d+2*b*c)*f*x*(d*x^2+c)^(1/2)/c/d^2/(f*x^2+e)^(1/2)+b*e^(3/2)*(1/(1+f*x^
2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2)
,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/c/d/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(
1/2)/(f*x^2+e)^(1/2)-(-a*d+2*b*c)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*E
llipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(
1/2)*(d*x^2+c)^(1/2)/c/d^2/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-
(-a*d+b*c)*x*(f*x^2+e)^(1/2)/c/d/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {540, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = -\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(2bc-ad)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{fx\sqrt{c+dx^2}(2bc-ad)}{cd^2\sqrt{e+fx^2}} - \frac{x\sqrt{e+fx^2}(bc-ad)}{cd\sqrt{c+dx^2}} + \frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{cd\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[In] Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]
```

```
[Out] ((2*b*c - a*d)*f*x*Sqrt[c + d*x^2])/(c*d^2*Sqrt[e + f*x^2]) - ((b*c - a*d)*
x*Sqrt[e + f*x^2])/(c*d*Sqrt[c + d*x^2]) - ((2*b*c - a*d)*Sqrt[e]*Sqrt[f]*S
qrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*
d^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b*e^(3/2)*Sqr
t[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*d*
Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}} - \frac{\int \frac{-bce-(2bc-ad)fx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{cd}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}} + \frac{(be)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{d} + \frac{((2bc-ad)f)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{cd}$$

$$= \frac{(2bc-ad)fx\sqrt{c+dx^2}}{cd^2\sqrt{e+fx^2}} - \frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}} + \frac{be^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{((2bc-ad)ef)\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{cd^2}$$

$$= \frac{(2bc-ad)fx\sqrt{c+dx^2}}{cd^2\sqrt{e+fx^2}} - \frac{(bc-ad)x\sqrt{e+fx^2}}{cd\sqrt{c+dx^2}} - \frac{(2bc-ad)\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{be^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{cd\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.51 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \frac{-i(2bc-ad)e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-(bc-ad)\left(\sqrt{\frac{d}{c}}x(e\right.}{c^2\left(\frac{d}{c}\right)^{3/2}\sqrt{c+dx^2}\sqrt{}}$$

[In] Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(3/2),x]

[Out] ((-I)*(2*b*c - a*d)*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (b*c - a*d)*(Sqrt[d/c]*x*(e + f*x^2) - I*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(c^2*(d/c)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])

Maple [A] (verified)

Time = 3.33 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.21

method	result
default	$\frac{\sqrt{f\,x^2+e}\,\sqrt{d\,x^2+c}\left(\sqrt{-\frac{d}{c}}\,adf\,x^3-\sqrt{-\frac{d}{c}}\,bcf\,x^3+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ade-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)b\right.}{d(df\,x^4+cf\,x^2+de\,x^2+c}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{\left(df\,x^2+de\right)(ad-bc)x}{c\,d^2\sqrt{\left(x^2+\frac{c}{d}\right)\left(df\,x^2+de\right)}}+\frac{\left(\frac{adf-bcf+bde}{d^2}-\frac{(ad-bc)(cf-de)}{d^2c}-\frac{e(ad-bc)}{dc}\right)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+d}{ed}}\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}\right.}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}$

[In] int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] (f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)*((-d/c)^(1/2)*a*d*f*x^3-(-d/c)^(1/2)*b*c*f*x^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e+(-d/c)^(1/2)*a*d*e*x-(-d/c)^(1/2)*b*c*e*x)/d/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)/c/(-d/c)^(1/2)

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{((2\,bcd-ad^2)ex^3+(2\,bc^2-acd)ex)\sqrt{df}\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right)\mid\frac{cf}{de})-((bcdf+(2\,bcd-ad^2)e)x^3+(bc^2f}{cd^3fx^3+c^2}$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -(((2*b*c*d - a*d^2)*e*x^3 + (2*b*c^2 - a*c*d)*e*x)*sqrt(d*f)*sqrt(-e/f)*el
liptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - ((b*c*d*f + (2*b*c*d - a*d^2)*e
)*x^3 + (b*c^2*f + (2*b*c^2 - a*c*d)*e)*x)*sqrt(d*f)*sqrt(-e/f)*elliptic_f(
arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (b*c*d*f*x^2 + (2*b*c^2 - a*c*d)*f)*sqrt
(d*x^2 + c)*sqrt(f*x^2 + e))/(c*d^3*f*x^3 + c^2*d^2*f*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{3/2}}\,dx$$

```
[In] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(3/2),x)
[Out] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(3/2), x)
```

3.27 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 274

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = -\frac{(bc-ad)x\sqrt{e+fx^2}}{3cd\,(c+dx^2)^{3/2}} + \frac{(d(bc+2ad)e-c(2bc+ad)f)\sqrt{e+fx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}d^{3/2}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{(bc-ad)e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2d(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] 1/3*(-a*d+b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*
f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2
)/c^2/d/(-c*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(-a*
d+b*c)*x*(f*x^2+e)^(1/2)/c/d/(d*x^2+c)^(3/2)+1/3*(d*(2*a*d+b*c)*e-c*(a*d+2*
b*c)*f)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)
/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(3/2)/d^(3/2)/(-c*f
+d*e)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {540, 539, 429, 422}

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{e+fx^2}(de(2ad+bc)-cf(ad+2bc))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}d^{3/2}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2d\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{x\sqrt{e+fx^2}(bc-ad)}{3cd\,(c+dx^2)^{3/2}}$$

```
[In] Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```

```
[Out] -1/3*((b*c - a*d)*x*Sqrt[e + f*x^2])/(c*d*(c + d*x^2)^(3/2)) + ((d*(b*c + 2
*a*d)*e - c*(2*b*c + a*d)*f)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/S
qrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*d^(3/2)*(d*e - c*f)*Sqrt[c + d*x^2]*S
qrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + ((b*c - a*d)*e^(3/2)*Sqrt[f]*Sqrt[c
 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*d
*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3cd\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-((bc+2ad)e)-(2bc+ad)fx^2}{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}\,dx}{3cd} \\
&= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3cd\left(c+dx^2\right)^{3/2}} + \frac{((bc-ad)ef)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3cd(de-cf)} \\
&\quad + \frac{(d(bc+2ad)e-c(2bc+ad)f)\int \frac{\sqrt{e+fx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{3cd(de-cf)} \\
&= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3cd\left(c+dx^2\right)^{3/2}} \\
&\quad + \frac{(d(bc+2ad)e-c(2bc+ad)f)\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}d^{3/2}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&\quad + \frac{(bc-ad)e^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.53 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{\frac{d}{c}}x(e+fx^2)\left(ad(-3cde+2c^2f-2d^2ex^2+cdfx^2)+bc(c^2f-d^2ex^2+2cdfx^2)\right.}{}$$

```
[In] Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(5/2),x]
```

```
[Out] (Sqrt[d/c]*x*(e + f*x^2)*(a*d*(-3*c*d*e + 2*c^2*f - 2*d^2*e*x^2 + c*d*f*x^2
) + b*c*(c^2*f - d^2*e*x^2 + 2*c*d*f*x^2)) + I*e*(a*d*(-2*d*e + c*f) + b*c*
(-(d*e) + 2*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ellip
ticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*(b*c + 2*a*d)*e*(-(d*e) + c*f
)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[S
qrt[d/c]*x], (c*f)/(d*e)])/(3*c^3*(d/c)^(3/2)*(-(d*e) + c*f)*(c + d*x^2)^(3
/2)*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 3.45 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.89

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(\frac{(ad-bc)x\sqrt{df x^4+cf x^2+de x^2+ce}}{3c d^3\left(x^2+\frac{c}{d}\right)^2}+\frac{\left(df x^2+de\right)x\left(acdf-2ae d^2+2c^2bf-bcde\right)}{3d^2c^2(cf-de)\sqrt{\left(x^2+\frac{c}{d}\right)\left(df x^2+de\right)}}+\frac{\left(\frac{bf}{d^2}+\frac{(ad-bc)f}{3d^2c}-\frac{acdf-2ae d^2+2c^2bf}{3d^2c^2}\right.}{}\right.}{}$
default	Expression too large to display

```
[In] int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/3*(a*d-b*c)/
c/d^3*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2+1/3*(d*f*x^2+d*e)/d
^2/c^2/(c*f-d*e)*x*(a*c*d*f-2*a*d^2*e+2*b*c^2*f-b*c*d*e)/((x^2+c/d)*(d*f*x^
2+d*e))^(1/2)+(b*f/d^2+1/3*(a*d-b*c)/d^2*f/c-1/3/d^2*(a*c*d*f-2*a*d^2*e+2*b
*c^2*f-b*c*d*e)/c^2-1/3/d*e/c^2/(c*f-d*e)*(a*c*d*f-2*a*d^2*e+2*b*c^2*f-b*c*
d*e))/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e
*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1/3/d*(a
*c*d*f-2*a*d^2*e+2*b*c^2*f-b*c*d*e)/c^2/(c*f-d*e)*e/(-d/c)^(1/2)*(1+d*x^2/c
)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*(EllipticF(x*
(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*
e)/e/d)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.88

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx =$$

$$-\frac{(((bcd^4+2\,ad^5)e-(2\,bc^2d^3+acd^4)f)x^4+2\,((bc^2d^3+2\,acd^4)e-(2\,bc^3d^2+ac^2d^3)f)x^2+(bc^3d^2+2\,ac^2d^3\cdots}{}$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/3*((((b*c*d^4 + 2*a*d^5)*e - (2*b*c^2*d^3 + a*c*d^4)*f)*x^4 + 2*((b*c^2*
d^3 + 2*a*c*d^4)*e - (2*b*c^3*d^2 + a*c^2*d^3)*f)*x^2 + (b*c^3*d^2 + 2*a*c^
2*d^3)*e - (2*b*c^4*d + a*c^3*d^2)*f)*sqrt(c*e)*sqrt(-d/c)*elliptic_e(arcsi
n(x*sqrt(-d/c)), c*f/(d*e)) - (((b*c*d^4 + 2*a*d^5)*e - (b*c^3*d^2 - (a - 2
*b)*c^2*d^3 + a*c*d^4)*f)*x^4 + 2*((b*c^2*d^3 + 2*a*c*d^4)*e - (b*c^4*d - (
a - 2*b)*c^3*d^2 + a*c^2*d^3)*f)*x^2 + (b*c^3*d^2 + 2*a*c^2*d^3)*e - (b*c^5
 - (a - 2*b)*c^4*d + a*c^3*d^2)*f)*sqrt(c*e)*sqrt(-d/c)*elliptic_f(arcsin(x
```

```
*sqrt(-d/c)), c*f/(d*e)) - (((b*c^2*d^3 + 2*a*c*d^4)*e - (2*b*c^3*d^2 + a*c
^2*d^3)*f)*x^3 + (3*a*c^2*d^3*e - (b*c^4*d + 2*a*c^3*d^2)*f)*x)*sqrt(d*x^2
+ c)*sqrt(f*x^2 + e))/(c^5*d^3*e - c^6*d^2*f + (c^3*d^5*e - c^4*d^4*f)*x^4
+ 2*(c^4*d^4*e - c^5*d^3*f)*x^2)
```

Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(5/2),x)
[Out] Integral((a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx=\int \frac{(b\,x^2+a)\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{5/2}}\,dx$$

```
[In] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(5/2),x)
[Out] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(5/2), x)
```

3.28 $\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 385

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = -\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\,(c+dx^2)^{5/2}}$$
$$+\frac{(ad(4de-3cf)+bc(de-2cf))x\sqrt{e+fx^2}}{15c^2d(de-cf)\,(c+dx^2)^{3/2}}$$
$$+\frac{(2bc(d^2e^2-cdef+c^2f^2)+ad(8d^2e^2-13cdef+3c^2f^2))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{e^{3/2}\sqrt{f}(2ad(2de-3cf)+bc(de+cf))\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3d(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] -1/15*e^(3/2)*(2*a*d*(-3*c*f+2*d*e)+b*c*(c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(1
+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(
1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^3/d/(-c*f+d*e)^2/(e*(d*x^2+c)/c/(f*x^2+e))^
(1/2)/(f*x^2+e)^(1/2)-1/5*(-a*d+b*c)*x*(f*x^2+e)^(1/2)/c/d/(d*x^2+c)^(5/2)+
1/15*(a*d*(-3*c*f+4*d*e)+b*c*(-2*c*f+d*e))*x*(f*x^2+e)^(1/2)/c^2/d/(-c*f+d*
e)/(d*x^2+c)^(3/2)+1/15*(2*b*c*(c^2*f^2-c*d*e*f+d^2*e^2)+a*d*(3*c^2*f^2-13*
c*d*e*f+8*d^2*e^2))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(
1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(5/2)/d
^(3/2)/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {540, 541, 539, 429, 422}

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx =$$
$$-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(2ad(2de-3cf)+bc(cf+de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3d\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{\sqrt{e+fx^2}(ad(3c^2f^2-13cdef+8d^2e^2)+2bc(c^2f^2-cdef+d^2e^2))\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{3/2}\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+\frac{x\sqrt{e+fx^2}(ad(4de-3cf)+bc(de-2cf))}{15c^2d\,(c+dx^2)^{3/2}\,(de-cf)}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5cd\,(c+dx^2)^{5/2}}$$

```
[In] Int[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

```
[Out] -1/5*((b*c - a*d)*x*Sqrt[e + f*x^2])/(c*d*(c + d*x^2)^(5/2)) + ((a*d*(4*d*e
 - 3*c*f) + b*c*(d*e - 2*c*f))*x*Sqrt[e + f*x^2])/(15*c^2*d*(d*e - c*f)*(c
+ d*x^2)^(3/2)) + ((2*b*c*(d^2*e^2 - c*d*e*f + c^2*f^2) + a*d*(8*d^2*e^2 -
13*c*d*e*f + 3*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (c*f)/(d*e)])/(15*c^(5/2)*d^(3/2)*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sq
rt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (e^(3/2)*Sqrt[f]*(2*a*d*(2*d*e - 3*c
*f) + b*c*(d*e + c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]
], 1 - (d*e)/(c*f)])/(15*c^3*d*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f
*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\left(c+dx^2\right)^{5/2}}-\frac{\int \frac{-((bc+4ad)e)-(2bc+3ad)fx^2}{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}\,dx}{5cd} \\
&= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\left(c+dx^2\right)^{5/2}}+\frac{(ad(4de-3cf)+bc(de-2cf))x\sqrt{e+fx^2}}{15c^2d(de-cf)\left(c+dx^2\right)^{3/2}} \\
&\quad +\frac{\int \frac{e(ad(8de-9cf)+bc(2de-cf))+f(d(bc+4ad)e-c(2bc+3ad)f)x^2}{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}\,dx}{15c^2d(de-cf)} \\
&= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\left(c+dx^2\right)^{5/2}}+\frac{(ad(4de-3cf)+bc(de-2cf))x\sqrt{e+fx^2}}{15c^2d(de-cf)\left(c+dx^2\right)^{3/2}} \\
&\quad -\frac{(ef(2ad(2de-3cf)+bc(de+cf)))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15c^2d(de-cf)^2} \\
&\quad +\frac{\left(2bc\left(d^2e^2-cdef+c^2f^2\right)+ad\left(8d^2e^2-13cdef+3c^2f^2\right)\right)\int \frac{\sqrt{e+fx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{15c^2d(de-cf)^2}
\end{aligned}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5cd\left(c+dx^2\right)^{5/2}} + \frac{(ad(4de-3cf)+bc(de-2cf))x\sqrt{e+fx^2}}{15c^2d(de-cf)\left(c+dx^2\right)^{3/2}}$$
$$+ \frac{(2bc(d^2e^2-cdef+c^2f^2)+ad(8d^2e^2-13cdef+3c^2f^2))\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{3/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$- \frac{e^{3/2}\sqrt{f}(2ad(2de-3cf)+bc(de+cf))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3d(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.86 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.98

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \frac{-\sqrt{\frac{d}{c}}x(e+fx^2)\left(3c^2(bc-ad)(de-cf)^2-c(de-cf)(ad(4de-3cf)+bc(de\right.}{}$$

```
[In] Integrate[((a + b*x^2)*Sqrt[e + f*x^2])/(c + d*x^2)^(7/2),x]
```

```
[Out] (-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c*f)^2 - c*(d*e - c*f)
*(a*d*(4*d*e - 3*c*f) + b*c*(d*e - 2*c*f))*(c + d*x^2) - (2*b*c*(d^2*e^2 -
c*d*e*f + c^2*f^2) + a*d*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*(c + d*x^2)^
2)) + I*e*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*((2*b*c*(d^
2*e^2 - c*d*e*f + c^2*f^2) + a*d*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*Elli
pticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (-(d*e) + c*f)*(b*c*(-2*d*e +
c*f) + a*d*(-8*d*e + 9*c*f))*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
))/(15*c^4*(d/c)^(3/2)*(d*e - c*f)^2*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 756, normalized size of antiderivative = 1.96

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{(ad-bc)x\sqrt{dfx^4+cfx^2+dex^2+ce}}{5cd^4\left(x^2+\frac{c}{d}\right)^3}+\frac{\left(3acdf-4aed^2+2c^2bf-bcde\right)x\sqrt{dfx^4+cfx^2+dex^2+ce}}{15d^3c^2(cf-de)\left(x^2+\frac{c}{d}\right)^2}+\frac{\left(dfx^2+de\right)x\left(3ac^2df^2-\right.}{15d^2c^3}\right.}{}$
default	Expression too large to display

```
[In] int((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/5*(a*d-b*c)/
c/d^4*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^3+1/15*(3*a*c*d*f-4*a
*d^2*e+2*b*c^2*f-b*c*d*e)/d^3/c^2/(c*f-d*e)*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
^(1/2)/(x^2+c/d)^2+1/15*(d*f*x^2+d*e)/d^2/c^3/(c*f-d*e)^2*x*(3*a*c^2*d*f^2-
13*a*c*d^2*e*f+8*a*d^3*e^2+2*b*c^3*f^2-2*b*c^2*d*e*f+2*b*c*d^2*e^2)/((x^2+c
/d)*(d*f*x^2+d*e))^(1/2)+(1/15*f*(3*a*c*d*f-4*a*d^2*e+2*b*c^2*f-b*c*d*e)/c^
2/(c*f-d*e)/d^2-1/15/d^2/(c*f-d*e)*(3*a*c^2*d*f^2-13*a*c*d^2*e*f+8*a*d^3*e^
2+2*b*c^3*f^2-2*b*c^2*d*e*f+2*b*c*d^2*e^2)/c^3-1/15/d*e/c^3/(c*f-d*e)^2*(3*
a*c^2*d*f^2-13*a*c*d^2*e*f+8*a*d^3*e^2+2*b*c^3*f^2-2*b*c^2*d*e*f+2*b*c*d^2*
e^2))/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e
*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1/15/d*(
3*a*c^2*d*f^2-13*a*c*d^2*e*f+8*a*d^3*e^2+2*b*c^3*f^2-2*b*c^2*d*e*f+2*b*c*d^
2*e^2)/c^3/(c*f-d*e)^2*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(
d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/
e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1105 vs. $2(367) = 734$.

Time = 0.13 (sec) , antiderivative size = 1105, normalized size of antiderivative = 2.87

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(((2*(b*c*d^6 + 4*a*d^7)*e^2 - (2*b*c^2*d^5 + 13*a*c*d^6)*e*f + (2*b*
c^3*d^4 + 3*a*c^2*d^5)*f^2)*x^6 + 3*(2*(b*c^2*d^5 + 4*a*c*d^6)*e^2 - (2*b*c
^3*d^4 + 13*a*c^2*d^5)*e*f + (2*b*c^4*d^3 + 3*a*c^3*d^4)*f^2)*x^4 + 2*(b*c^
4*d^3 + 4*a*c^3*d^4)*e^2 - (2*b*c^5*d^2 + 13*a*c^4*d^3)*e*f + (2*b*c^6*d +
3*a*c^5*d^2)*f^2 + 3*(2*(b*c^3*d^4 + 4*a*c^2*d^5)*e^2 - (2*b*c^4*d^3 + 13*a
*c^3*d^4)*e*f + (2*b*c^5*d^2 + 3*a*c^4*d^3)*f^2)*x^2)*sqrt(c*e)*sqrt(-d/c)*
elliptic_e(arcsin(x*sqrt(-d/c)), c*f/(d*e)) - ((2*(b*c*d^6 + 4*a*d^7)*e^2 +
 (b*c^3*d^4 + 2*(2*a - b)*c^2*d^5 - 13*a*c*d^6)*e*f + (b*c^4*d^3 - 2*(3*a -
 b)*c^3*d^4 + 3*a*c^2*d^5)*f^2)*x^6 + 3*(2*(b*c^2*d^5 + 4*a*c*d^6)*e^2 + (b
*c^4*d^3 + 2*(2*a - b)*c^3*d^4 - 13*a*c^2*d^5)*e*f + (b*c^5*d^2 - 2*(3*a -
b)*c^4*d^3 + 3*a*c^3*d^4)*f^2)*x^4 + 2*(b*c^4*d^3 + 4*a*c^3*d^4)*e^2 + (b*c
^6*d + 2*(2*a - b)*c^5*d^2 - 13*a*c^4*d^3)*e*f + (b*c^7 - 2*(3*a - b)*c^6*d
 + 3*a*c^5*d^2)*f^2 + 3*(2*(b*c^3*d^4 + 4*a*c^2*d^5)*e^2 + (b*c^5*d^2 + 2*(
2*a - b)*c^4*d^3 - 13*a*c^3*d^4)*e*f + (b*c^6*d - 2*(3*a - b)*c^5*d^2 + 3*a
*c^4*d^3)*f^2)*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_f(arcsin(x*sqrt(-d/c)), c
*f/(d*e)) - ((2*(b*c^2*d^5 + 4*a*c*d^6)*e^2 - (2*b*c^3*d^4 + 13*a*c^2*d^5)*
e*f + (2*b*c^4*d^3 + 3*a*c^3*d^4)*f^2)*x^5 + (5*(b*c^3*d^4 + 4*a*c^2*d^5)*e
^2 - (7*b*c^4*d^3 + 33*a*c^3*d^4)*e*f + 3*(2*b*c^5*d^2 + 3*a*c^4*d^3)*f^2)*
```

```
x^3 + (15*a*c^3*d^4*e^2 + (b*c^5*d^2 - 26*a*c^4*d^3)*e*f + (b*c^6*d + 9*a*c
^5*d^2)*f^2)*x)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(c^7*d^4*e^2 - 2*c^8*d^3*e
*f + c^9*d^2*f^2 + (c^4*d^7*e^2 - 2*c^5*d^6*e*f + c^6*d^5*f^2)*x^6 + 3*(c^5
*d^6*e^2 - 2*c^6*d^5*e*f + c^7*d^4*f^2)*x^4 + 3*(c^6*d^5*e^2 - 2*c^7*d^4*e*
f + c^8*d^3*f^2)*x^2)
```

Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(1/2)/(d*x**2+c)**(7/2),x)
[Out] Integral((a + b*x**2)*sqrt(e + f*x**2)/(c + d*x**2)**(7/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)\sqrt{fx^2+e}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(1/2)/(d*x^2+c)^(7/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*sqrt(f*x^2 + e)/(d*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx=\int \frac{(b\,x^2+a)\,\sqrt{f\,x^2+e}}{(d\,x^2+c)^{7/2}}\,dx$$

```
[In] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(7/2),x)
[Out] int(((a + b*x^2)*(e + f*x^2)^(1/2))/(c + d*x^2)^(7/2), x)
```

3.29 $\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 543

$$\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \frac{(7adf(3d^2e^2+7cdef-2c^2f^2)-b(6d^3e^3-9cd^2e^2f+19c^2def^2-8c^3f^3))\,x\sqrt{c+dx^2}}{105d^3f\sqrt{e+fx^2}}$$
$$+\frac{(14adf(3de-cf)+b(3d^2e^2-15cdef+8c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105d^2f}$$
$$+\frac{(3bde-4bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35d^2}+\frac{bx(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{7d}$$
$$-\frac{\sqrt{e}(7adf(3d^2e^2+7cdef-2c^2f^2)-b(6d^3e^3-9cd^2e^2f+19c^2def^2-8c^3f^3))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1\right.}{105d^3f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{e^{3/2}(7adf(9de-cf)-b(3d^2e^2+9cdef-4c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{105d^2f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] 1/7*b*x*(d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/d+1/105*(7*a*d*f*(-2*c^2*f^2+7*c*d*
e*f+3*d^2*e^2)-b*(-8*c^3*f^3+19*c^2*d*e*f^2-9*c*d^2*e^2*f+6*d^3*e^3))*x*(d*
x^2+c)^(1/2)/d^3/f/(f*x^2+e)^(1/2)+1/105*e^(3/2)*(7*a*d*f*(-c*f+9*d*e)-b*(-
4*c^2*f^2+9*c*d*e*f+3*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ell
ipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/
2)/d^2/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/105*(7*a*d
*f*(-2*c^2*f^2+7*c*d*e*f+3*d^2*e^2)-b*(-8*c^3*f^3+19*c^2*d*e*f^2-9*c*d^2*e^
2*f+6*d^3*e^3))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)
/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/d^3/f
^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/35*(7*a*d*f-4*b*c*
f+3*b*d*e)*x*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/d^2+1/105*(14*a*d*f*(-c*f+3*d*
```

```
e)+b*(8*c^2*f^2-15*c*d*e*f+3*d^2*e^2))*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/d^
2/f
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {542, 545, 429, 506, 422}

$$\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \frac{e^{3/2}\sqrt{c+dx^2}(7adf(9de-cf)-b(-4c^2f^2+9cdef+3d^2e^2))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{105d^2f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{\sqrt{e}\sqrt{c+dx^2}(7adf(-2c^2f^2+7cdef+3d^2e^2)-b(-8c^3f^3+19c^2def^2-9cd^2e^2f+6d^3e^3))\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)\right.}{105d^3f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(14adf(3de-cf)+b(8c^2f^2-15cdef+3d^2e^2))}{105d^2f} + \frac{x\sqrt{c+dx^2}(7adf(-2c^2f^2+7cdef+3d^2e^2)-b(-8c^3f^3+19c^2def^2-9cd^2e^2f+6d^3e^3))}{105d^3f\sqrt{e+fx^2}} + \frac{x(c+dx^2)^{3/2}\sqrt{e+fx^2}(7adf-4bcf+3bde)}{35d^2} + \frac{bx(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{7d}$$

```
[In] Int[(a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

```
[Out] ((7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2) - b*(6*d^3*e^3 - 9*c*d^2*e^2*
f + 19*c^2*d*e*f^2 - 8*c^3*f^3))*x*Sqrt[c + d*x^2])/(105*d^3*f*Sqrt[e + f*x
^2]) + ((14*a*d*f*(3*d*e - c*f) + b*(3*d^2*e^2 - 15*c*d*e*f + 8*c^2*f^2))*x
*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(105*d^2*f) + ((3*b*d*e - 4*b*c*f + 7*a*d
*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(35*d^2) + (b*x*(c + d*x^2)^(3/2)*
(e + f*x^2)^(3/2))/(7*d) - (Sqrt[e]*(7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2
*f^2) - b*(6*d^3*e^3 - 9*c*d^2*e^2*f + 19*c^2*d*e*f^2 - 8*c^3*f^3))*Sqrt[c
+ d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d^3*
f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(
7*a*d*f*(9*d*e - c*f) - b*(3*d^2*e^2 + 9*c*d*e*f - 4*c^2*f^2))*Sqrt[c + d*x
^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(105*d^2*f^(3/
2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
```

```
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\text{integral} = \frac{bx(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{7d} + \frac{\int \sqrt{c+dx^2}\sqrt{e+fx^2}(-((bc-7ad)e)+(3bde-4bcf+7adf)x^2)\,dx}{7d}$$

$$= \frac{(3bde-4bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35d^2} + \frac{bx(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{7d} + \frac{\int \frac{\sqrt{c+dx^2}\left(-e(4bc(2de-cf)-7ad(5de-cf))+\left(14adf(3de-cf)+b\left(3d^2e^2-15cdef+8c^2f^2\right)\right)x^2\right)}{\sqrt{e+fx^2}}\,dx}{35d^2}$$

$$= \frac{(14adf(3de-cf)+b(3d^2e^2-15cdef+8c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105d^2f} + \frac{(3bde-4bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35d^2} + \frac{bx(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{7d} + \frac{\int \frac{ce(7adf(9de-cf)-b(3d^2e^2+9cdef-4c^2f^2))+(7adf(3d^2e^2+7cdef-2c^2f^2)-b(6d^3e^3-9cd^2e^2f+19c^2def^2-8c^3f^3))x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{105d^2f}$$

$$= \frac{(14adf(3de-cf)+b(3d^2e^2-15cdef+8c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105d^2f} + \frac{(3bde-4bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35d^2} + \frac{bx(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{7d} + \frac{(ce(7adf(9de-cf)-b(3d^2e^2+9cdef-4c^2f^2)))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{105d^2f} + \frac{(7adf(3d^2e^2+7cdef-2c^2f^2)-b(6d^3e^3-9cd^2e^2f+19c^2def^2-8c^3f^3))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{105d^2f}$$

$$= \frac{(7adf(3d^2e^2+7cdef-2c^2f^2)-b(6d^3e^3-9cd^2e^2f+19c^2def^2-8c^3f^3))\,x\sqrt{c+dx^2}}{105d^3f\sqrt{e+fx^2}} + \frac{(14adf(3de-cf)+b(3d^2e^2-15cdef+8c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105d^2f} + \frac{(3bde-4bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35d^2} + \frac{bx(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{7d} + \frac{e^{3/2}(7adf(9de-cf)-b(3d^2e^2+9cdef-4c^2f^2))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105d^2f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{(e(7adf(3d^2e^2+7cdef-2c^2f^2)-b(6d^3e^3-9cd^2e^2f+19c^2def^2-8c^3f^3)))\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}dx}{105d^3f}$$

$$= \frac{(7adf(3d^2e^2+7cdef-2c^2f^2)-b(6d^3e^3-9cd^2e^2f+19c^2def^2-8c^3f^3))\,x\sqrt{c+dx^2}}{105d^3f\sqrt{e+fx^2}} + \frac{(14adf(3de-cf)+b(3d^2e^2-15cdef+8c^2f^2))\,x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105d^2f} + \frac{(3bde-4bcf+7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35d^2} + \frac{bx(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{7d} - \frac{\sqrt{e}(7adf(3d^2e^2+7cdef-2c^2f^2)-b(6d^3e^3-9cd^2e^2f+19c^2def^2-8c^3f^3))\sqrt{c+dx^2}E\left(\tan^{-1}\right.}{105d^3f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{e^{3/2}(7adf(9de-cf)-b(3d^2e^2+9cdef-4c^2f^2))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{105d^2f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.16 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.69

$$\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \frac{-\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)(4bc^2f^2-3bcdf(3e+fx^2)-7adf(6de+cf+3dfx^2)-3bd^2(e^2}{}$$

```
[In] Integrate[(a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2),x]
```

```
[Out] (-(Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(4*b*c^2*f^2 - 3*b*c*d*f*(3*e + f*
x^2) - 7*a*d*f*(6*d*e + c*f + 3*d*f*x^2) - 3*b*d^2*(e^2 + 8*e*f*x^2 + 5*f^2
*x^4))) - I*e*(7*a*d*f*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2) + b*(-6*d^3*e^3
+ 9*c*d^2*e^2*f - 19*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
 (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*e*(-(d*e) +
c*f)*(-7*a*d*f*(3*d*e + c*f) + b*(6*d^2*e^2 - 6*c*d*e*f + 4*c^2*f^2))*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)])/(105*c^2*(d/c)^(5/2)*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 695, normalized size of antiderivative = 1.28

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{bfx^5\sqrt{dfx^4+cfx^2+dex^2+ce}}{7}+\frac{\left(adf^2+bcf^2+2ebdf-\frac{bf(6cf+6de)}{7}\right)x^3\sqrt{dfx^4+cfx^2+dex^2+ce}}{5df}+\frac{\left(acf^2+2adef+\frac{9bc}{7}\right.}{}\right.}{}$
risch	$\frac{x(15bx^4d^2f^2+21ad^2f^2x^2+3bcdf^2x^2+24bd^2efx^2+7acdf^2+42ad^2ef-4bc^2f^2+9bcdef+3bd^2e^2)\sqrt{dx^2+c}\sqrt{fx^2+e}}{105fd^2}-\frac{\left(-\frac{4bc^3}{}\right.}{}$
default	Expression too large to display

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/7*b*f*x^5*(d
*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/5*(a*d*f^2+b*c*f^2+2*e*b*d*f-1/7*b*f*(6
*c*f+6*d*e))/d/f*x^3*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/3*(a*c*f^2+2*a*d
*e*f+9/7*b*c*e*f+b*d*e^2-1/5*(a*d*f^2+b*c*f^2+2*e*b*d*f-1/7*b*f*(6*c*f+6*d*
e))/d/f*(4*c*f+4*d*e))/d/f*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(a*c*e^2-1
/3*(a*c*f^2+2*a*d*e*f+9/7*b*c*e*f+b*d*e^2-1/5*(a*d*f^2+b*c*f^2+2*e*b*d*f-1/
7*b*f*(6*c*f+6*d*e))/d/f*(4*c*f+4*d*e))/d/f*c*e)/(-d/c)^(1/2)*(1+d*x^2/c)^(
1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(2*a*c*e*f+a*d*e^2+b*c*e^2-3/5*(a*d*f^2+
b*c*f^2+2*e*b*d*f-1/7*b*f*(6*c*f+6*d*e))/d/f*c*e-1/3*(a*c*f^2+2*a*d*e*f+9/7
*b*c*e*f+b*d*e^2-1/5*(a*d*f^2+b*c*f^2+2*e*b*d*f-1/7*b*f*(6*c*f+6*d*e))/d/f*
(4*c*f+4*d*e))/d/f*(2*c*f+2*d*e))*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2
/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(
-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))
))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 465, normalized size of antiderivative = 0.86

$$\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \frac{(6\,bd^3e^4-3\,(3\,bcd^2+7\,ad^3)e^3f+(19\,bc^2d-49\,acd^2)e^2f^2-2\,(4\,bc^3-7\,ac^2d)ef^3)\sqrt{df}x\sqrt{-\frac{e}{f}}}{}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/105*((6*b*d^3*e^4 - 3*(3*b*c*d^2 + 7*a*d^3)*e^3*f + (19*b*c^2*d - 49*a*c*
d^2)*e^2*f^2 - 2*(4*b*c^3 - 7*a*c^2*d)*e*f^3)*sqrt(d*f)*x*sqrt(-e/f)*ellipt
ic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (6*b*d^3*e^4 - 3*(3*b*c*d^2 + 7*a*d
^3)*e^3*f + (19*b*c^2*d - (49*a - 3*b)*c*d^2)*e^2*f^2 - (8*b*c^3 - (14*a +
9*b)*c^2*d + 63*a*c*d^2)*e*f^3 - (4*b*c^3 - 7*a*c^2*d)*f^4)*sqrt(d*f)*x*sqr
t(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)) + (15*b*d^3*f^4*x^6 - 6
*b*d^3*e^3*f + 3*(3*b*c*d^2 + 7*a*d^3)*e^2*f^2 - (19*b*c^2*d - 49*a*c*d^2)*
e*f^3 + 2*(4*b*c^3 - 7*a*c^2*d)*f^4 + 3*(8*b*d^3*e*f^3 + (b*c*d^2 + 7*a*d^3
)*f^4)*x^4 + (3*b*d^3*e^2*f^2 + 3*(3*b*c*d^2 + 14*a*d^3)*e*f^3 - (4*b*c^2*d
 - 7*a*c*d^2)*f^4)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d^3*f^3*x)
```

Sympy [F]

$$\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)*(f*x**2+e)**(3/2),x)
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \int (bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \int (bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)*(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx = \int (b\,x^2+a)\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}\,dx$$

```
[In] int((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2),x)
[Out] int((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2), x)
```

3.30 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{\sqrt{c+dx^2}} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 400

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{\sqrt{c+dx^2}} \, dx = \frac{(10adf(2de-cf)+b(3d^2e^2-13cdef+8c^2f^2))\,x\sqrt{c+dx^2}}{15d^3\sqrt{e+fx^2}} + \frac{(3bde-4bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15d^2} + \frac{bx\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5d} - \frac{\sqrt{e}(10adf(2de-cf)+b(3d^2e^2-13cdef+8c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15d^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{e^{3/2}(5ad(3de-cf)-b(6cde-4c^2f))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15cd^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] 1/5*b*x*(f*x^2+e)^(3/2)*(d*x^2+c)^(1/2)/d+1/15*(10*a*d*f*(-c*f+2*d*e)+b*(8*
c^2*f^2-13*c*d*e*f+3*d^2*e^2))*x*(d*x^2+c)^(1/2)/d^3/(f*x^2+e)^(1/2)+1/15*e
^(3/2)*(5*a*d*(-c*f+3*d*e)-b*(-4*c^2*f+6*c*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f
*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/
2))*(d*x^2+c)^(1/2)/c/d^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)
^(1/2)-1/15*(10*a*d*f*(-c*f+2*d*e)+b*(8*c^2*f^2-13*c*d*e*f+3*d^2*e^2))*(1/(
1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)
^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/d^3/f^(1/2)/(e*(d*x^2+c)/
c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/15*(5*a*d*f-4*b*c*f+3*b*d*e)*x*(d*x^2+
c)^(1/2)*(f*x^2+e)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {542, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{e^{3/2}\sqrt{c+dx^2}(5ad(3de-cf)-b(6cde-4c^2f))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\right.}{15cd^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{\sqrt{e}\sqrt{c+dx^2}(10adf(2de-cf)+b(8c^2f^2-13cdef+3d^2e^2))\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15d^3\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{x\sqrt{c+dx^2}(10adf(2de-cf)+b(8c^2f^2-13cdef+3d^2e^2))}{15d^3\sqrt{e+fx^2}}$$
$$+\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(5adf-4bcf+3bde)}{15d^2}+\frac{bx\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5d}$$

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] ((10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f + 8*c^2*f^2))*x*Sqrt[c
 + d*x^2])/(15*d^3*Sqrt[e + f*x^2]) + ((3*b*d*e - 4*b*c*f + 5*a*d*f)*x*Sqrt
[c + d*x^2]*Sqrt[e + f*x^2])/(15*d^2) + (b*x*Sqrt[c + d*x^2]*(e + f*x^2)^(3
/2))/(5*d) - (Sqrt[e]*(10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f +
 8*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*
e)/(c*f)])/(15*d^3*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f
*x^2]) + (e^(3/2)*(5*a*d*(3*d*e - c*f) - b*(6*c*d*e - 4*c^2*f))*Sqrt[c + d*
x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*c*d^2*Sqr
t[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5d} + \frac{\int \frac{\sqrt{e+fx^2}\left(-((bc-5ad)e)+(3bde-4bcf+5adf)x^2\right)}{\sqrt{c+dx^2}}\,dx}{5d} \\
&= \frac{(3bde-4bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15d^2} + \frac{bx\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5d} \\
&\quad + \frac{\int \frac{-e(2bc(3de-2cf)-5ad(3de-cf))+\left(10adf(2de-cf)+b\left(3d^2e^2-13cdef+8c^2f^2\right)\right)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15d^2} \\
&= \frac{(3bde-4bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15d^2} + \frac{bx\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5d} \\
&\quad - \frac{(e(2bc(3de-2cf)-5ad(3de-cf)))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15d^2} \\
&\quad + \frac{\left(10adf(2de-cf)+b\left(3d^2e^2-13cdef+8c^2f^2\right)\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15d^2}
\end{aligned}
$$

$$= \frac{(10adf(2de-cf)+b(3d^2e^2-13cdef+8c^2f^2))\,x\sqrt{c+dx^2}}{15d^3\sqrt{e+fx^2}} + \frac{(3bde-4bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15d^2} + \frac{bx\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5d} + \frac{e^{3/2}(5ad(3de-cf)-b(6cde-4c^2f))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15cd^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{(e(10adf(2de-cf)+b(3d^2e^2-13cdef+8c^2f^2)))\int\frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}dx}{15d^3}$$

$$= \frac{(10adf(2de-cf)+b(3d^2e^2-13cdef+8c^2f^2))\,x\sqrt{c+dx^2}}{15d^3\sqrt{e+fx^2}} + \frac{(3bde-4bcf+5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15d^2} + \frac{bx\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5d} - \frac{\sqrt{e}(10adf(2de-cf)+b(3d^2e^2-13cdef+8c^2f^2))\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15d^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{e^{3/2}(5ad(3de-cf)-b(6cde-4c^2f))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15cd^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.10 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \frac{-\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)(4bcf-5adf-3bd(2e+fx^2))-ie(10adf(2de-$$

```
[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/Sqrt[c + d*x^2],x]
```

```
[Out] (-(Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(4*b*c*f - 5*a*d*f - 3*b*d*(2*e +
f*x^2))) - I*e*(10*a*d*f*(2*d*e - c*f) + b*(3*d^2*e^2 - 13*c*d*e*f + 8*c^2*
f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)] + I*e*(-(d*e) + c*f)*(-3*b*d*e + 4*b*c*f - 5*a*d*f)*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)])/(15*c^2*(d/c)^(5/2)*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 6.17 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.12

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{bfx^3\sqrt{dfx^4+cfx^2+dex^2+ce}}{5d}+\frac{\left(af^2+2bfe-\frac{bf(4cf+4de)}{5d}\right)x\sqrt{dfx^4+cfx^2+dex^2+ce}}{3df}+\frac{\left(e^2a-\frac{\left(af^2+2bfe-\frac{bf(4cf+4c\cdots}{5d}\right)}{3df}\cdots\right.}{\sqrt{-\cdots}}\right.}{\cdots}$
risch	$\frac{x(3bdfx^2+5adf-4bcf+6bde)\sqrt{dx^2+c}\,\sqrt{fx^2+e}}{15d^2}-\frac{\left(-\frac{15ad^2e^2\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{dfx^4+cfx^2+dex^2+ce}}-\frac{4bc^2ef\sqrt{1+\frac{dx^2}{c}}\,\sqrt{1+\frac{fx\cdots}{e}}}{\sqrt{-\frac{d}{c}}\,\sqrt{dfx^4+\cdots}}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{fx^2+e}\,\sqrt{dx^2+c}\left(3\sqrt{-\frac{d}{c}}\,bd^2f^3x^7+5\sqrt{-\frac{d}{c}}\,ad^2f^3x^5-\sqrt{-\frac{d}{c}}\,bcdf^3x^5+9\sqrt{-\frac{d}{c}}\,bd^2ef^2x^5+5\sqrt{-\frac{d}{c}}\,acdf^3x^3+5\sqrt{-\frac{d}{c}}\,ad^2ef^2x^3-4\cdots\right.}{\cdots}$

```
[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/5*b*f/d*x^3*
(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/3*(a*f^2+2*b*f*e-1/5*b*f/d*(4*c*f+4*d
*e))/d/f*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(e^2*a-1/3*(a*f^2+2*b*f*e-1/
5*b*f/d*(4*c*f+4*d*e))/d/f*c*e)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^
(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f
+d*e)/e/d)^(1/2))-(2*a*e*f+e^2*b-3/5*b*f/d*c*e-1/3*(a*f^2+2*b*f*e-1/5*b*f/d
*(4*c*f+4*d*e))/d/f*(2*c*f+2*d*e))*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^
2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),
(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx =$$

$$-\frac{(3\,bd^2e^3-(13\,bcd-20\,ad^2)e^2f+2\,(4\,bc^2-5\,acd)ef^2)\sqrt{df}\,x\sqrt{-\frac{e}{f}}\,E\left(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right)\mid\frac{cf}{de}\right)-(3\,bd^2e^3-(13\,b\cdots}{\cdots}$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*((3*b*d^2*e^3 - (13*b*c*d - 20*a*d^2)*e^2*f + 2*(4*b*c^2 - 5*a*c*d)*e
*f^2)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) -
```

```
(3*b*d^2*e^3 - (13*b*c*d - 20*a*d^2)*e^2*f + (8*b*c^2 - 2*(5*a + 3*b)*c*d +
 15*a*d^2)*e*f^2 + (4*b*c^2 - 5*a*c*d)*f^3)*sqrt(d*f)*x*sqrt(-e/f)*elliptic
_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (3*b*d^2*f^3*x^4 + 3*b*d^2*e^2*f - (1
3*b*c*d - 20*a*d^2)*e*f^2 + 2*(4*b*c^2 - 5*a*c*d)*f^3 + (6*b*d^2*e*f^2 - (4
*b*c*d - 5*a*d^2)*f^3)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d^3*f^2*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(a+bx^2)\,(e+fx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}}\,dx$$

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(1/2),x)
[Out] Integral((a + b*x**2)*(e + f*x**2)**(3/2)/sqrt(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/sqrt(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{\sqrt{c+dx^2}}\,dx = \int \frac{(b\,x^2+a)\,\left(f\,x^2+e\right)^{3/2}}{\sqrt{d\,x^2+c}}\,dx$$

```
[In] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(1/2),x)
[Out] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(1/2), x)
```

3.31 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 369

$$\begin{aligned}\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx &= \frac{f(bc(7de-8cf)-3ad(de-2cf))x\sqrt{c+dx^2}}{3cd^3\sqrt{e+fx^2}} \\ &+ \frac{(4bc-3ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3cd^2} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{cd\sqrt{c+dx^2}} \\ &- \frac{\sqrt{e}\sqrt{f}(bc(7de-8cf)-3ad(de-2cf))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{e^{3/2}(3bde-4bcf+3adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3cd^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

```
[Out] -(-a*d+b*c)*x*(f*x^2+e)^(3/2)/c/d/(d*x^2+c)^(1/2)+1/3*f*(b*c*(-8*c*f+7*d*e)
-3*a*d*(-2*c*f+d*e))*x*(d*x^2+c)^(1/2)/c/d^3/(f*x^2+e)^(1/2)+1/3*e^(3/2)*(3
*a*d*f-4*b*c*f+3*b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x
*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/c/d^2
/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(b*c*(-8*c*f+7
*d*e)-3*a*d*(-2*c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE
(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*
x^2+c)^(1/2)/c/d^3/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*(-3*
a*d+4*b*c)*f*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/d^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {540, 542, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = -\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(bc(7de-8cf)-3ad(de-2cf))E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd^3\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{e^{3/2}\sqrt{c+dx^2}(3adf-4bcf+3bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3cd^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{fx\sqrt{c+dx^2}(bc(7de-8cf)-3ad(de-2cf))}{3cd^3\sqrt{e+fx^2}} + \frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}(4bc-3ad)}{3cd^2} - \frac{x(e+fx^2)^{3/2}(bc-ad)}{cd\sqrt{c+dx^2}}$$

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

```
[Out] (f*(b*c*(7*d*e - 8*c*f) - 3*a*d*(d*e - 2*c*f))*x*Sqrt[c + d*x^2])/(3*c*d^3*
Sqrt[e + f*x^2]) + ((4*b*c - 3*a*d)*f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3
*c*d^2) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(c*d*Sqrt[c + d*x^2]) - (Sqrt[e
]*Sqrt[f]*(b*c*(7*d*e - 8*c*f) - 3*a*d*(d*e - 2*c*f))*Sqrt[c + d*x^2]*Ellip
ticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c*d^3*Sqrt[(e*(c + d
*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(3*b*d*e - 4*b*c*f + 3*
a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(3*c*d^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]
)
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x(e+fx^2)^{3/2}}{cd\sqrt{c+dx^2}} - \frac{\int \frac{\sqrt{e+fx^2}\left(-bce-(4bc-3ad)fx^2\right)}{\sqrt{c+dx^2}}\,dx}{cd} \\
&= \frac{(4bc-3ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3cd^2} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{cd\sqrt{c+dx^2}} \\
&\quad - \frac{\int \frac{-ce(3bde-4bcf+3adf)-f(bc(7de-8cf)-3ad(de-2cf))x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3cd^2}
\end{aligned}
$$

$$= \frac{(4bc-3ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3cd^2} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{(e(3bde-4bcf+3adf))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{3d^2} + \frac{(f(bc(7de-8cf)-3ad(de-2cf)))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{3cd^2}$$

$$= \frac{f(bc(7de-8cf)-3ad(de-2cf))x\sqrt{c+dx^2}}{3cd^3\sqrt{e+fx^2}} + \frac{(4bc-3ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3cd^2} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{cd\sqrt{c+dx^2}} + \frac{e^{3/2}(3bde-4bcf+3adf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{(ef(bc(7de-8cf)-3ad(de-2cf)))\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}dx}{3cd^3}$$

$$= \frac{f(bc(7de-8cf)-3ad(de-2cf))x\sqrt{c+dx^2}}{3cd^3\sqrt{e+fx^2}} + \frac{(4bc-3ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3cd^2} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{cd\sqrt{c+dx^2}} - \frac{\sqrt{e}\sqrt{f}(bc(7de-8cf)-3ad(de-2cf))\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{e^{3/2}(3bde-4bcf+3adf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3cd^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.84 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.67

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}dx = \frac{\sqrt{\frac{d}{c}}\left(\sqrt{\frac{d}{c}}x(e+fx^2)(3ad(de-cf)+bc(-3de+4cf+dfx^2))+ie(3ad(de-2\right.}{}$$

```
[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*(Sqrt[d/c]*x*(e + f*x^2)*(3*a*d*(d*e - c*f) + b*c*(-3*d*e + 4*c*
f + d*f*x^2)) + I*e*(3*a*d*(d*e - 2*c*f) + b*c*(-7*d*e + 8*c*f))*Sqrt[1 + (
```

```
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)
] - I*(4*b*c - 3*a*d)*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*d^3*Sqrt[c + d*x^2]
*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 7.38 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.47

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(-\frac{(dfx^2+de)(acdf-aed^2-c^2bf+bcde)x}{d^3c\sqrt{\left(x^2+\frac{c}{d}\right)(dfx^2+de)}}+\frac{bfx\sqrt{dfx^4+cfx^2+dex^2+ce}}{3d^2}+\frac{\left(-\frac{acdf^2-2ad^2ef-bc^2f^2+2bcdef-bd^2e^2}{d^3}\right)}{}\right)}{}$
default	$-\frac{\sqrt{fx^2+e}\sqrt{dx^2+c}\left(-\sqrt{-\frac{d}{c}}bcdf^2x^5+3\sqrt{-\frac{d}{c}}acdf^2x^3-3\sqrt{-\frac{d}{c}}ad^2efx^3-4\sqrt{-\frac{d}{c}}bc^2f^2x^3+2\sqrt{-\frac{d}{c}}bcdefx^3+3\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2}{e}}\right)}{}$
risch	$\frac{bx\sqrt{dx^2+c}\sqrt{fx^2+e}f}{3d^2}+\frac{\left(-\frac{(3adf-5bcf+4bde)e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)-E\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)\right)}{\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+ce}}-\frac{(3acdf^2}{}\right)}{}$

```
[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-(d*f*x^2+d*e)
*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/d^3/c*x/((x^2+c/d)*(d*f*x^2+d*e))^(1/2)+
1/3*b*f/d^2*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(-(a*c*d*f^2-2*a*d^2*e*f-
b*c^2*f^2+2*b*c*d*e*f-b*d^2*e^2)/d^3+(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/d^3*
(c*f-d*e)/c+1/d^2*e*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/c-1/3*b*f/d^2*c*e)/(-
d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e
)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(1/d^2*f*(a*d*f-
b*c*f+2*b*d*e)+(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/d^2*f/c-1/3*b*f/d^2*(2*c*f
+2*d*e))*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^
2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-
EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{(((7\,bcd^2-3\,ad^3)e^2-2\,(4\,bc^2d-3\,acd^2)ef)x^3+((7\,bc^2d-3\,acd^2)e^2-2\,(4\,bc^3-3\,ac^2d)ef)x)\sqrt{df}\sqrt{-\frac{e}{f}}E(}{}$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/3*((((7*b*c*d^2 - 3*a*d^3)*e^2 - 2*(4*b*c^2*d - 3*a*c*d^2)*e*f)*x^3 + ((
7*b*c^2*d - 3*a*c*d^2)*e^2 - 2*(4*b*c^3 - 3*a*c^2*d)*e*f)*x)*sqrt(d*f)*sqrt
(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (((7*b*c*d^2 - 3*a*d^3
)*e^2 - (8*b*c^2*d - 3*(2*a + b)*c*d^2)*e*f - (4*b*c^2*d - 3*a*c*d^2)*f^2)*
x^3 + ((7*b*c^2*d - 3*a*c*d^2)*e^2 - (8*b*c^3 - 3*(2*a + b)*c^2*d)*e*f - (4
*b*c^3 - 3*a*c^2*d)*f^2)*x)*sqrt(d*f)*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-e/
f)/x), c*f/(d*e)) - (b*c*d^2*f^2*x^4 + (7*b*c^2*d - 3*a*c*d^2)*e*f - 2*(4*b
*c^3 - 3*a*c^2*d)*f^2 + (4*b*c*d^2*e*f - (4*b*c^2*d - 3*a*c*d^2)*f^2)*x^2)*
sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(c*d^4*f*x^3 + c^2*d^3*f*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)(e+fx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*(e + f*x**2)**(3/2)/(c + d*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{3/2}}\,dx$$

```
[In] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2),x)
[Out] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(3/2), x)
```

3.32 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 373

$$\begin{aligned}\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx &= -\frac{f(bc(de-8cf)+2ad(de+cf))x\sqrt{c+dx^2}}{3c^2d^3\sqrt{e+fx^2}} \\ &+ \frac{(bc(de-4cf)+ad(2de+cf))x\sqrt{e+fx^2}}{3c^2d^2\sqrt{c+dx^2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{3cd\,(c+dx^2)^{3/2}} \\ &+ \frac{\sqrt{e}\sqrt{f}(bc(de-8cf)+2ad(de+cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{(4bc-ad)e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2d^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

```
[Out] -1/3*(-a*d+b*c)*x*(f*x^2+e)^(3/2)/c/d/(d*x^2+c)^(3/2)-1/3*f*(b*c*(-8*c*f+d*
e)+2*a*d*(c*f+d*e))*x*(d*x^2+c)^(1/2)/c^2/d^3/(f*x^2+e)^(1/2)+1/3*(-a*d+4*b
*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(
1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^2/d^2/(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*(b*c*(-8*c*f+d*e)+2*a*d*
(c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1
/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c^
2/d^3/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*(b*c*(-4*c*f+d*e)
+a*d*(c*f+2*d*e))*x*(f*x^2+e)^(1/2)/c^2/d^2/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {540, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(2ad(cf+de)+bc(de-8cf))E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d^3\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(4bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2d^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{fx\sqrt{c+dx^2}(2ad(cf+de)+bc(de-8cf))}{3c^2d^3\sqrt{e+fx^2}}$$
$$+\frac{x\sqrt{e+fx^2}(ad(cf+2de)+bc(de-4cf))}{3c^2d^2\sqrt{c+dx^2}}-\frac{x(e+fx^2)^{3/2}\,(bc-ad)}{3cd\,(c+dx^2)^{3/2}}$$

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

```
[Out] -1/3*(f*(b*c*(d*e - 8*c*f) + 2*a*d*(d*e + c*f))*x*Sqrt[c + d*x^2])/(c^2*d^3
*Sqrt[e + f*x^2]) + ((b*c*(d*e - 4*c*f) + a*d*(2*d*e + c*f))*x*Sqrt[e + f*x
^2])/(3*c^2*d^2*Sqrt[c + d*x^2]) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(3*c*d
*(c + d*x^2)^(3/2)) + (Sqrt[e]*Sqrt[f]*(b*c*(d*e - 8*c*f) + 2*a*d*(d*e + c*
f))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)]
)/(3*c^2*d^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((4*b
*c - a*d)*e^(3/2)*Sqrt[f]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt
[e]], 1 - (d*e)/(c*f)])/(3*c^2*d^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sq
rt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x(e+fx^2)^{3/2}}{3cd\,(c+dx^2)^{3/2}} - \frac{\int \frac{\sqrt{e+fx^2}\left(-((bc+2ad)e)-(4bc-ad)fx^2\right)}{(c+dx^2)^{3/2}}\,dx}{3cd}$$

$$= \frac{(bc(de-4cf)+ad(2de+cf))x\sqrt{e+fx^2}}{3c^2d^2\sqrt{c+dx^2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{3cd\,(c+dx^2)^{3/2}} + \frac{\int \frac{c(4bc-ad)ef-f(bc(de-8cf)+2ad(de+cf))x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3c^2d^2}$$

$$= \frac{(bc(de-4cf)+ad(2de+cf))x\sqrt{e+fx^2}}{3c^2d^2\sqrt{c+dx^2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{3cd\,(c+dx^2)^{3/2}} + \frac{((4bc-ad)ef)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3cd^2} - \frac{(f(bc(de-8cf)+2ad(de+cf)))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3c^2d^2}$$

$$
\begin{aligned}
&= -\frac{f(bc(de-8cf)+2ad(de+cf))x\sqrt{c+dx^2}}{3c^2d^3\sqrt{e+fx^2}} \\
&\quad + \frac{(bc(de-4cf)+ad(2de+cf))x\sqrt{e+fx^2}}{3c^2d^2\sqrt{c+dx^2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{3cd\,(c+dx^2)^{3/2}} \\
&\quad + \frac{(4bc-ad)e^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{(ef(bc(de-8cf)+2ad(de+cf)))\int\frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{3c^2d^3} \\
&= -\frac{f(bc(de-8cf)+2ad(de+cf))x\sqrt{c+dx^2}}{3c^2d^3\sqrt{e+fx^2}} \\
&\quad + \frac{(bc(de-4cf)+ad(2de+cf))x\sqrt{e+fx^2}}{3c^2d^2\sqrt{c+dx^2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{3cd\,(c+dx^2)^{3/2}} \\
&\quad + \frac{\sqrt{e}\sqrt{f}(bc(de-8cf)+2ad(de+cf))\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{(4bc-ad)e^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2d^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.58 (sec) , antiderivative size = 296, normalized size of antiderivative = 0.79

$$
\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \frac{\left(\frac{d}{c}\right)^{3/2}\left(\sqrt{\frac{d}{c}}x(e+fx^2)\left(bc(-4c^2f+d^2ex^2-5cdfx^2)+ad(c^2f+2d^2ex^2+c\right.\right.}{}
$$

```
[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x]
```

```
[Out] ((d/c)^(3/2)*(Sqrt[d/c]*x*(e + f*x^2)*(b*c*(-4*c^2*f + d^2*e*x^2 - 5*c*d*f*
x^2) + a*d*(c^2*f + 2*d^2*e*x^2 + c*d*(3*e + 2*f*x^2))) - I*e*(-2*a*d*(d*e
+ c*f) + b*c*(-(d*e) + 8*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*
x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*e*(-(a*d*(2*d*e
+ c*f)) + b*c*(-(d*e) + 4*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f
*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*d^4*(c + d*x^2
)^(3/2)*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 4.50 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.50

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(-\frac{\left(acdf-ae d^2-c^2bf+bcde\right)x\sqrt{df x^4+cf x^2+de x^2+ce}}{3c d^4\left(x^2+\frac{c}{d}\right)^2}+\frac{\left(df x^2+de\right)\left(2acdf+2ae d^2-5c^2bf+bcde\right)x}{3c^2d^3\sqrt{\left(x^2+\frac{c}{d}\right)\left(df x^2+de\right)}}+\frac{\left(\frac{f(adf-2bcf+2bde}{d^3}\right.}{}\right.}{}$
default	Expression too large to display

[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/3*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/c/d^4*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2+1/3*(d*f*x^2+d*e)*(2*a*c*d*f+2*a*d^2*e-5*b*c^2*f+b*c*d*e)/c^2/d^3*x/((x^2+c/d)*(d*f*x^2+d*e))^(1/2)+(f*(a*d*f-2*b*c*f+2*b*d*e)/d^3-1/3*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/d^3*f/c-1/3*(2*a*c*d*f+2*a*d^2*e-5*b*c^2*f+b*c*d*e)/d^3*(c*f-d*e)/c^2-1/3/d^2*e*(2*a*c*d*f+2*a*d^2*e-5*b*c^2*f+b*c*d*e)/c^2)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(b*f^2/d^2-1/3*(2*a*c*d*f+2*a*d^2*e-5*b*c^2*f+b*c*d*e)/d^2*f/c^2)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))

Fricas [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \frac{(((bcd^3+2\,ad^4)e^2-2\,(4\,bc^2d^2-acd^3)ef)x^5+2\,((bc^2d^2+2\,acd^3)e^2-2\,(4\,bc^3c}{}$$

[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="fricas")

[Out] 1/3*((((b*c*d^3 + 2*a*d^4)*e^2 - 2*(4*b*c^2*d^2 - a*c*d^3)*e*f)*x^5 + 2*((b*c^2*d^2 + 2*a*c*d^3)*e^2 - 2*(4*b*c^3*d - a*c^2*d^2)*e*f)*x^3 + ((b*c^3*d + 2*a*c^2*d^2)*e^2 - 2*(4*b*c^4 - a*c^3*d)*e*f)*x)*sqrt(d*f)*sqrt(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (((b*c*d^3 + 2*a*d^4)*e^2 - 2*(4*b*c^2*d^2 - a*c*d^3)*e*f - (4*b*c^2*d^2 - a*c*d^3)*f^2)*x^5 + 2*((b*c^2*d^2 + 2*a*c*d^3)*e^2 - 2*(4*b*c^3*d - a*c^2*d^2)*e*f - (4*b*c^3*d - a*c^2*d^2)*f^2)*x^3 + ((b*c^3*d + 2*a*c^2*d^2)*e^2 - 2*(4*b*c^4 - a*c^3*d)*e*f - (4*b*c^4 - a*c^3*d)*f^2)*x)*sqrt(d*f)*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/

```
x), c*f/(d*e)) + (3*b*c^2*d^2*f^2*x^4 - (b*c^3*d + 2*a*c^2*d^2)*e*f + 2*(4*
b*c^4 - a*c^3*d)*f^2 - ((2*b*c^2*d^2 + a*c*d^3)*e*f - 3*(4*b*c^3*d - a*c^2*
d^2)*f^2)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(c^2*d^5*f*x^5 + 2*c^3*d^4*
f*x^3 + c^4*d^3*f*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(a+bx^2)(e+fx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(5/2),x)
[Out] Integral((a + b*x**2)*(e + f*x**2)**(3/2)/(c + d*x**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{5/2}}\,dx = \int \frac{(b\,x^2+a)\,\left(f\,x^2+e\right)^{3/2}}{\left(d\,x^2+c\right)^{5/2}}\,dx$$

```
[In] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2),x)
[Out] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(5/2), x)
```

3.33 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 376

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \frac{(d(bc+4ad)e-c(4bc+ad)f)x\sqrt{e+fx^2}}{15c^2d^2(c+dx^2)^{3/2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{5cd(c+dx^2)^{5/2}} + \frac{(bc(2d^2e^2+3cdef-8c^2f^2)+ad(8d^2e^2-3cdef-2c^2f^2))\sqrt{e+fx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{5/2}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{e^{3/2}\sqrt{f}(bc(de-4cf)+ad(4de-cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3d^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] -1/5*(-a*d+b*c)*x*(f*x^2+e)^(3/2)/c/d/(d*x^2+c)^(5/2)-1/15*e^(3/2)*(b*c*(-4
*c*f+d*e)+a*d*(-c*f+4*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellipti
cF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)
^(1/2)/c^3/d^2/(-c*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1
/15*(d*(4*a*d+b*c)*e-c*(a*d+4*b*c)*f)*x*(f*x^2+e)^(1/2)/c^2/d^2/(d*x^2+c)^(
3/2)+1/15*(b*c*(-8*c^2*f^2+3*c*d*e*f+2*d^2*e^2)+a*d*(-2*c^2*f^2-3*c*d*e*f+8
*d^2*e^2))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1
/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(5/2)/d^(5/2)/(-
c*f+d*e)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {540, 539, 429, 422}

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = -\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(ad(4de-cf)+bc(de-4cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3d^2\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{\sqrt{e+fx^2}(ad(-2c^2f^2-3cdef+8d^2e^2)+bc(-8c^2f^2+3cdef+2d^2e^2))\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{5/2}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{x\sqrt{e+fx^2}(de(4ad+bc)-cf(ad+4bc))}{15c^2d^2(c+dx^2)^{3/2}} - \frac{x(e+fx^2)^{3/2}(bc-ad)}{5cd(c+dx^2)^{5/2}}$$

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]
```

```
[Out] ((d*(b*c + 4*a*d)*e - c*(4*b*c + a*d)*f)*x*Sqrt[e + f*x^2])/(15*c^2*d^2*(c
+ d*x^2)^(3/2)) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(5*c*d*(c + d*x^2)^(5/2
)) + ((b*c*(2*d^2*e^2 + 3*c*d*e*f - 8*c^2*f^2) + a*d*(8*d^2*e^2 - 3*c*d*e*f
 - 2*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
c*f)/(d*e)])/(15*c^(5/2)*d^(5/2)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f
*x^2))/(e*(c + d*x^2))]) - (e^(3/2)*Sqrt[f]*(b*c*(d*e - 4*c*f) + a*d*(4*d*e
 - c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(15*c^3*d^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
 + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x(e+fx^2)^{3/2}}{5cd\,(c+dx^2)^{5/2}} - \frac{\int \frac{\sqrt{e+fx^2}\left(-((bc+4ad)e)-(4bc+ad)fx^2\right)}{(c+dx^2)^{5/2}}\,dx}{5cd} \\
&= \frac{(d(bc+4ad)e-c(4bc+ad)f)x\sqrt{e+fx^2}}{15c^2d^2\,(c+dx^2)^{3/2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{5cd\,(c+dx^2)^{5/2}} \\
&\quad + \frac{\int \frac{e(ad(8de+cf)+2bc(de+2cf))+f(2ad(2de+cf)+bc(de+8cf))x^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx}{15c^2d^2} \\
&= \frac{(d(bc+4ad)e-c(4bc+ad)f)x\sqrt{e+fx^2}}{15c^2d^2\,(c+dx^2)^{3/2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{5cd\,(c+dx^2)^{5/2}} \\
&\quad - \frac{(ef(bc(de-4cf)+ad(4de-cf)))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15c^2d^2(de-cf)} \\
&\quad + \frac{(bc(2d^2e^2+3cdef-8c^2f^2)+ad(8d^2e^2-3cdef-2c^2f^2))\int \frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx}{15c^2d^2(de-cf)} \\
&= \frac{(d(bc+4ad)e-c(4bc+ad)f)x\sqrt{e+fx^2}}{15c^2d^2\,(c+dx^2)^{3/2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{5cd\,(c+dx^2)^{5/2}} \\
&\quad + \frac{(bc(2d^2e^2+3cdef-8c^2f^2)+ad(8d^2e^2-3cdef-2c^2f^2))\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}d^{5/2}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&\quad - \frac{e^{3/2}\sqrt{f}(bc(de-4cf)+ad(4de-cf))\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3d^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.00 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(-\sqrt{\frac{d}{c}}x(e+fx^2)\left(3c^2(bc-ad)(de-cf)^2-c(de-cf)(bc(de-7cf)+2\right.\right.}{}$$

[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x]

[Out] (Sqrt[d/c]*(-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c*f)^2 - c*(d*e - c*f)*(b*c*(d*e - 7*c*f) + 2*a*d*(2*d*e + c*f))*(c + d*x^2) + (a*d*(-8*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2) + b*c*(-2*d^2*e^2 - 3*c*d*e*f + 8*c^2*f^2))*(c + d*x^2)^2)) - I*e*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*((a*d*(-8*d^2*e^2 + 3*c*d*e*f + 2*c^2*f^2) + b*c*(-2*d^2*e^2 - 3*c*d*e*f + 8*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*e - c*f)*(a*d*(8*d*e + c*f) + 2*b*c*(d*e + 2*c*f))*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])))/(15*c^2*d^3*(d*e - c*f)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 749, normalized size of antiderivative = 1.99

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(-\frac{(acdf-ae\,d^2-c^2bf+bcde)x\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{5c\,d^5\left(x^2+\frac{c}{d}\right)^3}+\frac{(2acdf+4ae\,d^2-7c^2bf+bcde)x\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{15c^2d^4\left(x^2+\frac{c}{d}\right)^2}+\frac{(df\,x^2\right.}{}$
default	Expression too large to display

[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)

[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/5*(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/c/d^5*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^3+1/15*(2*a*c*d*f+4*a*d^2*e-7*b*c^2*f+b*c*d*e)/c^2/d^4*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2+1/15*(d*f*x^2+d*e)/d^3/c^3/(c*f-d*e)*x*(2*a*c^2*d*f^2+3*a*c*d^2*e*f-8*a*d^3*e^2+8*b*c^3*f^2-3*b*c^2*d*e*f-2*b*c*d^2*e^2)/((x^2+c/d)*(d*f*x^2+d*e))^(1/2)+(b*f^2/d^3+1/15*f*(2*a*c*d*f+4*a*d^2*e-7*b*c^2*f+b*c*d*e)/c^2/d^3-1/15/d^3*(2*a*c^2*d*f^2+3*a*c*d^2*e*f-8*a*d^3*e^2+8*b*c^3*f^2-3*b*c^2*d*e*f-2*b*c*d^2*e^2)/c^3-1/15/d^2*e/c^3/(c*f-d*e)*(2*a*c^2*d*f^2+3*a*c*d^2*e*f-8*a*d^3*e^2+8*b*c^3*f^2-3*b*c^2*d*e*f-2*b*c*d^2*e^2))/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e

```
*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1/15/d^2
*(2*a*c^2*d*f^2+3*a*c*d^2*e*f-8*a*d^3*e^2+8*b*c^3*f^2-3*b*c^2*d*e*f-2*b*c*d
^2*e^2)/c^3/(c*f-d*e)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d
*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e
/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1040 vs. $2(358) = 716$.

Time = 0.14 (sec) , antiderivative size = 1040, normalized size of antiderivative = 2.77

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx =$$

$$-\frac{((2\,(bcd^6+4\,ad^7)e^2+3\,(bc^2d^5-acd^6)ef-2\,(4\,bc^3d^4+ac^2d^5)f^2)x^6+3\,(2\,(bc^2d^5+4\,acd^6)e^2+3\,(bc^3d^4-}{}$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(((2*(b*c*d^6 + 4*a*d^7)*e^2 + 3*(b*c^2*d^5 - a*c*d^6)*e*f - 2*(4*b*c
^3*d^4 + a*c^2*d^5)*f^2)*x^6 + 3*(2*(b*c^2*d^5 + 4*a*c*d^6)*e^2 + 3*(b*c^3*
d^4 - a*c^2*d^5)*e*f - 2*(4*b*c^4*d^3 + a*c^3*d^4)*f^2)*x^4 + 2*(b*c^4*d^3
+ 4*a*c^3*d^4)*e^2 + 3*(b*c^5*d^2 - a*c^4*d^3)*e*f - 2*(4*b*c^6*d + a*c^5*d
^2)*f^2 + 3*(2*(b*c^3*d^4 + 4*a*c^2*d^5)*e^2 + 3*(b*c^4*d^3 - a*c^3*d^4)*e*
f - 2*(4*b*c^5*d^2 + a*c^4*d^3)*f^2)*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_e(a
rcsin(x*sqrt(-d/c)), c*f/(d*e)) - ((2*(b*c*d^6 + 4*a*d^7)*e^2 + (b*c^3*d^4
+ (4*a + 3*b)*c^2*d^5 - 3*a*c*d^6)*e*f - (4*b*c^4*d^3 + (a + 8*b)*c^3*d^4 +
 2*a*c^2*d^5)*f^2)*x^6 + 3*(2*(b*c^2*d^5 + 4*a*c*d^6)*e^2 + (b*c^4*d^3 + (4
*a + 3*b)*c^3*d^4 - 3*a*c^2*d^5)*e*f - (4*b*c^5*d^2 + (a + 8*b)*c^4*d^3 + 2
*a*c^3*d^4)*f^2)*x^4 + 2*(b*c^4*d^3 + 4*a*c^3*d^4)*e^2 + (b*c^6*d + (4*a +
3*b)*c^5*d^2 - 3*a*c^4*d^3)*e*f - (4*b*c^7 + (a + 8*b)*c^6*d + 2*a*c^5*d^2)
*f^2 + 3*(2*(b*c^3*d^4 + 4*a*c^2*d^5)*e^2 + (b*c^5*d^2 + (4*a + 3*b)*c^4*d^
3 - 3*a*c^3*d^4)*e*f - (4*b*c^6*d + (a + 8*b)*c^5*d^2 + 2*a*c^4*d^3)*f^2)*x
^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_f(arcsin(x*sqrt(-d/c)), c*f/(d*e)) - ((2*
(b*c^2*d^5 + 4*a*c*d^6)*e^2 + 3*(b*c^3*d^4 - a*c^2*d^5)*e*f - 2*(4*b*c^4*d^
3 + a*c^3*d^4)*f^2)*x^5 + (5*(b*c^3*d^4 + 4*a*c^2*d^5)*e^2 - 2*(b*c^4*d^3 +
 4*a*c^3*d^4)*e*f - 3*(3*b*c^5*d^2 + 2*a*c^4*d^3)*f^2)*x^3 + (15*a*c^3*d^4*
e^2 + (b*c^5*d^2 - 11*a*c^4*d^3)*e*f - (4*b*c^6*d + a*c^5*d^2)*f^2)*x)*sqrt
(d*x^2 + c)*sqrt(f*x^2 + e))/(c^7*d^4*e - c^8*d^3*f + (c^4*d^7*e - c^5*d^6*
f)*x^6 + 3*(c^5*d^6*e - c^6*d^5*f)*x^4 + 3*(c^6*d^5*e - c^7*d^4*f)*x^2)
```

Sympy [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(a+bx^2)(e+fx^2)^{\frac{3}{2}}}{(c+dx^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(7/2),x)
[Out] Integral((a + b*x**2)*(e + f*x**2)**(3/2)/(c + d*x**2)**(7/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(7/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{7/2}}\,dx = \int \frac{(b\,x^2+a)\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{7/2}}\,dx$$

```
[In] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2),x)
[Out] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(7/2), x)
```

3.34 $\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}} dx$

Optimal result

Integrand size = 30, antiderivative size = 531

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}} dx = \frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2(c+dx^2)^{5/2}}$$
$$+\frac{(bc(4d^2e^2+cdef-8c^2f^2)+3ad(8d^2e^2-5cdef-2c^2f^2))\,x\sqrt{e+fx^2}}{105c^3d^2(de-cf)(c+dx^2)^{3/2}}$$
$$-\frac{(bc-ad)x(e+fx^2)^{3/2}}{7cd(c+dx^2)^{7/2}}$$
$$+\frac{(6ad(8d^3e^3-12cd^2e^2f+2c^2def^2+c^3f^3)+bc(8d^3e^3-5cd^2e^2f-5c^2def^2+8c^3f^3))\sqrt{e+fx^2}E\Big(\arctan}{105c^{7/2}d^{5/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{e^{3/2}\sqrt{f}(3ad(8d^2e^2-11cdef+c^2f^2)+2bc(2d^2e^2-cdef+2c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\right.}{105c^4d^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] -1/7*(-a*d+b*c)*x*(f*x^2+e)^(3/2)/c/d/(d*x^2+c)^(7/2)-1/105*e^(3/2)*(3*a*d*
(c^2*f^2-11*c*d*e*f+8*d^2*e^2)+2*b*c*(2*c^2*f^2-c*d*e*f+2*d^2*e^2))*(1/(1+f
*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1
/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^4/d^2/(-c*f+d*e)^2/(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/35*(d*(6*a*d+b*c)*e-c*(3*a*d+4*b
*c)*f)*x*(f*x^2+e)^(1/2)/c^2/d^2/(d*x^2+c)^(5/2)+1/105*(b*c*(-8*c^2*f^2+c*d
*e*f+4*d^2*e^2)+3*a*d*(-2*c^2*f^2-5*c*d*e*f+8*d^2*e^2))*x*(f*x^2+e)^(1/2)/c
^3/d^2/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/105*(6*a*d*(c^3*f^3+2*c^2*d*e*f^2-12*c*
d^2*e^2*f+8*d^3*e^3)+b*c*(8*c^3*f^3-5*c^2*d*e*f^2-5*c*d^2*e^2*f+8*d^3*e^3))
*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x
```

```
^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(7/2)/d^(5/2)/(-c*f+d*e)^2
/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {540, 541, 539, 429, 422}

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx =$$

$$-\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(3ad(c^2f^2-11cdef+8d^2e^2)+2bc(2c^2f^2-cdef+2d^2e^2))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\right.}{105c^4d^2\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$$+\frac{\sqrt{e+fx^2}(6ad(c^3f^3+2c^2def^2-12cd^2e^2f+8d^3e^3)+bc(8c^3f^3-5c^2def^2-5cd^2e^2f+8d^3e^3))\,E\left(\arctan\left(\right.\right.}{105c^{7/2}d^{5/2}\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

$$+\frac{x\sqrt{e+fx^2}(de(6ad+bc)-cf(3ad+4bc))}{35c^2d^2(c+dx^2)^{5/2}}$$

$$+\frac{x\sqrt{e+fx^2}(3ad(-2c^2f^2-5cdef+8d^2e^2)+bc(-8c^2f^2+cdef+4d^2e^2))}{105c^3d^2(c+dx^2)^{3/2}(de-cf)}$$

$$-\frac{x(e+fx^2)^{3/2}(bc-ad)}{7cd(c+dx^2)^{7/2}}$$

```
[In] Int[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

```
[Out] ((d*(b*c + 6*a*d)*e - c*(4*b*c + 3*a*d)*f)*x*Sqrt[e + f*x^2])/(35*c^2*d^2*(
c + d*x^2)^(5/2)) + ((b*c*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2) + 3*a*d*(8*d^2*
e^2 - 5*c*d*e*f - 2*c^2*f^2))*x*Sqrt[e + f*x^2])/(105*c^3*d^2*(d*e - c*f)*(
c + d*x^2)^(3/2)) - ((b*c - a*d)*x*(e + f*x^2)^(3/2))/(7*c*d*(c + d*x^2)^(7
/2)) + ((6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2*f + 2*c^2*d*e*f^2 + c^3*f^3) + b*c
*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))*Sqrt[e + f*x^2]*E
llipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(105*c^(7/2)*d^(5/2
)*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (e
^(3/2)*Sqrt[f]*(3*a*d*(8*d^2*e^2 - 11*c*d*e*f + c^2*f^2) + 2*b*c*(2*d^2*e^2
 - c*d*e*f + 2*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(105*c^4*d^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(
e + f*x^2))]*Sqrt[e + f*x^2])
```

<u>Rule 422</u>

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x\left(e+fx^2\right)^{3/2}}{7cd\left(c+dx^2\right)^{7/2}}-\frac{\int \frac{\sqrt{e+fx^2}\left(-((bc+6ad)e)-(4bc+3ad)fx^2\right)}{\left(c+dx^2\right)^{7/2}}\,dx}{7cd}\\
&= \frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2\left(c+dx^2\right)^{5/2}}-\frac{(bc-ad)x\left(e+fx^2\right)^{3/2}}{7cd\left(c+dx^2\right)^{7/2}}\\
&\quad+\frac{\int \frac{e(4bc(de+cf)+3ad(8de+cf))+f(6ad(3de+cf)+bc(3de+8cf))x^2}{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}\,dx}{35c^2d^2}
\end{aligned}
$$

$$= \frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2\left(c+dx^2\right)^{5/2}} + \frac{(bc(4d^2e^2+cdef-8c^2f^2)+3ad(8d^2e^2-5cdef-2c^2f^2))\,x\sqrt{e+fx^2}}{105c^3d^2(de-cf)\left(c+dx^2\right)^{3/2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{7cd\left(c+dx^2\right)^{7/2}} - \frac{\int \frac{-e\left(bc\left(8d^2e^2-cdef-4c^2f^2\right)+3ad\left(16d^2e^2-16cdef-c^2f^2\right)\right)-f\left(bc\left(4d^2e^2+cdef-8c^2f^2\right)+3ad\left(8d^2e^2-5cdef-2c^2f^2\right)\right)x^2}{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}\,dx}{105c^3d^2(de-cf)}$$

$$= \frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2\left(c+dx^2\right)^{5/2}} + \frac{(bc(4d^2e^2+cdef-8c^2f^2)+3ad(8d^2e^2-5cdef-2c^2f^2))\,x\sqrt{e+fx^2}}{105c^3d^2(de-cf)\left(c+dx^2\right)^{3/2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{7cd\left(c+dx^2\right)^{7/2}} - \frac{(ef(3ad(8d^2e^2-11cdef+c^2f^2)+2bc(2d^2e^2-cdef+2c^2f^2)))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{105c^3d^2(de-cf)^2} + \frac{(6ad(8d^3e^3-12cd^2e^2f+2c^2def^2+c^3f^3)+bc(8d^3e^3-5cd^2e^2f-5c^2def^2+8c^3f^3))\int \frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}}{105c^3d^2(de-cf)^2}$$

$$= \frac{(d(bc+6ad)e-c(4bc+3ad)f)x\sqrt{e+fx^2}}{35c^2d^2\left(c+dx^2\right)^{5/2}} + \frac{(bc(4d^2e^2+cdef-8c^2f^2)+3ad(8d^2e^2-5cdef-2c^2f^2))\,x\sqrt{e+fx^2}}{105c^3d^2(de-cf)\left(c+dx^2\right)^{3/2}} - \frac{(bc-ad)x(e+fx^2)^{3/2}}{7cd\left(c+dx^2\right)^{7/2}} + \frac{(6ad(8d^3e^3-12cd^2e^2f+2c^2def^2+c^3f^3)+bc(8d^3e^3-5cd^2e^2f-5c^2def^2+8c^3f^3))\sqrt{e+fx^2}E}{105c^{7/2}d^{5/2}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{e^{3/2}\sqrt{f}(3ad(8d^2e^2-11cdef+c^2f^2)+2bc(2d^2e^2-cdef+2c^2f^2))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\right.}{105c^4d^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.47 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.03

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(-\sqrt{\frac{d}{c}}x(e+fx^2)\left(15c^3(bc-ad)(de-cf)^3-3c^2(de-cf)^2(bc(de-9c\right.\right.}{}$$

```
[In] Integrate[((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x]
```

```
[Out] (Sqrt[d/c]*(-(Sqrt[d/c]*x*(e + f*x^2)*(15*c^3*(b*c - a*d)*(d*e - c*f)^3 - 3
*c^2*(d*e - c*f)^2*(b*c*(d*e - 9*c*f) + 2*a*d*(3*d*e + c*f))*(c + d*x^2) -
c*(d*e - c*f)*(b*c*(4*d^2*e^2 + c*d*e*f - 8*c^2*f^2) + 3*a*d*(8*d^2*e^2 - 5
*c*d*e*f - 2*c^2*f^2))*(c + d*x^2)^2 - (6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2*f +
 2*c^2*d*e*f^2 + c^3*f^3) + b*c*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2
+ 8*c^3*f^3))*(c + d*x^2)^3)) + I*e*(c + d*x^2)^3*Sqrt[1 + (d*x^2)/c]*Sqrt[
1 + (f*x^2)/e]*((6*a*d*(8*d^3*e^3 - 12*c*d^2*e^2*f + 2*c^2*d*e*f^2 + c^3*f^
3) + b*c*(8*d^3*e^3 - 5*c*d^2*e^2*f - 5*c^2*d*e*f^2 + 8*c^3*f^3))*EllipticE
[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (-(d*e) + c*f)*(3*a*d*(-16*d^2*e^2
+ 16*c*d*e*f + c^2*f^2) + b*c*(-8*d^2*e^2 + c*d*e*f + 4*c^2*f^2))*EllipticF
[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])))/(105*c^3*d^3*(d*e - c*f)^2*(c + d*
x^2)^(7/2)*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 1023, normalized size of antiderivative = 1.93

method	result	size
elliptic	Expression too large to display	1023
default	Expression too large to display	5113

```
[In] int((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/7*(a*c*d*f-
a*d^2*e-b*c^2*f+b*c*d*e)/c/d^6*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c
/d)^4+1/35*(2*a*c*d*f+6*a*d^2*e-9*b*c^2*f+b*c*d*e)/c^2/d^5*x*(d*f*x^4+c*f*x
^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^3+1/105*(6*a*c^2*d*f^2+15*a*c*d^2*e*f-24*a*
d^3*e^2+8*b*c^3*f^2-b*c^2*d*e*f-4*b*c*d^2*e^2)/d^4/c^3/(c*f-d*e)*x*(d*f*x^4
+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2+1/105*(d*f*x^2+d*e)/d^3/c^4/(c*f-d*
e)^2*x*(6*a*c^3*d*f^3+12*a*c^2*d^2*e*f^2-72*a*c*d^3*e^2*f+48*a*d^4*e^3+8*b*
c^4*f^3-5*b*c^3*d*e*f^2-5*b*c^2*d^2*e^2*f+8*b*c*d^3*e^3)/((x^2+c/d)*(d*f*x^
2+d*e))^(1/2)+(1/105*f*(6*a*c^2*d*f^2+15*a*c*d^2*e*f-24*a*d^3*e^2+8*b*c^3*f
^2-b*c^2*d*e*f-4*b*c*d^2*e^2)/d^3/c^3/(c*f-d*e)-1/105/d^3/(c*f-d*e)*(6*a*c^
```

```
3*d*f^3+12*a*c^2*d^2*e*f^2-72*a*c*d^3*e^2*f+48*a*d^4*e^3+8*b*c^4*f^3-5*b*c^
3*d*e*f^2-5*b*c^2*d^2*e^2*f+8*b*c*d^3*e^3)/c^4-1/105/d^2*e/c^4/(c*f-d*e)^2*
(6*a*c^3*d*f^3+12*a*c^2*d^2*e*f^2-72*a*c*d^3*e^2*f+48*a*d^4*e^3+8*b*c^4*f^3
-5*b*c^3*d*e*f^2-5*b*c^2*d^2*e^2*f+8*b*c*d^3*e^3))/(-d/c)^(1/2)*(1+d*x^2/c)
^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-
d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1/105/d^2*(6*a*c^3*d*f^3+12*a*c^2*d^2*
e*f^2-72*a*c*d^3*e^2*f+48*a*d^4*e^3+8*b*c^4*f^3-5*b*c^3*d*e*f^2-5*b*c^2*d^2
*e^2*f+8*b*c*d^3*e^3)/c^4/(c*f-d*e)^2*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f
*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*(EllipticF(x*(-d/c)^(1/2)
,(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2
))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1847 vs. $2(507) = 1014$.

Time = 0.17 (sec) , antiderivative size = 1847, normalized size of antiderivative = 3.48

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="fricas")
```

```
[Out] -1/105*(((8*(b*c*d^8 + 6*a*d^9)*e^3 - (5*b*c^2*d^7 + 72*a*c*d^8)*e^2*f - (5
*b*c^3*d^6 - 12*a*c^2*d^7)*e*f^2 + 2*(4*b*c^4*d^5 + 3*a*c^3*d^6)*f^3)*x^8 +
 4*(8*(b*c^2*d^7 + 6*a*c*d^8)*e^3 - (5*b*c^3*d^6 + 72*a*c^2*d^7)*e^2*f - (5
*b*c^4*d^5 - 12*a*c^3*d^6)*e*f^2 + 2*(4*b*c^5*d^4 + 3*a*c^4*d^5)*f^3)*x^6 +
 6*(8*(b*c^3*d^6 + 6*a*c^2*d^7)*e^3 - (5*b*c^4*d^5 + 72*a*c^3*d^6)*e^2*f -
(5*b*c^5*d^4 - 12*a*c^4*d^5)*e*f^2 + 2*(4*b*c^6*d^3 + 3*a*c^5*d^4)*f^3)*x^4
 + 8*(b*c^5*d^4 + 6*a*c^4*d^5)*e^3 - (5*b*c^6*d^3 + 72*a*c^5*d^4)*e^2*f - (
5*b*c^7*d^2 - 12*a*c^6*d^3)*e*f^2 + 2*(4*b*c^8*d + 3*a*c^7*d^2)*f^3 + 4*(8*
(b*c^4*d^5 + 6*a*c^3*d^6)*e^3 - (5*b*c^5*d^4 + 72*a*c^4*d^5)*e^2*f - (5*b*c
^6*d^3 - 12*a*c^5*d^4)*e*f^2 + 2*(4*b*c^7*d^2 + 3*a*c^6*d^3)*f^3)*x^2)*sqrt
(c*e)*sqrt(-d/c)*elliptic_e(arcsin(x*sqrt(-d/c)), c*f/(d*e)) - ((8*(b*c*d^8
 + 6*a*d^9)*e^3 + (4*b*c^3*d^6 + (24*a - 5*b)*c^2*d^7 - 72*a*c*d^8)*e^2*f -
 (2*b*c^4*d^5 + (33*a + 5*b)*c^3*d^6 - 12*a*c^2*d^7)*e*f^2 + (4*b*c^5*d^4 +
 (3*a + 8*b)*c^4*d^5 + 6*a*c^3*d^6)*f^3)*x^8 + 4*(8*(b*c^2*d^7 + 6*a*c*d^8)
*e^3 + (4*b*c^4*d^5 + (24*a - 5*b)*c^3*d^6 - 72*a*c^2*d^7)*e^2*f - (2*b*c^5
*d^4 + (33*a + 5*b)*c^4*d^5 - 12*a*c^3*d^6)*e*f^2 + (4*b*c^6*d^3 + (3*a + 8
*b)*c^5*d^4 + 6*a*c^4*d^5)*f^3)*x^6 + 6*(8*(b*c^3*d^6 + 6*a*c^2*d^7)*e^3 +
(4*b*c^5*d^4 + (24*a - 5*b)*c^4*d^5 - 72*a*c^3*d^6)*e^2*f - (2*b*c^6*d^3 +
(33*a + 5*b)*c^5*d^4 - 12*a*c^4*d^5)*e*f^2 + (4*b*c^7*d^2 + (3*a + 8*b)*c^6
*d^3 + 6*a*c^5*d^4)*f^3)*x^4 + 8*(b*c^5*d^4 + 6*a*c^4*d^5)*e^3 + (4*b*c^7*d
^2 + (24*a - 5*b)*c^6*d^3 - 72*a*c^5*d^4)*e^2*f - (2*b*c^8*d + (33*a + 5*b)
*c^7*d^2 - 12*a*c^6*d^3)*e*f^2 + (4*b*c^9 + (3*a + 8*b)*c^8*d + 6*a*c^7*d^2
```

```
)*f^3 + 4*(8*(b*c^4*d^5 + 6*a*c^3*d^6)*e^3 + (4*b*c^6*d^3 + (24*a - 5*b)*c^
5*d^4 - 72*a*c^4*d^5)*e^2*f - (2*b*c^7*d^2 + (33*a + 5*b)*c^6*d^3 - 12*a*c^
5*d^4)*e*f^2 + (4*b*c^8*d + (3*a + 8*b)*c^7*d^2 + 6*a*c^6*d^3)*f^3)*x^2)*sq
rt(c*e)*sqrt(-d/c)*elliptic_f(arcsin(x*sqrt(-d/c)), c*f/(d*e)) - ((8*(b*c^2
*d^7 + 6*a*c*d^8)*e^3 - (5*b*c^3*d^6 + 72*a*c^2*d^7)*e^2*f - (5*b*c^4*d^5 -
 12*a*c^3*d^6)*e*f^2 + 2*(4*b*c^5*d^4 + 3*a*c^4*d^5)*f^3)*x^7 + (28*(b*c^3*
d^6 + 6*a*c^2*d^7)*e^3 - 3*(6*b*c^4*d^5 + 85*a*c^3*d^6)*e^2*f - 3*(8*b*c^5*
d^4 - 15*a*c^4*d^5)*e*f^2 + 8*(4*b*c^6*d^3 + 3*a*c^5*d^4)*f^3)*x^5 + (35*(b
*c^4*d^5 + 6*a*c^3*d^6)*e^3 - 54*(b*c^5*d^4 + 6*a*c^4*d^5)*e^2*f + 12*(2*b*
c^6*d^3 + 5*a*c^5*d^4)*e*f^2 + (13*b*c^7*d^2 + 36*a*c^6*d^3)*f^3)*x^3 + (10
5*a*c^4*d^5*e^3 + 2*(2*b*c^6*d^3 - 93*a*c^5*d^4)*e^2*f - 2*(b*c^7*d^2 - 36*
a*c^6*d^3)*e*f^2 + (4*b*c^8*d + 3*a*c^7*d^2)*f^3)*x)*sqrt(d*x^2 + c)*sqrt(f
*x^2 + e))/(c^9*d^5*e^2 - 2*c^10*d^4*e*f + c^11*d^3*f^2 + (c^5*d^9*e^2 - 2*
c^6*d^8*e*f + c^7*d^7*f^2)*x^8 + 4*(c^6*d^8*e^2 - 2*c^7*d^7*e*f + c^8*d^6*f
^2)*x^6 + 6*(c^7*d^7*e^2 - 2*c^8*d^6*e*f + c^9*d^5*f^2)*x^4 + 4*(c^8*d^6*e^
2 - 2*c^9*d^5*e*f + c^10*d^4*f^2)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)*(f*x**2+e)**(3/2)/(d*x**2+c)**(9/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(a+bx^2)\,(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}{(dx^2+c)^{\frac{9}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(f*x^2+e)^(3/2)/(d*x^2+c)^(9/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*(f*x^2 + e)^(3/2)/(d*x^2 + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(e+fx^2)^{3/2}}{(c+dx^2)^{9/2}}\,dx = \int \frac{(b\,x^2+a)\,(f\,x^2+e)^{3/2}}{(d\,x^2+c)^{9/2}}\,dx$$

```
[In] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2),x)
[Out] int(((a + b*x^2)*(e + f*x^2)^(3/2))/(c + d*x^2)^(9/2), x)
```

3.35 $\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}} dx$

Optimal result

Integrand size = 30, antiderivative size = 551

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}} dx = \frac{(7adf(8d^2e^2-23cdef+23c^2f^2)-b(48d^3e^3-128cd^2e^2f+103c^2def^2-15c}{105df^3\sqrt{e+fx^2}}$$

$$-\frac{(28adf(de-2cf)-b(24d^2e^2-43cdef+15c^2f^2))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105f^3}$$

$$-\frac{(6bde-5bcf-7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35f^2}+\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f}$$

$$-\frac{\sqrt{e}(7adf(8d^2e^2-23cdef+23c^2f^2)-b(48d^3e^3-128cd^2e^2f+103c^2def^2-15c^3f^3))\sqrt{c+dx^2}E\Big(\arctan}{105df^{7/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

$$+\frac{\sqrt{e}(7af(4d^2e^2-11cdef+15c^2f^2)-be(24d^2e^2-61cdef+45c^2f^2))\sqrt{c+dx^2}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1\right.}{105f^{7/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] 1/105*(7*a*d*f*(23*c^2*f^2-23*c*d*e*f+8*d^2*e^2)-b*(-15*c^3*f^3+103*c^2*d*e
*f^2-128*c*d^2*e^2*f+48*d^3*e^3))*x*(d*x^2+c)^(1/2)/d/f^3/(f*x^2+e)^(1/2)-1
/105*(7*a*d*f*(23*c^2*f^2-23*c*d*e*f+8*d^2*e^2)-b*(-15*c^3*f^3+103*c^2*d*e*
f^2-128*c*d^2*e^2*f+48*d^3*e^3))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*El
lipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x
^2+c)^(1/2)/d/f^(7/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/105
*(7*a*f*(15*c^2*f^2-11*c*d*e*f+4*d^2*e^2)-b*e*(45*c^2*f^2-61*c*d*e*f+24*d^2
*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/
(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^(7/2)/(e*(d*
x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/35*(-7*a*d*f-5*b*c*f+6*b*d*e)*x
*(d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/f^2+1/7*b*x*(d*x^2+c)^(5/2)*(f*x^2+e)^(1/2
```

```
)/f-1/105*(28*a*d*f*(-2*c*f+d*e)-b*(15*c^2*f^2-43*c*d*e*f+24*d^2*e^2))*x*(d
*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f^3
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 551, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {542, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e}\sqrt{c+dx^2}(7af(15c^2f^2-11cdef+4d^2e^2)-be(45c^2f^2-61cdef+24d^2e^2))}{105f^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{\sqrt{e}\sqrt{c+dx^2}(7adf(23c^2f^2-23cdef+8d^2e^2)-b(-15c^3f^3+103c^2def^2-128cd^2e^2f+48d^3e^3))\,E\Big(\arctan}{105df^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(28adf(de-2cf)-b(15c^2f^2-43cdef+24d^2e^2))}{105f^3}$$
$$+\frac{x\sqrt{c+dx^2}(7adf(23c^2f^2-23cdef+8d^2e^2)-b(-15c^3f^3+103c^2def^2-128cd^2e^2f+48d^3e^3))}{105df^3\sqrt{e+fx^2}}$$
$$-\frac{x(c+dx^2)^{3/2}\sqrt{e+fx^2}(-7adf-5bcf+6bde)}{35f^2}+\frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2)^(5/2))/Sqrt[e + f*x^2],x]
```

```
[Out] ((7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2) - b*(48*d^3*e^3 - 128*c*d^2
*e^2*f + 103*c^2*d*e*f^2 - 15*c^3*f^3))*x*Sqrt[c + d*x^2])/(105*d*f^3*Sqrt[
e + f*x^2]) - ((28*a*d*f*(d*e - 2*c*f) - b*(24*d^2*e^2 - 43*c*d*e*f + 15*c^
2*f^2))*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(105*f^3) - ((6*b*d*e - 5*b*c*f
- 7*a*d*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(35*f^2) + (b*x*(c + d*x^2)
^(5/2)*Sqrt[e + f*x^2])/(7*f) - (Sqrt[e]*(7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f +
 23*c^2*f^2) - b*(48*d^3*e^3 - 128*c*d^2*e^2*f + 103*c^2*d*e*f^2 - 15*c^3*f
^3))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)
])/(105*d*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) +
(Sqrt[e]*(7*a*f*(4*d^2*e^2 - 11*c*d*e*f + 15*c^2*f^2) - b*e*(24*d^2*e^2 - 6
1*c*d*e*f + 45*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(105*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*S
qrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f} + \frac{\int \frac{(c+dx^2)^{3/2}(-c(be-7af)+(-6bde+5bcf+7adf)x^2)}{\sqrt{e+fx^2}}\,dx}{7f} \\
&= -\frac{(6bde-5bcf-7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35f^2} + \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f} \\
&\quad + \frac{\int \frac{\sqrt{c+dx^2}(-c(7af(de-5cf)-2be(3de-5cf))+(-28adf(de-2cf)+b(24d^2e^2-43cdef+15c^2f^2))x^2)}{\sqrt{e+fx^2}}\,dx}{35f^2}
\end{aligned}$$

$$= -\frac{\left(28adf(de-2cf)-b(24d^2e^2-43cdef+15c^2f^2)\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105f^3} - \frac{(6bde-5bcf-7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35f^2} + \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f} + \frac{\int \frac{c\left(7af\left(4d^2e^2-11cdef+15c^2f^2\right)-be\left(24d^2e^2-61cdef+45c^2f^2\right)\right)+\left(7adf\left(8d^2e^2-23cdef+23c^2f^2\right)-b\left(48d^3e^3-128cd^2e^2f+103c^2de\right.\right.}{\sqrt{c+dx^2}\sqrt{e+fx^2}}}{105f^3}$$

$$= -\frac{\left(28adf(de-2cf)-b(24d^2e^2-43cdef+15c^2f^2)\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105f^3} - \frac{(6bde-5bcf-7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35f^2} + \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f} + \frac{\left(c\left(7af(4d^2e^2-11cdef+15c^2f^2)-be(24d^2e^2-61cdef+45c^2f^2)\right)\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{105f^3} + \frac{\left(7adf(8d^2e^2-23cdef+23c^2f^2)-b(48d^3e^3-128cd^2e^2f+103c^2def^2-15c^3f^3)\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx}}}{105f^3}$$

$$= \frac{\left(7adf(8d^2e^2-23cdef+23c^2f^2)-b(48d^3e^3-128cd^2e^2f+103c^2def^2-15c^3f^3)\right)x\sqrt{c+dx^2}}{105df^3\sqrt{e+fx^2}} - \frac{\left(28adf(de-2cf)-b(24d^2e^2-43cdef+15c^2f^2)\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105f^3} - \frac{(6bde-5bcf-7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35f^2} + \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f} + \frac{\sqrt{e}\left(7af(4d^2e^2-11cdef+15c^2f^2)-be(24d^2e^2-61cdef+45c^2f^2)\right)\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|\right.}{105f^{7/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{\left(e\left(7adf(8d^2e^2-23cdef+23c^2f^2)-b(48d^3e^3-128cd^2e^2f+103c^2def^2-15c^3f^3)\right)\right)\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}}{105df^3}$$

$$= \frac{\left(7adf(8d^2e^2-23cdef+23c^2f^2)-b(48d^3e^3-128cd^2e^2f+103c^2def^2-15c^3f^3)\right)x\sqrt{c+dx^2}}{105df^3\sqrt{e+fx^2}} - \frac{\left(28adf(de-2cf)-b(24d^2e^2-43cdef+15c^2f^2)\right)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{105f^3} - \frac{(6bde-5bcf-7adf)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{35f^2} + \frac{bx(c+dx^2)^{5/2}\sqrt{e+fx^2}}{7f} - \frac{\sqrt{e}\left(7adf(8d^2e^2-23cdef+23c^2f^2)-b(48d^3e^3-128cd^2e^2f+103c^2def^2-15c^3f^3)\right)\sqrt{c+dx^2}\,E}{105df^{7/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{\sqrt{e}\left(7af(4d^2e^2-11cdef+15c^2f^2)-be(24d^2e^2-61cdef+45c^2f^2)\right)\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|\right.}{105f^{7/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.00 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \frac{\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)(7adf(-4de+11cf+3dfx^2)+b(45c^2f^2+cdf(-6\cdots}{\cdots}$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^(5/2))/Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(7*a*d*f*(-4*d*e + 11*c*f + 3*d*f*x^
2) + b*(45*c^2*f^2 + c*d*f*(-61*e + 45*f*x^2) + 3*d^2*(8*e^2 - 6*e*f*x^2 +
5*f^2*x^4))) - I*e*(7*a*d*f*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2) + b*(-48*
d^3*e^3 + 128*c*d^2*e^2*f - 103*c^2*d*e*f^2 + 15*c^3*f^3))*Sqrt[1 + (d*x^2)
/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*
(-(d*e) + c*f)*(4*b*e*(12*d^2*e^2 - 26*c*d*e*f + 15*c^2*f^2) - 7*a*f*(8*d^2
*e^2 - 19*c*d*e*f + 15*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*El
lipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(105*Sqrt[d/c]*f^4*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 7.48 (sec) , antiderivative size = 691, normalized size of antiderivative = 1.25

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{b\,d^2x^5\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{7f}+\frac{\left(a\,d^3+3bc\,d^2-\frac{b\,d^2(6cf+6de)}{7f}\right)x^3\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{5df}+\frac{\left(3ac\,d^2+3b\,c^2d-\frac{5b\,d^2c\cdots}{7f}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
risch	$\frac{x(15b\,x^4d^2f^2+21a\,d^2f^2x^2+45bcd\,f^2x^2-18b\,d^2ef\,x^2+77acd\,f^2-28a\,d^2ef+45b\,c^2f^2-61bcdef+24b\,d^2e^2)\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{105f^3}+\frac{\left(\cdots\right.}{\cdots}$
default	Expression too large to display

```
[In] int((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/7*b*d^2/f*x^
5*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/5*(a*d^3+3*b*c*d^2-1/7*b*d^2/f*(6*c
*f+6*d*e))/d/f*x^3*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/3*(3*a*c*d^2+3*b*c
^2*d-5/7*b*d^2/f*c*e-1/5*(a*d^3+3*b*c*d^2-1/7*b*d^2/f*(6*c*f+6*d*e))/d/f*(4
*c*f+4*d*e))/d/f*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(c^3*a-1/3*(3*a*c*d^
2+3*b*c^2*d-5/7*b*d^2/f*c*e-1/5*(a*d^3+3*b*c*d^2-1/7*b*d^2/f*(6*c*f+6*d*e))
/d/f*(4*c*f+4*d*e))/d/f*c*e)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/
2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*
e)/e/d)^(1/2))-(3*a*c^2*d+c^3*b-3/5*(a*d^3+3*b*c*d^2-1/7*b*d^2/f*(6*c*f+6*d
*e))/d/f*c*e-1/3*(3*a*c*d^2+3*b*c^2*d-5/7*b*d^2/f*c*e-1/5*(a*d^3+3*b*c*d^2-
1/7*b*d^2/f*(6*c*f+6*d*e))/d/f*(4*c*f+4*d*e))/d/f*(2*c*f+2*d*e))*e/(-d/c)^(
1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2
)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^
(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 489, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx^2)\,(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \frac{(48\,bd^3e^5-8\,(16\,bcd^2+7\,ad^3)e^4f+(103\,bc^2d+161\,acd^2)e^3f^2-(15\,bc^3+161}{}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/105*((48*b*d^3*e^5 - 8*(16*b*c*d^2 + 7*a*d^3)*e^4*f + (103*b*c^2*d + 161*
a*c*d^2)*e^3*f^2 - (15*b*c^3 + 161*a*c^2*d)*e^2*f^3)*sqrt(d*f)*x*sqrt(-e/f)
*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (48*b*d^3*e^5 - 105*a*c^3*f^
5 - 8*(16*b*c*d^2 + 7*a*d^3)*e^4*f + (103*b*c^2*d + (161*a + 24*b)*c*d^2)*e
^3*f^2 - (15*b*c^3 + (161*a + 61*b)*c^2*d + 28*a*c*d^2)*e^2*f^3 + (45*b*c^3
 + 77*a*c^2*d)*e*f^4)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x
), c*f/(d*e)) + (15*b*d^3*e*f^4*x^6 - 48*b*d^3*e^4*f + 8*(16*b*c*d^2 + 7*a*
d^3)*e^3*f^2 - (103*b*c^2*d + 161*a*c*d^2)*e^2*f^3 + (15*b*c^3 + 161*a*c^2*
d)*e*f^4 - 3*(6*b*d^3*e^2*f^3 - (15*b*c*d^2 + 7*a*d^3)*e*f^4)*x^4 + (24*b*d
^3*e^3*f^2 - (61*b*c*d^2 + 28*a*d^3)*e^2*f^3 + (45*b*c^2*d + 77*a*c*d^2)*e*
f^4)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d*e*f^5*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}{\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)*(c + d*x**2)**(5/2)/sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}{\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}{\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)\,(d\,x^2+c)^{5/2}}{\sqrt{f\,x^2+e}}\,dx$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(1/2),x)
[Out] int(((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(1/2), x)
```

3.36 $\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 396

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = -\frac{(10adf(de-2cf)-b(8d^2e^2-13cdef+3c^2f^2))\,x\sqrt{c+dx^2}}{15df^2\sqrt{e+fx^2}} - \frac{(4bde-3bcf-5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15f^2} + \frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5f} + \frac{\sqrt{e}(10adf(de-2cf)-b(8d^2e^2-13cdef+3c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15df^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{\sqrt{e}(5af(de-3cf)-b(4de^2-6cef))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15f^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] -1/15*(10*a*d*f*(-2*c*f+d*e)-b*(3*c^2*f^2-13*c*d*e*f+8*d^2*e^2))*x*(d*x^2+c
)^(1/2)/d/f^2/(f*x^2+e)^(1/2)+1/15*(10*a*d*f*(-2*c*f+d*e)-b*(3*c^2*f^2-13*c
*d*e*f+8*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1
/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/d/
f^(5/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/15*(5*a*f*(-3*c*f
+d*e)-b*(-6*c*e*f+4*d*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellipti
cF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)
^(1/2)/f^(5/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/5*b*x*(d*x
^2+c)^(3/2)*(f*x^2+e)^(1/2)/f-1/15*(-5*a*d*f-3*b*c*f+4*b*d*e)*x*(d*x^2+c)^(
1/2)*(f*x^2+e)^(1/2)/f^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {542, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e}\sqrt{c+dx^2}(10adf(de-2cf)-b(3c^2f^2-13cdef+8d^2e^2))\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right.\right.}{15df^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{\sqrt{e}\sqrt{c+dx^2}(5af(de-3cf)-b(4de^2-6cef))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{x\sqrt{c+dx^2}(10adf(de-2cf)-b(3c^2f^2-13cdef+8d^2e^2))}{15df^2\sqrt{e+fx^2}}$$
$$-\frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}(-5adf-3bcf+4bde)}{15f^2}+\frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5f}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2)^(3/2))/Sqrt[e + f*x^2],x]
```

```
[Out] -1/15*((10*a*d*f*(d*e - 2*c*f) - b*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2))*x*
Sqrt[c + d*x^2])/(d*f^2*Sqrt[e + f*x^2]) - ((4*b*d*e - 3*b*c*f - 5*a*d*f)*x
*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*f^2) + (b*x*(c + d*x^2)^(3/2)*Sqrt[e
+ f*x^2])/(5*f) + (Sqrt[e]*(10*a*d*f*(d*e - 2*c*f) - b*(8*d^2*e^2 - 13*c*d*
e*f + 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
- (d*e)/(c*f)])/(15*d*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2]) - (Sqrt[e]*(5*a*f*(d*e - 3*c*f) - b*(4*d*e^2 - 6*c*e*f))*Sqrt[c +
 d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*f^(5/2
)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5f}+\frac{\int \frac{\sqrt{c+dx^2}\left(-c(be-5af)+(-4bde+3bcf+5adf)x^2\right)}{\sqrt{e+fx^2}}\,dx}{5f} \\
&= -\frac{(4bde-3bcf-5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15f^2}+\frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5f} \\
&\quad+\frac{\int \frac{-c(5af(de-3cf)-2be(2de-3cf))+\left(-10adf(de-2cf)+b\left(8d^2e^2-13cdef+3c^2f^2\right)\right)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15f^2} \\
&= -\frac{(4bde-3bcf-5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15f^2}+\frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5f} \\
&\quad-\frac{\left(c(5af(de-3cf)-b(4de^2-6cef))\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15f^2} \\
&\quad-\frac{\left(10adf(de-2cf)-b(8d^2e^2-13cdef+3c^2f^2)\right)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15f^2}
\end{aligned}$$

$$= -\frac{(10adf(de-2cf)-b(8d^2e^2-13cdef+3c^2f^2))\,x\sqrt{c+dx^2}}{15df^2\sqrt{e+fx^2}} - \frac{(4bde-3bcf-5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15f^2} + \frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5f} - \frac{\sqrt{e}(5af(de-3cf)-b(4de^2-6cef))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15f^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(e(10adf(de-2cf)-b(8d^2e^2-13cdef+3c^2f^2)))\int\frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{15df^2}$$

$$= -\frac{(10adf(de-2cf)-b(8d^2e^2-13cdef+3c^2f^2))\,x\sqrt{c+dx^2}}{15df^2\sqrt{e+fx^2}} - \frac{(4bde-3bcf-5adf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15f^2} + \frac{bx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5f} + \frac{\sqrt{e}(10adf(de-2cf)-b(8d^2e^2-13cdef+3c^2f^2))\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15df^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{\sqrt{e}(5af(de-3cf)-b(4de^2-6cef))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15f^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \frac{\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)(5adf+b(-4de+6cf+3dfx^2))-ie(-10adf(de\cdots}{\cdots}$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^(3/2))/Sqrt[e + f*x^2],x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2)*(5*a*d*f + b*(-4*d*e + 6*c*f + 3*d*f
*x^2)) - I*e*(-10*a*d*f*(d*e - 2*c*f) + b*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f
^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*
x], (c*f)/(d*e)] + I*(-(d*e) + c*f)*(5*a*f*(2*d*e - 3*c*f) + b*e*(-8*d*e +
9*c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/
c]*x], (c*f)/(d*e)])/(15*Sqrt[d/c]*f^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 6.12 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.13

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{bdx^3\sqrt{dfx^4+cfx^2+dex^2+ce}}{5f}+\frac{\left(ad^2+2bcd-\frac{bd(4cf+4de)}{5f}\right)x\sqrt{dfx^4+cfx^2+dex^2+ce}}{3df}+\frac{\left(c^2a-\frac{\left(ad^2+2bcd-\frac{bd(4cf+4de}{5f}\cdots\right.}{3df}\cdots\right.}{\sqrt{-\cdots}}\cdots\right.}{\cdots}$
risch	$\frac{x(3bdfx^2+5adf+6bcf-4bde)\sqrt{dx^2+c}\sqrt{fx^2+e}}{15f^2}+\frac{\left(\frac{15c^2af^2\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+ce}}-\frac{6bc^2ef\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\cdots}{\sqrt{-\frac{d}{c}}\sqrt{dfx^4+c\cdots}}\cdots\right.}{\cdots}$
default	$\frac{\sqrt{dx^2+c}\sqrt{fx^2+e}\left(3\sqrt{-\frac{d}{c}}bd^2f^3x^7+5\sqrt{-\frac{d}{c}}ad^2f^3x^5+9\sqrt{-\frac{d}{c}}bcdf^3x^5-\sqrt{-\frac{d}{c}}bd^2ef^2x^5+5\sqrt{-\frac{d}{c}}acdf^3x^3+5\sqrt{-\frac{d}{c}}ad^2ef^2x^3+6\cdots\right.}{\cdots}$

```
[In] int((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/5*b*d/f*x^3*
(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+1/3*(a*d^2+2*b*c*d-1/5*b*d/f*(4*c*f+4*d
*e))/d/f*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(c^2*a-1/3*(a*d^2+2*b*c*d-1/
5*b*d/f*(4*c*f+4*d*e))/d/f*c*e)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^
(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f
+d*e)/e/d)^(1/2))-(2*a*c*d+b*c^2-3/5*b*d/f*c*e-1/3*(a*d^2+2*b*c*d-1/5*b*d/f
*(4*c*f+4*d*e))/d/f*(2*c*f+2*d*e))*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^
2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),
(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)
)))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx =$$

$$-\frac{(8\,bd^2e^4-(13\,bcd+10\,ad^2)e^3f+(3\,bc^2+20\,acd)e^2f^2)\sqrt{df}x\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right)\mid\frac{cf}{de})-(8\,bd^2e^4+15\,a\cdots}{\cdots}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*((8*b*d^2*e^4 - (13*b*c*d + 10*a*d^2)*e^3*f + (3*b*c^2 + 20*a*c*d)*e^
2*f^2)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) -
```

```
(8*b*d^2*e^4 + 15*a*c^2*f^4 - (13*b*c*d + 10*a*d^2)*e^3*f + (3*b*c^2 + 4*(
5*a + b)*c*d)*e^2*f^2 - (6*b*c^2 + 5*a*c*d)*e*f^3)*sqrt(d*f)*x*sqrt(-e/f)*e
lliptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (3*b*d^2*e*f^3*x^4 + 8*b*d^2*e
^3*f - (13*b*c*d + 10*a*d^2)*e^2*f^2 + (3*b*c^2 + 20*a*c*d)*e*f^3 - (4*b*d^
2*e^2*f^2 - (6*b*c*d + 5*a*d^2)*e*f^3)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)
)/(d*e*f^4*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)(c+dx^2)^{\frac{3}{2}}}{\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)*(c + d*x**2)**(3/2)/sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}{\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}{\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\,(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)\,\left(d\,x^2+c\right)^{3/2}}{\sqrt{f\,x^2+e}}\,dx$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(1/2),x)
[Out] int(((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(1/2), x)
```

3.37 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 282

$$\begin{aligned}\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx &= -\frac{(2bde-bcf-3adf)x\sqrt{c+dx^2}}{3df\sqrt{e+fx^2}}+\frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}\\ &\quad+\frac{\sqrt{e}(2bde-bcf-3adf)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3df^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\\ &\quad-\frac{\sqrt{e}(be-3af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

```
[Out] -1/3*(-3*a*d*f-b*c*f+2*b*d*e)*x*(d*x^2+c)^(1/2)/d/f/(f*x^2+e)^(1/2)+1/3*(-3
*a*d*f-b*c*f+2*b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f
^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)
/d/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(-3*a*f+b*e)
*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x
^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^(3/2)/(e*(d*x^2+c)
/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*b*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)
/f
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {542, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = -\frac{\sqrt{e}\sqrt{c+dx^2}(be-3af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{\sqrt{e}\sqrt{c+dx^2}(-3adf-bcf+2bde)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3df^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{x\sqrt{c+dx^2}(-3adf-bcf+2bde)}{3df\sqrt{e+fx^2}} + \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f}$$

```
[In] Int[((a + b*x^2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] -1/3*((2*b*d*e - b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(d*f*Sqrt[e + f*x^2])
+ (b*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*f) + (Sqrt[e]*(2*b*d*e - b*c*f -
 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/
(c*f)])/(3*d*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
 - (Sqrt[e]*(b*e - 3*a*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt
[e]], 1 - (d*e)/(c*f)])/(3*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sq
rt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f} + \frac{\int \frac{-c(be-3af)+(-2bde+bcf+3adf)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3f} \\
&= \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f} - \frac{(c(be-3af))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3f} \\
&\quad + \frac{(-2bde+bcf+3adf)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3f} \\
&= -\frac{(2bde-bcf-3adf)x\sqrt{c+dx^2}}{3df\sqrt{e+fx^2}} + \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f} \\
&\quad - \frac{\sqrt{e}(be-3af)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{(e(-2bde+bcf+3adf))\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{3df} \\
&= -\frac{(2bde-bcf-3adf)x\sqrt{c+dx^2}}{3df\sqrt{e+fx^2}} + \frac{bx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3f} \\
&\quad + \frac{\sqrt{e}(2bde-bcf-3adf)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3df^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{\sqrt{e}(be-3af)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.37 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.76

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \frac{b\sqrt{\frac{d}{c}}fx(c+dx^2)(e+fx^2)-ie(-2bde+bcf+3adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+i(2be-}{3\sqrt{\frac{d}{c}}f^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[In] Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] (b*Sqrt[d/c]*f*x*(c + d*x^2)*(e + f*x^2) - I*e*(-2*b*d*e + b*c*f + 3*a*d*f)
*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x],
(c*f)/(d*e)] + I*(2*b*e - 3*a*f)*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*Sqrt[d/c]*f
^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 5.26 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.11

method	result
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{bx\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{3f}+\frac{\left(ac-\frac{ceb}{3f}\right)\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{\left(ad+bc-\frac{b(2cf+2de)}{3f}\right)e\sqrt{1+\frac{d\,x^2}{c}}}{}\right.}{\sqrt{d\,x^2+c}\sqrt{f\,x^2+e}}$
risch	$\frac{bx\sqrt{d\,x^2+c}\sqrt{f\,x^2+e}}{3f}+\frac{\left(\frac{3acf\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{bce\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{(3adf+bc}{}\right.}{3f\sqrt{d\,x^2+c}\,\sqrt{}}$
default	$\frac{\sqrt{d\,x^2+c}\sqrt{f\,x^2+e}\left(\sqrt{-\frac{d}{c}}\,bd\,f^2x^5+\sqrt{-\frac{d}{c}}\,bc\,f^2x^3+\sqrt{-\frac{d}{c}}\,bdef\,x^3+3\sqrt{\frac{d\,x^2+c}{c}}\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ac\,f^2-3\sqrt{\frac{d\,x^2+c}{c}}\sqrt{\frac{f\,x^2+e}{e}}\right.}{}$

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/3*b/f*x*(d*f
*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+(a*c-1/3*c*e/f*b)/(-d/c)^(1/2)*(1+d*x^2/c)^
(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d
/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(a*d+b*c-1/3*b/f*(2*c*f+2*d*e))*e/(-d/c
)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(
1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/
c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \frac{(2\,bde^3-(bc+3\,ad)e^2f)\sqrt{df}x\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right)\mid\frac{cf}{de})-(2\,bde^3+bcef^2-3\,acf^3-(bc+3\,ad)e^2f)\sqrt{\cdots}}{3\,def^3x}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*b*d*e^3 - (b*c + 3*a*d)*e^2*f)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_e(ar
csin(sqrt(-e/f)/x), c*f/(d*e)) - (2*b*d*e^3 + b*c*e*f^2 - 3*a*c*f^3 - (b*c
+ 3*a*d)*e^2*f)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x), c*f
/(d*e)) + (b*d*e*f^2*x^2 - 2*b*d*e^2*f + (b*c + 3*a*d)*e*f^2)*sqrt(d*x^2 +
c)*sqrt(f*x^2 + e))/(d*e*f^3*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
[Out] int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

3.38 $\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}} \, dx$

Optimal result

Integrand size = 30, antiderivative size = 206

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}} \, dx = \frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}} - \frac{b\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] b*x*(d*x^2+c)^(1/2)/d/(f*x^2+e)^(1/2)-b*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(
1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/
2)*(d*x^2+c)^(1/2)/d/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2
)+a*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+
f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/f^(1/2)/(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {545, 429, 506, 422}

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}} \, dx = \frac{a\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{b\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}}$$

```
[In] Int[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(d*Sqrt[e + f*x^2]) - (b*Sqrt[e]*Sqrt[c + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(d*Sqrt[f]*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (a*Sqrt[e]*Sqrt[c + d*x^2]*El
lipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= a\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx + b\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx \\ &= \frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}} + \frac{a\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{(be)\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{d}\end{aligned}$$

$$= \frac{bx\sqrt{c+dx^2}}{d\sqrt{e+fx^2}} - \frac{b\sqrt{e}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{d\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{a\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.59 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = -\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(beE\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+(-be+af)\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right)\right)}{\sqrt{\frac{d}{c}}f\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[In] Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(b*e*EllipticE[I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)] + (-(b*e) + a*f)*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)]))/(Sqrt[d/c]*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.77

method	result
default	$\frac{\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)af-F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)be+E\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)be\right)\sqrt{\frac{f\,x^2+e}{e}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{f\sqrt{-\frac{d}{c}}\,(df\,x^4+cf\,x^2+de\,x^2+ce)}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{a\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{be\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)-E\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}\,f}\right.}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}$

```
[In] int((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*f-EllipticF(x*(-d/c)^(1/2),(c*
f/d/e)^(1/2))*b*e+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*e)*((f*x^2+e)
/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/f/(-d/c)^(1/2
)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.63

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = -\frac{\sqrt{df}be^2x\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right) \mid \frac{cf}{de}) - \sqrt{dx^2+c}\sqrt{fx^2+e}bef - (be^2+af^2)\sqrt{df}x\sqrt{-\frac{e}{f}}F(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right)}{def^2x}$$

[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")

[Out] -(sqrt(d*f)*b*e^2*x*sqrt(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*b*e*f - (b*e^2 + a*f^2)*sqrt(d*f)*x*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)))/(d*e*f^2*x)

Sympy [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

[In] integrate((b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)

[Out] Integral((a + b*x**2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)

Maxima [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{bx^2+a}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)

Giac [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{bx^2+a}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{b\,x^2+a}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
[Out] int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.39 $\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 209

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = -\frac{(bc-ad)\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{\sqrt{e}(be-af)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] (-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1
/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/(-c*f+d*
e)/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(-a*d+b*c)*(1/(1
+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^
(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/(-c*f+d*e)/c^(1/2)/d^(1/2)/(d*x^2+
c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {539, 429, 422}

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e}\sqrt{c+dx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{\sqrt{e+fx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -(((b*c - a*d)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^
2))/(e*(c + d*x^2))])) + (Sqrt[e]*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticF[Arc
Tan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt[(e*
(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(bc-ad)\int \frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx}{de-cf}+\frac{(be-af)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{de-cf}\\
&= -\frac{(bc-ad)\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}\\
&\quad+\frac{\sqrt{e}(be-af)\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.62 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.99

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(\sqrt{\frac{d}{c}}(bc-ad)x(e+fx^2)+i(bc-ad)e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{\right.\right.\right.}}{d(-de+cf)\sqrt{}}$$

[In] Integrate[(a + b*x^2)/((c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]

[Out] (Sqrt[d/c]*(Sqrt[d/c]*(b*c - a*d)*x*(e + f*x^2) + I*(b*c - a*d)*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(d*(-(d*e) + c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])

Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.60

method	result
default	$\frac{\left(-\sqrt{-\frac{d}{c}}\,adf\,x^3+\sqrt{-\frac{d}{c}}\,bcf\,x^3+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)acf-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ade+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{f\,x}\right.}{\sqrt{-\frac{d}{c}}\,c(cf-de)(df\,x^4+cf\,x^2+}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(-\frac{\left(df\,x^2+de\right)x(ad-bc)}{dc(cf-de)\sqrt{\left(x^2+\frac{c}{d}\right)\left(df\,x^2+de\right)}}+\frac{\left(\frac{b}{d}+\frac{ad-bc}{dc}+\frac{e(ad-bc)}{c(cf-de)}\right)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{(ad-bc)e\sqrt{}}{}\right.}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}$

[In] int((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-(-d/c)^(1/2)*a*d*f*x^3+(-d/c)^(1/2)*b*c*f*x^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e-(-d/c)^(1/2)*a*d*e*x+(-d/c)^(1/2)*b*c*e*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/(-d/c)^(1/2)/c/(c*f-d*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.22

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = -\frac{(bc^2d-acd^2)\sqrt{dx^2+c}\sqrt{fx^2+e}ex-((bcd^2-ad^3)ex^2+(bc^2d-acd^2)e)\sqrt{ce}\sqrt{-\frac{d}{c}}E(\arcsin\left(x\sqrt{-\frac{d}{c}}\right) \mid \frac{cf}{de}}{c^3d^2e^2-c^4def+(c}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] -((b*c^2*d - a*c*d^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*e*x - ((b*c*d^2 - a*d
^3)*e*x^2 + (b*c^2*d - a*c*d^2)*e)*sqrt(c*e)*sqrt(-d/c)*elliptic_e(arcsin(x
*sqrt(-d/c)), c*f/(d*e)) - (a*c^3*f + (a*c^2*d*f - (b*c^2*d + b*c*d^2 - a*d
^3)*e)*x^2 - (b*c^3 + b*c^2*d - a*c*d^2)*e)*sqrt(c*e)*sqrt(-d/c)*elliptic_f
(arcsin(x*sqrt(-d/c)), c*f/(d*e)))/(c^3*d^2*e^2 - c^4*d*e*f + (c^2*d^3*e^2
- c^3*d^2*e*f)*x^2)
```

Sympy [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{a+bx^2}{(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/((c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{b\,x^2+a}{(d\,x^2+c)^{3/2}\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int((a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
[Out] int((a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

3.40 $\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 284

$$\begin{aligned}\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx &= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3c(de-cf)\left(c+dx^2\right)^{3/2}} \\ &+ \frac{(2ad(de-2cf)+bc(de+cf))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\ &- \frac{\sqrt{e}\sqrt{f}(2bce+ade-3acf)\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

```
[Out] -1/3*(-3*a*c*f+a*d*e+2*b*c*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellip
ticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)
*(d*x^2+c)^(1/2)/c^2/(-c*f+d*e)^2/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)
^(1/2)-1/3*(-a*d+b*c)*x*(f*x^2+e)^(1/2)/c/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/3*(2
*a*d*(-2*c*f+d*e)+b*c*(c*f+d*e))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*El
lipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1
/2)/c^(3/2)/(-c*f+d*e)^2/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^
(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {541, 539, 429, 422}

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e+fx^2}(2ad(de-2cf)+bc(cf+de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}\sqrt{c+dx^2}(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(-3acf+ade+2bce)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{x\sqrt{e+fx^2}(bc-ad)}{3c\,(c+dx^2)^{3/2}\,(de-cf)}$$

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -1/3*((b*c - a*d)*x*Sqrt[e + f*x^2])/(c*(d*e - c*f)*(c + d*x^2)^(3/2)) + ((
2*a*d*(d*e - 2*c*f) + b*c*(d*e + c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*Sqrt[d]*(d*e - c*f)^2*Sqrt[
c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (Sqrt[e]*Sqrt[f]*(2*b*c
*e + a*d*e - 3*a*c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]]
, 1 - (d*e)/(c*f)])/(3*c^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2
))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
```

```
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3c(de-cf)\left(c+dx^2\right)^{3/2}} - \frac{\int \frac{-bce-2ade+3acf+(bc-ad)fx^2}{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}\,dx}{3c(de-cf)} \\
&= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3c(de-cf)\left(c+dx^2\right)^{3/2}} - \frac{(f(2bce+ade-3acf))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3c(de-cf)^2} \\
&\quad + \frac{(2ad(de-2cf)+bc(de+cf))\int \frac{\sqrt{e+fx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{3c(de-cf)^2} \\
&= -\frac{(bc-ad)x\sqrt{e+fx^2}}{3c(de-cf)\left(c+dx^2\right)^{3/2}} \\
&\quad + \frac{(2ad(de-2cf)+bc(de+cf))\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&\quad - \frac{\sqrt{e}\sqrt{f}(2bce+ade-3acf)\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.14 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.06

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{\frac{d}{c}}x(e+fx^2)\left(bc(2c^2f+d^2ex^2+cdfx^2)+ad(-5c^2f+2d^2ex^2+cd(3e-4\right.}{}$$

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[d/c]*x*(e + f*x^2)*(b*c*(2*c^2*f + d^2*e*x^2 + c*d*f*x^2) + a*d*(-5*c
^2*f + 2*d^2*e*x^2 + c*d*(3*e - 4*f*x^2))) + I*e*(2*a*d*(d*e - 2*c*f) + b*c
*(d*e + c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE
[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*(-(d*e) + c*f)*(b*c*e + 2*a*d*e -
 3*a*c*f)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*A
rcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*c^2*Sqrt[d/c]*(d*e - c*f)^2*(c + d*x^
2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 5.56 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.90

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(-\frac{x(ad-bc)\sqrt{df x^4+cf x^2+de x^2+ce}}{3d^2 c(cf-de)\left(x^2+\frac{c}{d}\right)^2}-\frac{\left(df x^2+de\right)x\left(4acdf-2ae d^2-c^2bf-bcde\right)}{3d c^2(cf-de)^2\sqrt{\left(x^2+\frac{c}{d}\right)\left(df x^2+de\right)}}+\frac{\left(-\frac{(ad-bc)f}{3dc(cf-de)}+\frac{4acdf-2ae d^2-c^2bf-}{3d(cf-de)c^2}\right.}{}\right.}{}$
default	Expression too large to display

```
[In] int((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/3/d^2/c/(c*
f-d*e)*x*(a*d-b*c)*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2-1/3*(d*f
*x^2+d*e)/d/c^2/(c*f-d*e)^2*x*(4*a*c*d*f-2*a*d^2*e-b*c^2*f-b*c*d*e)/((x^2+c
/d)*(d*f*x^2+d*e))^(1/2)+(-1/3*(a*d-b*c)/d*f/c/(c*f-d*e)+1/3/d/(c*f-d*e)*(4
*a*c*d*f-2*a*d^2*e-b*c^2*f-b*c*d*e)/c^2+1/3*e/c^2/(c*f-d*e)^2*(4*a*c*d*f-2*
a*d^2*e-b*c^2*f-b*c*d*e))/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/
(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/
e/d)^(1/2))-1/3*(4*a*c*d*f-2*a*d^2*e-b*c^2*f-b*c*d*e)/c^2/(c*f-d*e)^2*e/(-d
/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
^(1/2)*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/
c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. 2(272) = 544.

Time = 0.11 (sec) , antiderivative size = 611, normalized size of antiderivative = 2.15

$$\int \frac{a+bx^2}{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}\,dx =$$

$$-\frac{(((bcd^4+2\,ad^5)e^2+(bc^2d^3-4\,acd^4)ef)x^4+(bc^3d^2+2\,ac^2d^3)e^2+(bc^4d-4\,ac^3d^2)ef+2\,((bc^2d^3+2\,acd^4)}{}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*((((b*c*d^4 + 2*a*d^5)*e^2 + (b*c^2*d^3 - 4*a*c*d^4)*e*f)*x^4 + (b*c^3
*d^2 + 2*a*c^2*d^3)*e^2 + (b*c^4*d - 4*a*c^3*d^2)*e*f + 2*((b*c^2*d^3 + 2*a
*c*d^4)*e^2 + (b*c^3*d^2 - 4*a*c^2*d^3)*e*f)*x^2)*sqrt(c*e)*sqrt(-d/c)*elli
ptic_e(arcsin(x*sqrt(-d/c)), c*f/(d*e)) + (3*a*c^5*f^2 + (3*a*c^3*d^2*f^2 -
 (b*c*d^4 + 2*a*d^5)*e^2 - (2*b*c^3*d^2 + (a + b)*c^2*d^3 - 4*a*c*d^4)*e*f)
*x^4 - (b*c^3*d^2 + 2*a*c^2*d^3)*e^2 - (2*b*c^5 + (a + b)*c^4*d - 4*a*c^3*d
^2)*e*f + 2*(3*a*c^4*d*f^2 - (b*c^2*d^3 + 2*a*c*d^4)*e^2 - (2*b*c^4*d + (a
+ b)*c^3*d^2 - 4*a*c^2*d^3)*e*f)*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_f(arcsi
n(x*sqrt(-d/c)), c*f/(d*e)) - (((b*c^2*d^3 + 2*a*c*d^4)*e^2 + (b*c^3*d^2 -
4*a*c^2*d^3)*e*f)*x^3 + (3*a*c^2*d^3*e^2 + (2*b*c^4*d - 5*a*c^3*d^2)*e*f)*x
)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(c^5*d^3*e^3 - 2*c^6*d^2*e^2*f + c^7*d*e
*f^2 + (c^3*d^5*e^3 - 2*c^4*d^4*e^2*f + c^5*d^3*e*f^2)*x^4 + 2*(c^4*d^4*e^3
 - 2*c^5*d^3*e^2*f + c^6*d^2*e*f^2)*x^2)
```

Sympy [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx=\int \frac{a+bx^2}{(c+dx^2)^{\frac{5}{2}}\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)/((c + d*x**2)**(5/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx=\int \frac{bx^2+a}{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx=\int \frac{bx^2+a}{(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx=\int \frac{b\,x^2+a}{(d\,x^2+c)^{5/2}\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int((a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)),x)
[Out] int((a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)), x)
```

3.41 $\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 401

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = -\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)\left(c+dx^2\right)^{5/2}}$$
$$+\frac{(4ad(de-2cf)+bc(de+3cf))x\sqrt{e+fx^2}}{15c^2(de-cf)^2\left(c+dx^2\right)^{3/2}}$$
$$+\frac{(bc(2d^2e^2-7cdef-3c^2f^2)+ad(8d^2e^2-23cdef+23c^2f^2))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{d}(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{\sqrt{e}\sqrt{f}(bce(de-9cf)+a(4d^2e^2-11cdef+15c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] -1/15*(b*c*e*(-9*c*f+d*e)+a*(15*c^2*f^2-11*c*d*e*f+4*d^2*e^2))*(1/(1+f*x^2/
e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(
1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c^3/(-c*f+d*e)^3/(e*(d*x^
2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/5*(-a*d+b*c)*x*(f*x^2+e)^(1/2)/c/
(-c*f+d*e)/(d*x^2+c)^(5/2)+1/15*(4*a*d*(-2*c*f+d*e)+b*c*(3*c*f+d*e))*x*(f*x
^2+e)^(1/2)/c^2/(-c*f+d*e)^2/(d*x^2+c)^(3/2)+1/15*(b*c*(-3*c^2*f^2-7*c*d*e*
f+2*d^2*e^2)+a*d*(23*c^2*f^2-23*c*d*e*f+8*d^2*e^2))*(1/(1+d*x^2/c))^(1/2)*(
1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^
(1/2))*(f*x^2+e)^(1/2)/c^(5/2)/(-c*f+d*e)^3/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x
^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {541, 539, 429, 422}

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e+fx^2}(ad(23c^2f^2-23cdef+8d^2e^2)+bc(-3c^2f^2-7cdef+2d^2e^2))\,E\Big(\text{ar}}{15c^{5/2}\sqrt{d}\sqrt{c+dx^2}(de-cf)^3\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(a(15c^2f^2-11cdef+4d^2e^2)+bce(de-9cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{x\sqrt{e+fx^2}(4ad(de-2cf)+bc(3cf+de))}{15c^2\,(c+dx^2)^{3/2}\,(de-cf)^2}-\frac{x\sqrt{e+fx^2}(bc-ad)}{5c\,(c+dx^2)^{5/2}\,(de-cf)}$$

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -1/5*((b*c - a*d)*x*Sqrt[e + f*x^2])/(c*(d*e - c*f)*(c + d*x^2)^(5/2)) + ((
4*a*d*(d*e - 2*c*f) + b*c*(d*e + 3*c*f))*x*Sqrt[e + f*x^2])/(15*c^2*(d*e -
c*f)^2*(c + d*x^2)^(3/2)) + ((b*c*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2) + a*d
*(8*d^2*e^2 - 23*c*d*e*f + 23*c^2*f^2))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(S
qrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c^(5/2)*Sqrt[d]*(d*e - c*f)^3*Sqr
t[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (Sqrt[e]*Sqrt[f]*(b*c
*e*(d*e - 9*c*f) + a*(4*d^2*e^2 - 11*c*d*e*f + 15*c^2*f^2))*Sqrt[c + d*x^2]
*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*c^3*(d*e - c*
f)^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
```

```
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\text{integral} = -\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)\left(c+dx^2\right)^{5/2}} - \frac{\int \frac{-bce-4ade+5acf+3(bc-ad)fx^2}{\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}\,dx}{5c(de-cf)}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)\left(c+dx^2\right)^{5/2}} + \frac{(4ad(de-2cf)+bc(de+3cf))x\sqrt{e+fx^2}}{15c^2(de-cf)^2\left(c+dx^2\right)^{3/2}} + \frac{\int \frac{2bce(de-3cf)+a\left(8d^2e^2-19cdef+15c^2f^2\right)+f(4ad(de-2cf)+bc(de+3cf))x^2}{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}\,dx}{15c^2(de-cf)^2}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)\left(c+dx^2\right)^{5/2}} + \frac{(4ad(de-2cf)+bc(de+3cf))x\sqrt{e+fx^2}}{15c^2(de-cf)^2\left(c+dx^2\right)^{3/2}} - \frac{\left(f\left(bce(de-9cf)+a\left(4d^2e^2-11cdef+15c^2f^2\right)\right)\right)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15c^2(de-cf)^3} + \frac{\left(bc\left(2d^2e^2-7cdef-3c^2f^2\right)+ad\left(8d^2e^2-23cdef+23c^2f^2\right)\right)\int \frac{\sqrt{e+fx^2}}{\left(c+dx^2\right)^{3/2}}\,dx}{15c^2(de-cf)^3}$$

$$= -\frac{(bc-ad)x\sqrt{e+fx^2}}{5c(de-cf)\left(c+dx^2\right)^{5/2}} + \frac{(4ad(de-2cf)+bc(de+3cf))x\sqrt{e+fx^2}}{15c^2(de-cf)^2\left(c+dx^2\right)^{3/2}} + \frac{\left(bc\left(2d^2e^2-7cdef-3c^2f^2\right)+ad\left(8d^2e^2-23cdef+23c^2f^2\right)\right)\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{d}(de-cf)^3\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{\sqrt{e}\sqrt{f}\left(bce(de-9cf)+a\left(4d^2e^2-11cdef+15c^2f^2\right)\right)\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.23 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.98

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \frac{-\sqrt{\frac{d}{c}}x(e+fx^2)\left(3c^2(bc-ad)(de-cf)^2+c(-de+cf)(4ad(de-2cf)+bc(\right.}{}$$

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)^(7/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (-(Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)*(d*e - c*f)^2 + c*(-(d*e) + c
*f)*(4*a*d*(d*e - 2*c*f) + b*c*(d*e + 3*c*f))*(c + d*x^2) + (a*d*(-8*d^2*e^
2 + 23*c*d*e*f - 23*c^2*f^2) + b*c*(-2*d^2*e^2 + 7*c*d*e*f + 3*c^2*f^2))*(c
 + d*x^2)^2)) - I*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(e*
(a*d*(-8*d^2*e^2 + 23*c*d*e*f - 23*c^2*f^2) + b*c*(-2*d^2*e^2 + 7*c*d*e*f +
 3*c^2*f^2))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*e - c*f)*(
2*b*c*e*(d*e - 3*c*f) + a*(8*d^2*e^2 - 19*c*d*e*f + 15*c^2*f^2))*EllipticF[
I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(15*c^3*Sqrt[d/c]*(d*e - c*f)^3*(c +
 d*x^2)^(5/2)*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 6.75 (sec) , antiderivative size = 760, normalized size of antiderivative = 1.90

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(-\frac{x(ad-bc)\sqrt{dfx^4+cfx^2+dex^2+ce}}{5d^3c(cf-de)\left(x^2+\frac{c}{d}\right)^3}-\frac{\left(8acdf-4ae\,d^2-3c^2bf-bcde\right)x\sqrt{dfx^4+cfx^2+dex^2+ce}}{15c^2(cf-de)^2d^2\left(x^2+\frac{c}{d}\right)^2}-\frac{\left(dfx^2+de\right)x\left(23a\,c^2d\,f^2\right.}{15d\,c^3}\right.}{}$
default	Expression too large to display

```
[In] int((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/5/d^3/c/(c*
f-d*e)*x*(a*d-b*c)*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^3-1/15*(8*
a*c*d*f-4*a*d^2*e-3*b*c^2*f-b*c*d*e)/c^2/(c*f-d*e)^2/d^2*x*(d*f*x^4+c*f*x^2
+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2-1/15*(d*f*x^2+d*e)/d/c^3/(c*f-d*e)^3*x*(23*
a*c^2*d*f^2-23*a*c*d^2*e*f+8*a*d^3*e^2-3*b*c^3*f^2-7*b*c^2*d*e*f+2*b*c*d^2*
e^2)/((x^2+c/d)*(d*f*x^2+d*e))^(1/2)+(-1/15*f*(8*a*c*d*f-4*a*d^2*e-3*b*c^2*
f-b*c*d*e)/d/c^2/(c*f-d*e)^2+1/15/d/(c*f-d*e)^2*(23*a*c^2*d*f^2-23*a*c*d^2*
e*f+8*a*d^3*e^2-3*b*c^3*f^2-7*b*c^2*d*e*f+2*b*c*d^2*e^2)/c^3+1/15*e/c^3/(c*
f-d*e)^3*(23*a*c^2*d*f^2-23*a*c*d^2*e*f+8*a*d^3*e^2-3*b*c^3*f^2-7*b*c^2*d*e
*f+2*b*c*d^2*e^2))/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^
```

```
4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1
/2))-1/15*(23*a*c^2*d*f^2-23*a*c*d^2*e*f+8*a*d^3*e^2-3*b*c^3*f^2-7*b*c^2*d*
e*f+2*b*c*d^2*e^2)/c^3/(c*f-d*e)^3*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^
2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*(EllipticF(x*(-d/c)^(1/2),(-
1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1257 vs. $2(383) = 766$.

Time = 0.14 (sec) , antiderivative size = 1257, normalized size of antiderivative = 3.13

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(((2*(b*c*d^6 + 4*a*d^7)*e^3 - (7*b*c^2*d^5 + 23*a*c*d^6)*e^2*f - (3*
b*c^3*d^4 - 23*a*c^2*d^5)*e*f^2)*x^6 + 3*(2*(b*c^2*d^5 + 4*a*c*d^6)*e^3 - (
7*b*c^3*d^4 + 23*a*c^2*d^5)*e^2*f - (3*b*c^4*d^3 - 23*a*c^3*d^4)*e*f^2)*x^4
 + 2*(b*c^4*d^3 + 4*a*c^3*d^4)*e^3 - (7*b*c^5*d^2 + 23*a*c^4*d^3)*e^2*f - (
3*b*c^6*d - 23*a*c^5*d^2)*e*f^2 + 3*(2*(b*c^3*d^4 + 4*a*c^2*d^5)*e^3 - (7*b
*c^4*d^3 + 23*a*c^3*d^4)*e^2*f - (3*b*c^5*d^2 - 23*a*c^4*d^3)*e*f^2)*x^2)*s
qrt(c*e)*sqrt(-d/c)*elliptic_e(arcsin(x*sqrt(-d/c)), c*f/(d*e)) - (15*a*c^7
*f^3 + (15*a*c^4*d^3*f^3 + 2*(b*c*d^6 + 4*a*d^7)*e^3 + (b*c^3*d^4 + (4*a -
7*b)*c^2*d^5 - 23*a*c*d^6)*e^2*f - (9*b*c^4*d^3 + (11*a + 3*b)*c^3*d^4 - 23
*a*c^2*d^5)*e*f^2)*x^6 + 3*(15*a*c^5*d^2*f^3 + 2*(b*c^2*d^5 + 4*a*c*d^6)*e^
3 + (b*c^4*d^3 + (4*a - 7*b)*c^3*d^4 - 23*a*c^2*d^5)*e^2*f - (9*b*c^5*d^2 +
 (11*a + 3*b)*c^4*d^3 - 23*a*c^3*d^4)*e*f^2)*x^4 + 2*(b*c^4*d^3 + 4*a*c^3*d
^4)*e^3 + (b*c^6*d + (4*a - 7*b)*c^5*d^2 - 23*a*c^4*d^3)*e^2*f - (9*b*c^7 +
 (11*a + 3*b)*c^6*d - 23*a*c^5*d^2)*e*f^2 + 3*(15*a*c^6*d*f^3 + 2*(b*c^3*d^
4 + 4*a*c^2*d^5)*e^3 + (b*c^5*d^2 + (4*a - 7*b)*c^4*d^3 - 23*a*c^3*d^4)*e^2
*f - (9*b*c^6*d + (11*a + 3*b)*c^5*d^2 - 23*a*c^4*d^3)*e*f^2)*x^2)*sqrt(c*e
)*sqrt(-d/c)*elliptic_f(arcsin(x*sqrt(-d/c)), c*f/(d*e)) - ((2*(b*c^2*d^5 +
 4*a*c*d^6)*e^3 - (7*b*c^3*d^4 + 23*a*c^2*d^5)*e^2*f - (3*b*c^4*d^3 - 23*a*
c^3*d^4)*e*f^2)*x^5 + (5*(b*c^3*d^4 + 4*a*c^2*d^5)*e^3 - 2*(6*b*c^4*d^3 + 2
9*a*c^3*d^4)*e^2*f - 9*(b*c^5*d^2 - 6*a*c^4*d^3)*e*f^2)*x^3 + (15*a*c^3*d^4
*e^3 + (b*c^5*d^2 - 41*a*c^4*d^3)*e^2*f - (9*b*c^6*d - 34*a*c^5*d^2)*e*f^2)
*x)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(c^7*d^4*e^4 - 3*c^8*d^3*e^3*f + 3*c^9
*d^2*e^2*f^2 - c^10*d*e*f^3 + (c^4*d^7*e^4 - 3*c^5*d^6*e^3*f + 3*c^6*d^5*e^
2*f^2 - c^7*d^4*e*f^3)*x^6 + 3*(c^5*d^6*e^4 - 3*c^6*d^5*e^3*f + 3*c^7*d^4*e
^2*f^2 - c^8*d^3*e*f^3)*x^4 + 3*(c^6*d^5*e^4 - 3*c^7*d^4*e^3*f + 3*c^8*d^3*
e^2*f^2 - c^9*d^2*e*f^3)*x^2)
```

Sympy [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{a+bx^2}{(c+dx^2)^{\frac{7}{2}}\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(7/2)/(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)/((c + d*x**2)**(7/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{7}{2}}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{7}{2}}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(7/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(7/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^{7/2}\sqrt{e+fx^2}}\,dx = \int \frac{b\,x^2+a}{(d\,x^2+c)^{7/2}\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int((a + b*x^2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(1/2)),x)
[Out] int((a + b*x^2)/((c + d*x^2)^(7/2)*(e + f*x^2)^(1/2)), x)
```

3.42 $\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 501

$$\begin{aligned}
&\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx = \\
&-\frac{(5af(8d^2e^2-13cdef+3c^2f^2)-2be(24d^2e^2-44cdef+19c^2f^2))\,x\sqrt{c+dx^2}}{15ef^3\sqrt{e+fx^2}} \\
&-\frac{(be-af)x(c+dx^2)^{5/2}}{ef\sqrt{e+fx^2}}-\frac{d(be(24de-23cf)-5af(4de-3cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15ef^3} \\
&+\frac{d(6be-5af)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5ef^2} \\
&+\frac{(5af(8d^2e^2-13cdef+3c^2f^2)-2be(24d^2e^2-44cdef+19c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15\sqrt{e}f^{7/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&-\frac{\sqrt{e}(10adf(2de-3cf)-b(24d^2e^2-41cdef+15c^2f^2))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15f^{7/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

```
[Out] -(-a*f+b*e)*x*(d*x^2+c)^(5/2)/e/f/(f*x^2+e)^(1/2)-1/15*(5*a*f*(3*c^2*f^2-13
*c*d*e*f+8*d^2*e^2)-2*b*e*(19*c^2*f^2-44*c*d*e*f+24*d^2*e^2))*x*(d*x^2+c)^(
1/2)/e/f^3/(f*x^2+e)^(1/2)+1/15*(5*a*f*(3*c^2*f^2-13*c*d*e*f+8*d^2*e^2)-2*b
*e*(19*c^2*f^2-44*c*d*e*f+24*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1
/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2
+c)^(1/2)/f^(7/2)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1
/15*(10*a*d*f*(-3*c*f+2*d*e)-b*(15*c^2*f^2-41*c*d*e*f+24*d^2*e^2))*(1/(1+f*
x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/
2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^(7/2)/(e*(d*x^2+c)/c/(f*x^2
```

```
+e))^(1/2)/(f*x^2+e)^(1/2)+1/5*d*(-5*a*f+6*b*e)*x*(d*x^2+c)^(3/2)*(f*x^2+e)
^(1/2)/e/f^2-1/15*d*(b*e*(-23*c*f+24*d*e)-5*a*f*(-3*c*f+4*d*e))*x*(d*x^2+c)
^(1/2)*(f*x^2+e)^(1/2)/e/f^3
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {540, 542, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx =$$
$$-\frac{\sqrt{e}\sqrt{c+dx^2}(10adf(2de-3cf)-b(15c^2f^2-41cdef+24d^2e^2))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15f^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{\sqrt{c+dx^2}(5af(3c^2f^2-13cdef+8d^2e^2)-2be(19c^2f^2-44cdef+24d^2e^2))\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15\sqrt{e}f^{7/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{x\sqrt{c+dx^2}(5af(3c^2f^2-13cdef+8d^2e^2)-2be(19c^2f^2-44cdef+24d^2e^2))}{15ef^3\sqrt{e+fx^2}}$$
$$-\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(be(24de-23cf)-5af(4de-3cf))}{15ef^3}$$
$$+\frac{dx(c+dx^2)^{3/2}\sqrt{e+fx^2}(6be-5af)}{5ef^2}-\frac{x(c+dx^2)^{5/2}(be-af)}{ef\sqrt{e+fx^2}}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2),x]
```

```
[Out] -1/15*((5*a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) - 2*b*e*(24*d^2*e^2 - 44
*c*d*e*f + 19*c^2*f^2))*x*Sqrt[c + d*x^2])/(e*f^3*Sqrt[e + f*x^2]) - ((b*e
- a*f)*x*(c + d*x^2)^(5/2))/(e*f*Sqrt[e + f*x^2]) - (d*(b*e*(24*d*e - 23*c*
f) - 5*a*f*(4*d*e - 3*c*f))*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*e*f^3) +
 (d*(6*b*e - 5*a*f)*x*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*e*f^2) + ((5*a*
f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) - 2*b*e*(24*d^2*e^2 - 44*c*d*e*f + 1
9*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(15*Sqrt[e]*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2]) - (Sqrt[e]*(10*a*d*f*(2*d*e - 3*c*f) - b*(24*d^2*e^2 - 41*c*d*e*f
 + 15*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 -
(d*e)/(c*f)])/(15*f^(7/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*
x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2))]]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\text{integral} = -\frac{(be-af)x(c+dx^2)^{5/2}}{ef\sqrt{e+fx^2}} - \frac{\int \frac{(c+dx^2)^{3/2}(-bce-d(6be-5af)x^2)}{\sqrt{e+fx^2}}\,dx}{ef}$$

$$= -\frac{(be-af)x(c+dx^2)^{5/2}}{ef\sqrt{e+fx^2}} + \frac{d(6be-5af)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5ef^2} - \frac{\int \frac{\sqrt{c+dx^2}(ce(6bde-5bcf-5adf)+d(be(24de-23cf)-5af(4de-3cf))x^2)}{\sqrt{e+fx^2}}dx}{5ef^2}$$

$$= -\frac{(be-af)x(c+dx^2)^{5/2}}{ef\sqrt{e+fx^2}} - \frac{d(be(24de-23cf)-5af(4de-3cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15ef^3} + \frac{d(6be-5af)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5ef^2} - \frac{\int \frac{ce(10adf(2de-3cf)-b(24d^2e^2-41cdef+15c^2f^2))+d(5af(8d^2e^2-13cdef+3c^2f^2)-2be(24d^2e^2-44cdef+19c^2f^2))x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{15ef^3}$$

$$= -\frac{(be-af)x(c+dx^2)^{5/2}}{ef\sqrt{e+fx^2}} - \frac{d(be(24de-23cf)-5af(4de-3cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15ef^3} + \frac{d(6be-5af)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5ef^2} - \frac{(c(10adf(2de-3cf)-b(24d^2e^2-41cdef+15c^2f^2)))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{15f^3} - \frac{(d(5af(8d^2e^2-13cdef+3c^2f^2)-2be(24d^2e^2-44cdef+19c^2f^2)))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{15ef^3}$$

$$= -\frac{(5af(8d^2e^2-13cdef+3c^2f^2)-2be(24d^2e^2-44cdef+19c^2f^2))x\sqrt{c+dx^2}}{15ef^3\sqrt{e+fx^2}} - \frac{(be-af)x(c+dx^2)^{5/2}}{ef\sqrt{e+fx^2}} - \frac{d(be(24de-23cf)-5af(4de-3cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15ef^3} + \frac{d(6be-5af)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5ef^2} - \frac{\sqrt{e}(10adf(2de-3cf)-b(24d^2e^2-41cdef+15c^2f^2))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15f^{7/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(5af(8d^2e^2-13cdef+3c^2f^2)-2be(24d^2e^2-44cdef+19c^2f^2))\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}dx}{15f^3}$$

$$= -\frac{(5af(8d^2e^2 - 13cdef + 3c^2f^2) - 2be(24d^2e^2 - 44cdef + 19c^2f^2))\, x\sqrt{c+dx^2}}{15ef^3\sqrt{e+fx^2}}$$
$$- \frac{(be-af)x(c+dx^2)^{5/2}}{ef\sqrt{e+fx^2}}$$
$$- \frac{d(be(24de-23cf) - 5af(4de-3cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15ef^3}$$
$$+ \frac{d(6be-5af)x(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5ef^2}$$
$$+ \frac{(5af(8d^2e^2 - 13cdef + 3c^2f^2) - 2be(24d^2e^2 - 44cdef + 19c^2f^2))\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1 \right.}{15\sqrt{e}f^{7/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{\sqrt{e}(10adf(2de-3cf) - b(24d^2e^2 - 41cdef + 15c^2f^2))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1 - \frac{de}{cf}\right)}{15f^{7/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.81 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.74

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}fx(c+dx^2)(5af(-6cdef + 3c^2f^2 + d^2e(4e+fx^2)) + be(-15c^2f^2 + cdf}{}$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(5*a*f*(-6*c*d*e*f + 3*c^2*f^2 + d^2*e*(4*e + f*
x^2)) + b*e*(-15*c^2*f^2 + c*d*f*(41*e + 11*f*x^2) - 3*d^2*(8*e^2 + 2*e*f*x
^2 - f^2*x^4))) - I*d*e*(-5*a*f*(8*d^2*e^2 - 13*c*d*e*f + 3*c^2*f^2) + 2*b*
e*(24*d^2*e^2 - 44*c*d*e*f + 19*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x
^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) + c*f)*
(5*a*d*f*(-8*d*e + 9*c*f) + b*(48*d^2*e^2 - 64*c*d*e*f + 15*c^2*f^2))*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)])/(15*Sqrt[d/c]*e*f^4*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 9.68 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.58

method	result
risch	$\frac{xd(3bdf\,x^2+5adf+11bcf-9bde)\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{15f^3}+\frac{\left(-\frac{d\left(35acd\,f^2-25a\,d^2ef+23b\,c^2f^2-58bcdef+33b\,d^2e^2\right)e\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\left(F\left(x\sqrt{\ldots}\right.\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}\ldots}\right.}{\ldots}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{\left(df\,x^2+cf\right)\left(c^2a\,f^3-2acde\,f^2+a\,d^2e^2f-b\,c^2e\,f^2+2bcd\,e^2f-b\,d^2e^3\right)x}{e\,f^4\sqrt{\left(x^2+\frac{e}{f}\right)\left(df\,x^2+cf\right)}}+\frac{b\,d^2x^3\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}{5f^2}+\frac{\left(\frac{d^2(adf+3bcf-\ldots}{f^2}\right.}{\ldots}\right.}{\ldots}$
default	Expression too large to display

```
[In] int((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*x*d*(3*b*d*f*x^2+5*a*d*f+11*b*c*f-9*b*d*e)*(d*x^2+c)^(1/2)*(f*x^2+e)^(
1/2)/f^3+1/15/f^3*(-d*(35*a*c*d*f^2-25*a*d^2*e*f+23*b*c^2*f^2-58*b*c*d*e*f+
33*b*d^2*e^2)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c
*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1
/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)))+(45*a*c^2*d*f^3-50
*a*c*d^2*e*f^2+15*a*d^3*e^2*f+15*b*c^3*f^3-56*b*c^2*d*e*f^2+54*b*c*d^2*e^2*
f-15*b*d^3*e^3)/f/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4
+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/
2))+(15*a*c^3*f^4-45*a*c^2*d*e*f^3+45*a*c*d^2*e^2*f^2-15*a*d^3*e^3*f-15*b*c
^3*e*f^3+45*b*c^2*d*e^2*f^2-45*b*c*d^2*e^3*f+15*b*d^3*e^4)/f*((d*f*x^2+c*f)
/e/(c*f-d*e)*x/((x^2+e/f)*(d*f*x^2+c*f))^(1/2)+(1/e-c*f/e/(c*f-d*e))/(-d/c)
^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1
/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+d/(c*f-d*e)/(-d/c)^(
1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2
)*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1
/2),(-1+(c*f+d*e)/e/d)^(1/2)))))*((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2
)/(f*x^2+e)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx =$$

$$-\frac{((48\,bd^3e^4f - 15\,ac^2def^4 - 8\,(11\,bcd^2 + 5\,ad^3)e^3f^2 + (38\,bc^2d + 65\,acd^2)e^2f^3)x^3 + (48\,bd^3e^5 - 15\,ac^2de^2f}{}$$

[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")

[Out] -1/15*(((48*b*d^3*e^4*f - 15*a*c^2*d*e*f^4 - 8*(11*b*c*d^2 + 5*a*d^3)*e^3*f^2 + (38*b*c^2*d + 65*a*c*d^2)*e^2*f^3)*x^3 + (48*b*d^3*e^5 - 15*a*c^2*d*e^2*f^3 - 8*(11*b*c*d^2 + 5*a*d^3)*e^4*f + (38*b*c^2*d + 65*a*c*d^2)*e^3*f^2)*x)*sqrt(d*f)*sqrt(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - ((48*b*d^3*e^4*f - 8*(11*b*c*d^2 + 5*a*d^3)*e^3*f^2 + (38*b*c^2*d + (65*a + 24*b)*c*d^2)*e^2*f^3 - ((15*a + 41*b)*c^2*d + 20*a*c*d^2)*e*f^4 + 15*(b*c^3 + 2*a*c^2*d)*f^5)*x^3 + (48*b*d^3*e^5 - 8*(11*b*c*d^2 + 5*a*d^3)*e^4*f + (38*b*c^2*d + (65*a + 24*b)*c*d^2)*e^3*f^2 - ((15*a + 41*b)*c^2*d + 20*a*c*d^2)*e^2*f^3 + 15*(b*c^3 + 2*a*c^2*d)*e*f^4)*x)*sqrt(d*f)*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (3*b*d^3*e*f^4*x^6 + 48*b*d^3*e^4*f - 15*a*c^2*d*e*f^4 - 8*(11*b*c*d^2 + 5*a*d^3)*e^3*f^2 + (38*b*c^2*d + 65*a*c*d^2)*e^2*f^3 - (6*b*d^3*e^2*f^3 - (11*b*c*d^2 + 5*a*d^3)*e*f^4)*x^4 + (24*b*d^3*e^3*f^2 - (47*b*c*d^2 + 20*a*d^3)*e^2*f^3 + (23*b*c^2*d + 35*a*c*d^2)*e*f^4)*x^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d*e*f^6*x^3 + d*e^2*f^5*x)

Sympy [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)(c+dx^2)^{\frac{5}{2}}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

[In] integrate((b*x**2+a)*(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)

[Out] Integral((a + b*x**2)*(c + d*x**2)**(5/2)/(e + f*x**2)**(3/2), x)

Maxima [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(5/2)/(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(c+dx^2)^{5/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)\,(d\,x^2+c)^{5/2}}{(f\,x^2+e)^{3/2}}\,dx$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2),x)
[Out] int(((a + b*x^2)*(c + d*x^2)^(5/2))/(e + f*x^2)^(3/2), x)
```

3.43 $\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 358

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = -\frac{(be(8de-7cf)-3af(2de-cf))x\sqrt{c+dx^2}}{3ef^2\sqrt{e+fx^2}} - \frac{(be-af)x(c+dx^2)^{3/2}}{ef\sqrt{e+fx^2}} + \frac{d(4be-3af)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef^2} + \frac{(be(8de-7cf)-3af(2de-cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3\sqrt{e}f^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{\sqrt{e}(4bde-3bcf-3adf)\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3f^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] -(-a*f+b*e)*x*(d*x^2+c)^(3/2)/e/f/(f*x^2+e)^(1/2)-1/3*(b*e*(-7*c*f+8*d*e)-3
*a*f*(-c*f+2*d*e))*x*(d*x^2+c)^(1/2)/e/f^2/(f*x^2+e)^(1/2)+1/3*(b*e*(-7*c*f
+8*d*e)-3*a*f*(-c*f+2*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellipti
cE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/f
^(5/2)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(-3*a*d*
f-3*b*c*f+4*b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1
/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^
(5/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*(-3*a*f+4*b*e)*
x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/e/f^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {540, 542, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = -\frac{\sqrt{e}\sqrt{c+dx^2}(-3adf-3bcf+4bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{\sqrt{c+dx^2}(be(8de-7cf)-3af(2de-cf))E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3\sqrt{e}f^{5/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(4be-3af)}{3ef^2} - \frac{x\sqrt{c+dx^2}(be(8de-7cf)-3af(2de-cf))}{3ef^2\sqrt{e+fx^2}} - \frac{x(c+dx^2)^{3/2}(be-af)}{ef\sqrt{e+fx^2}}$$

```
[In] Int[((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

```
[Out] -1/3*((b*e*(8*d*e - 7*c*f) - 3*a*f*(2*d*e - c*f))*x*Sqrt[c + d*x^2])/(e*f^2
*Sqrt[e + f*x^2]) - ((b*e - a*f)*x*(c + d*x^2)^(3/2))/(e*f*Sqrt[e + f*x^2])
 + (d*(4*b*e - 3*a*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*e*f^2) + ((b*e*
(8*d*e - 7*c*f) - 3*a*f*(2*d*e - c*f))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sq
rt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*Sqrt[e]*f^(5/2)*Sqrt[(e*(c + d*x^2)
)/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*(4*b*d*e - 3*b*c*f - 3*a*d*f
)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(3*f^(5/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(be-af)x(c+dx^2)^{3/2}}{ef\sqrt{e+fx^2}} - \frac{\int \frac{\sqrt{c+dx^2}\left(-bce-d(4be-3af)x^2\right)}{\sqrt{e+fx^2}}\,dx}{ef} \\
&= -\frac{(be-af)x(c+dx^2)^{3/2}}{ef\sqrt{e+fx^2}} + \frac{d(4be-3af)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef^2} \\
&\quad - \frac{\int \frac{ce(4bde-3bcf-3adf)+d(be(8de-7cf)-3af(2de-cf))x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3ef^2}
\end{aligned}$$

$$= -\frac{(be-af)x(c+dx^2)^{3/2}}{ef\sqrt{e+fx^2}} + \frac{d(4be-3af)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef^2} - \frac{(c(4bde-3bcf-3adf))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3f^2} - \frac{(d(be(8de-7cf)-3af(2de-cf)))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3ef^2}$$

$$= -\frac{(be(8de-7cf)-3af(2de-cf))x\sqrt{c+dx^2}}{3ef^2\sqrt{e+fx^2}} - \frac{(be-af)x(c+dx^2)^{3/2}}{ef\sqrt{e+fx^2}} + \frac{d(4be-3af)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef^2} - \frac{\sqrt{e}(4bde-3bcf-3adf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(be(8de-7cf)-3af(2de-cf))\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{3f^2}$$

$$= -\frac{(be(8de-7cf)-3af(2de-cf))x\sqrt{c+dx^2}}{3ef^2\sqrt{e+fx^2}} - \frac{(be-af)x(c+dx^2)^{3/2}}{ef\sqrt{e+fx^2}} + \frac{d(4be-3af)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef^2} + \frac{(be(8de-7cf)-3af(2de-cf))\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3\sqrt{e}f^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{\sqrt{e}(4bde-3bcf-3adf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{5/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.87 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.73

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}fx(c+dx^2)(3af(-de+cf)+be(4de-3cf+dfx^2))-ide(-3af(-2de+$$

```
[In] Integrate[((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*f*x*(c + d*x^2)*(3*a*f*(-(d*e) + c*f) + b*e*(4*d*e - 3*c*f + d*f
*x^2)) - I*d*e*(-3*a*f*(-2*d*e + c*f) + b*e*(-8*d*e + 7*c*f))*Sqrt[1 + (d*x
```

```
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] -
 I*e*(-(d*e) + c*f)*(-8*b*d*e + 3*b*c*f + 6*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt
[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*Sqrt[d/c
]*e*f^3*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 7.41 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.52

method	result
elliptic	$\frac{\sqrt{(d x^2+c)(f x^2+e)}\left(\frac{\left(df x^2+cf\right)\left(ac f^2-adef-bcef+bd e^2\right)x}{f^3 e\sqrt{\left(x^2+\frac{e}{f}\right)\left(df x^2+cf\right)}}+\frac{bdx\sqrt{df x^4+cf x^2+de x^2+ce}}{3f^2}+\frac{\left(\frac{2acd f^2-a d^2 ef+b c^2 f^2-2bcdef+b d^2 e^2}{f^3}+\right.}{}\right.}{}$
risch	$\frac{bx\sqrt{d x^2+c}\sqrt{f x^2+e}\,d}{3f^2}+\frac{\left(-\frac{d(3adf+4bcf-5bde)e\sqrt{1+\frac{d x^2}{c}}\sqrt{1+\frac{f x^2}{e}}\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)-E\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)\right)}{\sqrt{-\frac{d}{c}}\sqrt{df x^4+cf x^2+de x^2+ce}\,f}+\frac{\left(6acd f^2\right.}{}\right.}{}$
default	$\frac{\sqrt{d x^2+c}\sqrt{f x^2+e}\left(\sqrt{-\frac{d}{c}}\,b\,d^2 e\,f^2 x^5+3\sqrt{-\frac{d}{c}}\,acd\,f^3 x^3-3\sqrt{-\frac{d}{c}}\,a\,d^2 e\,f^2 x^3-2\sqrt{-\frac{d}{c}}\,bcde\,f^2 x^3+4\sqrt{-\frac{d}{c}}\,b\,d^2 e^2 f\,x^3+6\sqrt{\frac{d x^2+c}{c}}\sqrt{\frac{f x}{}}\right.}{}$

```
[In] int((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*((d*f*x^2+c*f)*
(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/f^3/e*x/((x^2+e/f)*(d*f*x^2+c*f))^(1/2)+1
/3*b*d/f^2*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)+((2*a*c*d*f^2-a*d^2*e*f+b*
c^2*f^2-2*b*c*d*e*f+b*d^2*e^2)/f^3+(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/f^3*(c
*f-d*e)/e-c/f^2*(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/e-1/3*b*d/f^2*c*e)/(-d/c)
^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1
/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(d/f^2*(a*d*f+2*b*c*
f-b*d*e)-(a*c*f^2-a*d*e*f-b*c*e*f+b*d*e^2)/f^2*d/e-1/3*b*d/f^2*(2*c*f+2*d*e
))*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*
x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-Ellipt
icE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^2)\,(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \frac{((8\,bd^2e^3f+3\,acdef^3-(7\,bcd+6\,ad^2)e^2f^2)x^3+(8\,bd^2e^4+3\,acde^2f^2-(7\,bcd}{}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(((8*b*d^2*e^3*f + 3*a*c*d*e*f^3 - (7*b*c*d + 6*a*d^2)*e^2*f^2)*x^3 + (
8*b*d^2*e^4 + 3*a*c*d*e^2*f^2 - (7*b*c*d + 6*a*d^2)*e^3*f)*x)*sqrt(d*f)*sqr
t(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - ((8*b*d^2*e^3*f + (3*
a + 4*b)*c*d*e*f^3 - (7*b*c*d + 6*a*d^2)*e^2*f^2 - 3*(b*c^2 + a*c*d)*f^4)*x
^3 + (8*b*d^2*e^4 + (3*a + 4*b)*c*d*e^2*f^2 - (7*b*c*d + 6*a*d^2)*e^3*f - 3
*(b*c^2 + a*c*d)*e*f^3)*x)*sqrt(d*f)*sqrt(-e/f)*elliptic_f(arcsin(sqrt(-e/f
)/x), c*f/(d*e)) + (b*d^2*e*f^3*x^4 - 8*b*d^2*e^3*f - 3*a*c*d*e*f^3 + (7*b*
c*d + 6*a*d^2)*e^2*f^2 - (4*b*d^2*e^2*f^2 - (4*b*c*d + 3*a*d^2)*e*f^3)*x^2)
*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d*e*f^5*x^3 + d*e^2*f^4*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)\,(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*(c + d*x**2)**(3/2)/(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)\,(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*(d*x^2 + c)^(3/2)/(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)(c+dx^2)^{3/2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)\,(d\,x^2+c)^{3/2}}{(f\,x^2+e)^{3/2}}\,dx$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2),x)
[Out] int(((a + b*x^2)*(c + d*x^2)^(3/2))/(e + f*x^2)^(3/2), x)
```

3.44 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 258

$$\begin{aligned}\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx &= -\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} \\ &+ \frac{(2be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} - \frac{(2be-af)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{b\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

```
[Out] -(-a*f+b*e)*x*(d*x^2+c)^(1/2)/e/f/(f*x^2+e)^(1/2)+(-a*f+2*b*e)*x*(d*x^2+c)^
(1/2)/e/f/(f*x^2+e)^(1/2)-(-a*f+2*b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1
/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2
+c)^(1/2)/f^(3/2)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+b
*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x
^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^(3/2)/(e*(d*x^2+c)
/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {540, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = -\frac{\sqrt{c+dx^2}(2be-af)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{x\sqrt{c+dx^2}(be-af)}{ef\sqrt{e+fx^2}} + \frac{x\sqrt{c+dx^2}(2be-af)}{ef\sqrt{e+fx^2}} + \frac{b\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{f^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[In] Int[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] -(((b*e - a*f)*x*Sqrt[c + d*x^2])/(e*f*Sqrt[e + f*x^2])) + ((2*b*e - a*f)*x
*Sqrt[c + d*x^2])/(e*f*Sqrt[e + f*x^2]) - ((2*b*e - a*f)*Sqrt[c + d*x^2]*El
lipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(Sqrt[e]*f^(3/2)*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b*Sqrt[e]*Sqrt[c + d
*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(f^(3/2)*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\text{integral} = -\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} - \frac{\int \frac{-bce-d(2be-af)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{ef}$$

$$= -\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} + \frac{(bc)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{f} + \frac{(d(2be-af))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{ef}$$

$$= -\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} + \frac{(2be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} + \frac{b\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{(2be-af)\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{f}$$

$$= -\frac{(be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} + \frac{(2be-af)x\sqrt{c+dx^2}}{ef\sqrt{e+fx^2}} - \frac{(2be-af)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{b\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.51 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.81

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}f(-be+af)x(c+dx^2)-ide(2be-af)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\Big(i\text{arcsinh}\Big(\sqrt{}}{\sqrt{\frac{d}{c}}ef^2\sqrt{}}$$

```
[In] Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] (Sqrt[d/c]*f*(-(b*e) + a*f)*x*(c + d*x^2) - I*d*e*(2*b*e - a*f)*Sqrt[1 + (d
*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
 - I*e*(-2*b*d*e + b*c*f + a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(Sqrt[d/c]*e*f^2*Sqrt[c + d*
x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.47

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{(dfx^2+cf)(af-be)x}{ef^2\sqrt{\left(x^2+\frac{e}{f}\right)(dfx^2+cf)}}+\frac{\left(\frac{adf+bcf-bde}{f^2}+\frac{(cf-de)(af-be)}{f^2e}-\frac{c(af-be)}{fe}\right)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+}{ed}}\right.}{\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+ce}}\right.}{\sqrt{dx^2+c}\sqrt{fx^2+e}}$
default	$\sqrt{dx^2+c}\sqrt{fx^2+e}\left(\sqrt{-\frac{d}{c}}adf^2x^3-\sqrt{-\frac{d}{c}}bdefx^3+\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)adef+\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{c}{d}}\right.\right.$

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*((d*f*x^2+c*f)*
(a*f-b*e)/e/f^2*x/((x^2+e/f)*(d*f*x^2+c*f))^(1/2)+((a*d*f+b*c*f-b*d*e)/f^2+
(c*f-d*e)*(a*f-b*e)/f^2/e-c/f*(a*f-b*e)/e)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(
1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/
2),(-1+(c*f+d*e)/e/d)^(1/2))-(b*d/f-(a*f-b*e)/f*d/e)*e/(-d/c)^(1/2)*(1+d*x^
2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(Ellipti
cF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c
*f+d*e)/e/d)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx =$$

$$-\frac{((2\,bde^2f-adef^2)x^3+(2\,bde^3-ade^2f)x)\sqrt{df}\sqrt{-\frac{e}{f}}E(\arcsin\left(\frac{\sqrt{-\frac{e}{f}}}{x}\right)\mid\frac{cf}{de})-((2\,bde^2f-adef^2+bcf^3)x^3}{def^4}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] -(((2*b*d*e^2*f - a*d*e*f^2)*x^3 + (2*b*d*e^3 - a*d*e^2*f)*x)*sqrt(d*f)*sqr
t(-e/f)*elliptic_e(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - ((2*b*d*e^2*f - a*d*e
*f^2 + b*c*f^3)*x^3 + (2*b*d*e^3 - a*d*e^2*f + b*c*e*f^2)*x)*sqrt(d*f)*sqrt
(-e/f)*elliptic_f(arcsin(sqrt(-e/f)/x), c*f/(d*e)) - (b*d*e*f^2*x^2 + 2*b*d
*e^2*f - a*d*e*f^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(d*e*f^4*x^3 + d*e^2*f
^3*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{3/2}}\,dx$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
[Out] int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

3.45 $\int \frac{a+bx^2}{\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 209

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{(be-af)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{(bc-ad)\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] (-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1
/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/(-c*f+d*e)/e^(1/2)
/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(-a*d+b*c)*(1/(1+f
*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1
/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/(-c*f+d*e)/f^(1/2)/(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {539, 429, 422}

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{c+dx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[In] Int[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((b*e - a*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*
e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2]) - ((b*c - a*d)*Sqrt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan
[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(bc-ad)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{de-cf}+\frac{(be-af)\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{de-cf}\\
&= \frac{(be-af)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}-\frac{(bc-ad)\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.87 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}f(-be+af)x(c+dx^2)-ide(be-af)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\Big(i\operatorname{arcsinh}\Big(\sqrt{}}{\sqrt{\frac{d}{c}}ef(-de+cf)}$$

[In] Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]

[Out] (Sqrt[d/c]*f*(-(b*e) + a*f)*x*(c + d*x^2) - I*d*e*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*b*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(Sqrt[d/c]*e*f*(-(d*e) + c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])

Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.67

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\,ad\,f^2x^3-\sqrt{-\frac{d}{c}}\,bdef\,x^3+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)bcef-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)bd\,e^2-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{}\right.}{\sqrt{-\frac{d}{c}}\,ef(cf-de)(df\,x^4+c}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{\left(df\,x^2+cf\right)x(af-be)}{fe(cf-de)\sqrt{\left(x^2+\frac{e}{f}\right)\left(df\,x^2+cf\right)}}+\frac{\left(\frac{b}{f}+\frac{af-be}{fe}-\frac{c(af-be)}{e(cf-de)}\right)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{d(af-be)\sqrt{}}{}\right.}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}$

[In] int((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)

[Out] ((-d/c)^(1/2)*a*d*f^2*x^3-(-d/c)^(1/2)*b*d*e*f*x^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*c*e*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*e^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*b*d*e^2+(-d/c)^(1/2)*a*c*f^2*x-(-d/c)^(1/2)*b*c*e*f*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/(-d/c)^(1/2)/e/f/(c*f-d*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.23

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{(bcdef-acdf^2)\sqrt{dx^2+c}\sqrt{fx^2+e}x-(bd^2e^2-ad^2ef+(bd^2ef-ad^2f^2)x^2)}{}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] ((b*c*d*e*f - a*c*d*f^2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)*x - (b*d^2*e^2 - a
*d^2*e*f + (b*d^2*e*f - a*d^2*f^2)*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_e(arc
sin(x*sqrt(-d/c)), c*f/(d*e)) + (b*d^2*e^2 + (b*c^2 - a*c*d - a*d^2)*e*f +
(b*d^2*e*f + (b*c^2 - a*c*d - a*d^2)*f^2)*x^2)*sqrt(c*e)*sqrt(-d/c)*ellipti
c_f(arcsin(x*sqrt(-d/c)), c*f/(d*e)))/(c*d^2*e^3*f - c^2*d*e^2*f^2 + (c*d^2
*e^2*f^2 - c^2*d*e*f^3)*x^2)
```

Sympy [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral((a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{bx^2+a}{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{bx^2+a}{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{b\,x^2+a}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}\,dx$$

```
[In] int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
[Out] int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

3.46 $\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 272

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = -\frac{(bc-ad)x}{c(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{\sqrt{f}(2bce-ade-acf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{\sqrt{e}(bde+bcf-2adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] -(-a*d+b*c)*x/c/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+(-2*a*d*f+b*c*f+
b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/
(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/(-c*f+d*e)^2
/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(-a*c*f-a*d*e+2*b*
c*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1
+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c/(-c*f+d*e)^2/e
^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {541, 539, 429, 422}

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{e}\sqrt{c+dx^2}(-2adf+bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{\sqrt{f}\sqrt{c+dx^2}(-acf-ade+2bce)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e}\sqrt{e+fx^2}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{x(bc-ad)}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)}$$

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -(((b*c - a*d)*x)/(c*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])) - (Sqrt[
f]*(2*b*c*e - a*d*e - a*c*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[e]*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c
*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[e]*(b*d*e + b*c*f - 2*a*d*f)*Sqrt[c
 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*Sqrt[
f]*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x}{c(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{\int \frac{-c(be-af)+(bc-ad)fx^2}{\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx}{c(de-cf)} \\
&= -\frac{(bc-ad)x}{c(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{(f(2bce-ade-acf))\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{c(de-cf)^2} \\
&\quad + \frac{(bde+bcf-2adf)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{(de-cf)^2} \\
&= -\frac{(bc-ad)x}{c(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} \\
&\quad - \frac{\sqrt{f}(2bce-ade-acf)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{\sqrt{e}(bde+bcf-2adf)\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{f}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.50 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.96

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(\sqrt{\frac{d}{c}}x(a(c^2f^2+cdf^2x^2+d^2e(e+fx^2))-bce(cf+d(e+2fx^2)))-i\right.}{}$$

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*(Sqrt[d/c]*x*(a*(c^2*f^2 + c*d*f^2*x^2 + d^2*e*(e + f*x^2)) - b*
c*e*(c*f + d*(e + 2*f*x^2))) - I*d*e*(2*b*c*e - a*(d*e + c*f))*Sqrt[1 + (d*
x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
- I*(b*c - a*d)*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*El
lipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(d*e*(d*e - c*f)^2*Sqrt[c +
d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 4.98 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.89

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(-\frac{2df\left(-\frac{(acf+ade-2bce)x^3}{2ce\left(c^2f^2-2cdef+d^2e^2\right)}-\frac{\left(c^2af^2+ad^2e^2-bc^2ef-bcde^2\right)x}{2ce\left(c^2f^2-2cdef+d^2e^2\right)df}\right)}{\sqrt{\left(x^4+\frac{(cf+de)x^2}{df}+\frac{ce}{df}\right)df}}+\frac{\left(\frac{a}{ce}-\frac{c^2af^2+ad^2e^2-bc^2ef-bcde^2}{ce\left(c^2f^2-2cdef+d^2e^2\right)}\right)\sqrt{1+\frac{dx^2}{c}}}{\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2}}\right.}{\sqrt{dx^2+c}\sqrt{f}}$
default	$\left(\sqrt{-\frac{d}{c}}\,acdf^2x^3+\sqrt{-\frac{d}{c}}\,ad^2efx^3-2\sqrt{-\frac{d}{c}}\,bcdefx^3-\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)acdef+\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\right.\right.$

```
[In] int((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-2*d*f*(-1/2*(
a*c*f+a*d*e-2*b*c*e)/c/e/(c^2*f^2-2*c*d*e*f+d^2*e^2)*x^3-1/2*(a*c^2*f^2+a*d
^2*e^2-b*c^2*e*f-b*c*d*e^2)/c/e/(c^2*f^2-2*c*d*e*f+d^2*e^2)/d/f*x)/((x^4+(c
*f+d*e)/d/f*x^2+c*e/d/f)*d*f)^(1/2)+(a/c/e-(a*c^2*f^2+a*d^2*e^2-b*c^2*e*f-b
*c*d*e^2)/c/e/(c^2*f^2-2*c*d*e*f+d^2*e^2))/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(
1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/
2),(-1+(c*f+d*e)/e/d)^(1/2))+d*(a*c*f+a*d*e-2*b*c*e)/c/(c^2*f^2-2*c*d*e*f+d
^2*e^2)/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d
*e*x^2+c*e)^(1/2)*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-Ellip
ticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 618 vs. $2(266) = 532$.

Time = 0.11 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.27

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx =$$

$$-\frac{(ac^2d^2ef+(acd^3f^2-(2\,bcd^3-ad^4)ef)x^4-(2\,bc^2d^2-acd^3)e^2+(ac^2d^2f^2-(2\,bcd^3-ad^4)e^2-2\,(bc^2d^2-}{}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] -((a*c^2*d^2*e*f + (a*c*d^3*f^2 - (2*b*c*d^3 - a*d^4)*e*f)*x^4 - (2*b*c^2*d
^2 - a*c*d^3)*e^2 + (a*c^2*d^2*f^2 - (2*b*c*d^3 - a*d^4)*e^2 - 2*(b*c^2*d^2
 - a*c*d^3)*e*f)*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_e(arcsin(x*sqrt(-d/c)),
 c*f/(d*e)) + (((b*c^2*d^2 + 2*b*c*d^3 - a*d^4)*e*f + (b*c^3*d - 2*a*c^2*d^
2 - a*c*d^3)*f^2)*x^4 + (b*c^3*d + 2*b*c^2*d^2 - a*c*d^3)*e^2 + (b*c^4 - 2*
a*c^3*d - a*c^2*d^2)*e*f + ((b*c^2*d^2 + 2*b*c*d^3 - a*d^4)*e^2 + 2*(b*c^3*
```

```
d - (a - b)*c^2*d^2 - a*c*d^3)*e*f + (b*c^4 - 2*a*c^3*d - a*c^2*d^2)*f^2)*x
^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_f(arcsin(x*sqrt(-d/c)), c*f/(d*e)) - ((a*
c^2*d^2*f^2 - (2*b*c^2*d^2 - a*c*d^3)*e*f)*x^3 - (b*c^3*d*e*f - a*c^3*d*f^2
 + (b*c^2*d^2 - a*c*d^3)*e^2)*x)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(c^3*d^3*
e^4 - 2*c^4*d^2*e^3*f + c^5*d*e^2*f^2 + (c^2*d^4*e^3*f - 2*c^3*d^3*e^2*f^2
+ c^4*d^2*e*f^3)*x^4 + (c^2*d^4*e^4 - c^3*d^3*e^3*f - c^4*d^2*e^2*f^2 + c^5
*d*e*f^3)*x^2)
```

Sympy [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{a+bx^2}{(c+dx^2)^{\frac{3}{2}}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
[Out] Integral((a + b*x**2)/((c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{bx^2+a}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx=\int \frac{bx^2+a}{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{b\,x^2+a}{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{3/2}}\,dx$$

```
[In] int((a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
[Out] int((a + b*x^2)/((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

3.47 $\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 375

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = -\frac{(bc-ad)x}{3c(de-cf)(c+dx^2)^{3/2}\sqrt{e+fx^2}} + \frac{(2ad(de-3cf)+bc(de+3cf))x}{3c^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{\sqrt{f}(bce(de+7cf)+a(2d^2e^2-7cdef-3c^2f^2))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{\sqrt{e}\sqrt{f}(ad(de-9cf)+bc(5de+3cf))\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] -1/3*(-a*d+b*c)*x/c/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2)+1/3*(2*a*d*(
-3*c*f+d*e)+b*c*(3*c*f+d*e))*x/c^2/(-c*f+d*e)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(
1/2)+1/3*(b*c*e*(7*c*f+d*e)+a*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2))*(1/(1+f*x^2
/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),
(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-c*f+d*e)^3/e^(1/2)/(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(a*d*(-9*c*f+d*e)+b*c*(3*c*f+5
*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/
(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-
c*f+d*e)^3/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {541, 539, 429, 422}

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{f}\sqrt{c+dx^2}(a(-3c^2f^2-7cdef+2d^2e^2)+bce(7cf+de))\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right.\right.}{3c^2\sqrt{e}\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(ad(de-9cf)+bc(3cf+5de))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{x(2ad(de-3cf)+bc(3cf+de))}{3c^2\sqrt{c+dx^2}\sqrt{e+fx^2}(de-cf)^2}-\frac{x(bc-ad)}{3c\,(c+dx^2)^{3/2}\sqrt{e+fx^2}(de-cf)}$$

```
[In] Int[(a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -1/3*((b*c - a*d)*x)/(c*(d*e - c*f)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]) + ((
2*a*d*(d*e - 3*c*f) + b*c*(d*e + 3*c*f))*x)/(3*c^2*(d*e - c*f)^2*Sqrt[c + d
*x^2]*Sqrt[e + f*x^2]) + (Sqrt[f]*(b*c*e*(d*e + 7*c*f) + a*(2*d^2*e^2 - 7*c
*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
 1 - (d*e)/(c*f)])/(3*c^2*Sqrt[e]*(d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e
+ f*x^2))]*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*(a*d*(d*e - 9*c*f) + b*c*(5*
d*e + 3*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d
*e)/(c*f)])/(3*c^2*(d*e - c*f)^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
```

```
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(bc-ad)x}{3c(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} - \frac{\int \frac{-bce-2ade+3acf+3(bc-ad)fx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx}{3c(de-cf)} \\
&= -\frac{(bc-ad)x}{3c(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} + \frac{(2ad(de-3cf)+bc(de+3cf))x}{3c^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}} \\
&\quad + \frac{\int \frac{-cf(4bce-ade-3acf)+f(2ad(de-3cf)+bc(de+3cf))x^2}{\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx}{3c^2(de-cf)^2} \\
&= -\frac{(bc-ad)x}{3c(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} + \frac{(2ad(de-3cf)+bc(de+3cf))x}{3c^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}} \\
&\quad - \frac{(f(ad(de-9cf)+bc(5de+3cf)))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3c(de-cf)^3} \\
&\quad + \frac{(f(bce(de+7cf)+a(2d^2e^2-7cdef-3c^2f^2)))\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{3c^2(de-cf)^3} \\
&= -\frac{(bc-ad)x}{3c(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} + \frac{(2ad(de-3cf)+bc(de+3cf))x}{3c^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}} \\
&\quad + \frac{\sqrt{f}(bce(de+7cf)+a(2d^2e^2-7cdef-3c^2f^2))\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2\sqrt{e}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{\sqrt{e}\sqrt{f}(ad(de-9cf)+bc(5de+3cf))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.39 (sec) , antiderivative size = 428, normalized size of antiderivative = 1.14

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}x(-bce(3c^3f^2+d^3ex^2(e+fx^2)+cd^2fx^2(4e+7fx^2)+c^2df(5e+11fx\ldots}{\ldots}$$

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*x*(-(b*c*e*(3*c^3*f^2 + d^3*e*x^2*(e + f*x^2) + c*d^2*f*x^2*(4*e
 + 7*f*x^2) + c^2*d*f*(5*e + 11*f*x^2))) + a*(3*c^4*f^3 + 6*c^3*d*f^3*x^2 -
 2*d^4*e^2*x^2*(e + f*x^2) + c^2*d^2*f*(8*e^2 + 8*e*f*x^2 + 3*f^2*x^4) + c*
d^3*e*(-3*e^2 + 4*e*f*x^2 + 7*f^2*x^4))) - I*d*e*(b*c*e*(d*e + 7*c*f) + a*(
2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*e*(-(d*e) +
 c*f)*(2*a*d*(d*e - 3*c*f) + b*c*(d*e + 3*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3
*c^2*Sqrt[d/c]*e*(-(d*e) + c*f)^3*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 6.53 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.79

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{(dfx^2+cf)fx(af-be)}{e(cf-de)^3\sqrt{\left(x^2+\frac{e}{f}\right)(dfx^2+cf)}}+\frac{x(ad-bc)\sqrt{dfx^4+cfx^2+dex^2+ce}}{3dc(cf-de)^2\left(x^2+\frac{c}{d}\right)^2}+\frac{(dfx^2+de)x(7acdf-2aed^2-4c^2bf-bcde)}{3c^2(cf-de)^3\sqrt{\left(x^2+\frac{c}{d}\right)(dfx^2+de)}}+\ldots\right.}{\ldots}$
default	Expression too large to display

```
[In] int((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*((d*f*x^2+c*f)*
f/e/(c*f-d*e)^3*x*(a*f-b*e)/((x^2+e/f)*(d*f*x^2+c*f))^(1/2)+1/3/d/c/(c*f-d*
e)^2*x*(a*d-b*c)*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2+1/3*(d*f*x
^2+d*e)/c^2/(c*f-d*e)^3*x*(7*a*c*d*f-2*a*d^2*e-4*b*c^2*f-b*c*d*e)/((x^2+c/d
)*(d*f*x^2+d*e))^(1/2)+(1/(c*f-d*e)^2*f*(a*f-b*e)/e-c*f^2/e/(c*f-d*e)^3*(a*
f-b*e)+1/3*(a*d-b*c)*f/c/(c*f-d*e)^2-1/3/(c*f-d*e)^2*(7*a*c*d*f-2*a*d^2*e-4
*b*c^2*f-b*c*d*e)/c^2-1/3*d*e/c^2/(c*f-d*e)^3*(7*a*c*d*f-2*a*d^2*e-4*b*c^2*
f-b*c*d*e))/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x
^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(-
d*f^2*(a*f-b*e)/e/(c*f-d*e)^3-1/3*d*f*(7*a*c*d*f-2*a*d^2*e-4*b*c^2*f-b*c*d*
```

```
e)/c^2/(c*f-d*e)^3)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f
*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e
/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1259 vs. $2(357) = 714$.

Time = 0.15 (sec) , antiderivative size = 1259, normalized size of antiderivative = 3.36

$$\int \frac{a + bx^2}{(c + dx^2)^{5/2} (e + fx^2)^{3/2}} \, dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*((3*a*c^4*d^2*e*f^2 + (3*a*c^2*d^4*f^3 - (b*c*d^5 + 2*a*d^6)*e^2*f - 7*
(b*c^2*d^4 - a*c*d^5)*e*f^2)*x^6 + (6*a*c^3*d^3*f^3 - (b*c*d^5 + 2*a*d^6)*e
^3 - 3*(3*b*c^2*d^4 - a*c*d^5)*e^2*f - (14*b*c^3*d^3 - 17*a*c^2*d^4)*e*f^2)
*x^4 - (b*c^3*d^3 + 2*a*c^2*d^4)*e^3 - 7*(b*c^4*d^2 - a*c^3*d^3)*e^2*f + (3
*a*c^4*d^2*f^3 - 2*(b*c^2*d^4 + 2*a*c*d^5)*e^3 - 3*(5*b*c^3*d^3 - 4*a*c^2*d
^4)*e^2*f - (7*b*c^4*d^2 - 13*a*c^3*d^3)*e*f^2)*x^2)*sqrt(c*e)*sqrt(-d/c)*e
lliptic_e(arcsin(x*sqrt(-d/c)), c*f/(d*e)) + (((b*c*d^5 + 2*a*d^6)*e^2*f +
(5*b*c^3*d^3 + (a + 7*b)*c^2*d^4 - 7*a*c*d^5)*e*f^2 + 3*(b*c^4*d^2 - 3*a*c^
3*d^3 - a*c^2*d^4)*f^3)*x^6 + ((b*c*d^5 + 2*a*d^6)*e^3 + (5*b*c^3*d^3 + (a
+ 9*b)*c^2*d^4 - 3*a*c*d^5)*e^2*f + (13*b*c^4*d^2 - 7*(a - 2*b)*c^3*d^3 - 1
7*a*c^2*d^4)*e*f^2 + 6*(b*c^5*d - 3*a*c^4*d^2 - a*c^3*d^3)*f^3)*x^4 + (b*c^
3*d^3 + 2*a*c^2*d^4)*e^3 + (5*b*c^5*d + (a + 7*b)*c^4*d^2 - 7*a*c^3*d^3)*e^
2*f + 3*(b*c^6 - 3*a*c^5*d - a*c^4*d^2)*e*f^2 + (2*(b*c^2*d^4 + 2*a*c*d^5)*
e^3 + (10*b*c^4*d^2 + (2*a + 15*b)*c^3*d^3 - 12*a*c^2*d^4)*e^2*f + (11*b*c^
5*d - (17*a - 7*b)*c^4*d^2 - 13*a*c^3*d^3)*e*f^2 + 3*(b*c^6 - 3*a*c^5*d - a
*c^4*d^2)*f^3)*x^2)*sqrt(c*e)*sqrt(-d/c)*elliptic_f(arcsin(x*sqrt(-d/c)), c
*f/(d*e)) - ((3*a*c^3*d^3*f^3 - (b*c^2*d^4 + 2*a*c*d^5)*e^2*f - 7*(b*c^3*d^
3 - a*c^2*d^4)*e*f^2)*x^5 + (6*a*c^4*d^2*f^3 - (b*c^2*d^4 + 2*a*c*d^5)*e^3
- 4*(b*c^3*d^3 - a*c^2*d^4)*e^2*f - (11*b*c^4*d^2 - 8*a*c^3*d^3)*e*f^2)*x^3
 - (3*a*c^2*d^4*e^3 + 3*b*c^5*d*e*f^2 - 3*a*c^5*d*f^3 + (5*b*c^4*d^2 - 8*a*
c^3*d^3)*e^2*f)*x)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e))/(c^5*d^4*e^5 - 3*c^6*d^
3*e^4*f + 3*c^7*d^2*e^3*f^2 - c^8*d*e^2*f^3 + (c^3*d^6*e^4*f - 3*c^4*d^5*e^
3*f^2 + 3*c^5*d^4*e^2*f^3 - c^6*d^3*e*f^4)*x^6 + (c^3*d^6*e^5 - c^4*d^5*e^4
*f - 3*c^5*d^4*e^3*f^2 + 5*c^6*d^3*e^2*f^3 - 2*c^7*d^2*e*f^4)*x^4 + (2*c^4*
d^5*e^5 - 5*c^5*d^4*e^4*f + 3*c^6*d^3*e^3*f^2 + c^7*d^2*e^2*f^3 - c^8*d*e*f
^4)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{bx^2+a}{(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)/((d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{b\,x^2+a}{(d\,x^2+c)^{5/2}\,(f\,x^2+e)^{3/2}}\,dx$$

```
[In] int((a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x)
[Out] int((a + b*x^2)/((c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x)
```

3.48 $\int \frac{e+fx^2}{\sqrt{a+bx^2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 209

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = -\frac{(de-cf)\sqrt{a+bx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{\sqrt{c}(be-af)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

```
[Out] -(-c*f+d*e)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(
1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)/(-a*d+b*c)/c^(1/2
)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)+(-a*f+b*e)*(1/(1+
d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticF(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(
1/2),(1-b*c/a/d)^(1/2))*c^(1/2)*(b*x^2+a)^(1/2)/a/(-a*d+b*c)/d^(1/2)/(c*(b*
x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {539, 429, 422}

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{c}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{a+bx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[In] Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(((d*e - c*f)*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^
2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(b*e - a*f)*Sqrt[a + b*x^2]*EllipticF[Arc
Tan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*(b*c - a*d)*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(be-af)\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}}\,dx}{bc-ad} - \frac{(de-cf)\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}}\,dx}{bc-ad} \\
&= -\frac{(de-cf)\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad + \frac{\sqrt{c}(be-af)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a\sqrt{d}(bc-ad)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.63 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.01

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}d(de-cf)x(a+bx^2)-ibc(-de+cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsinh}\Big(\sqrt{}}{\sqrt{\frac{b}{a}}cd(-bc+ad)}$$

```
[In] Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*d*(d*e - c*f)*x*(a + b*x^2) - I*b*c*(-(d*e) + c*f)*Sqrt[1 + (b*x
^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] -
 I*c*(-(b*c) + a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(Sqrt[b/a]*c*d*(-(b*c) + a*d)*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 3.91 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.67

method	result
default	$\frac{\left(-\sqrt{-\frac{b}{a}}\,bcdf\,x^3+\sqrt{-\frac{b}{a}}\,b\,d^2e\,x^3+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)acdf-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)b\,c^2f+\sqrt{\frac{b\,x^2+a}{a}}\right.}{\sqrt{-\frac{b}{a}}\,cd(ad-bc)(bd\,x^4}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(d\,x^2+c)}\left(-\frac{\left(bd\,x^2+ad\right)x(cf-de)}{dc(ad-bc)\sqrt{\left(x^2+\frac{c}{d}\right)\left(bd\,x^2+ad\right)}}+\frac{\left(\frac{f}{d}-\frac{cf-de}{dc}+\frac{a(cf-de)}{c(ad-bc)}\right)\sqrt{1+\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad+bc}{cb}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bd\,x^4+ad\,x^2+cb\,x^2+ac}}-\frac{(cf-de)}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}$

```
[In] int((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-b/a)^(1/2)*b*c*d*f*x^3+(-b/a)^(1/2)*b*d^2*e*x^3+((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d*f-((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b
*c^2*f+((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*
d/b/c)^(1/2))*b*c^2*f-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(
-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c*d*e-(-b/a)^(1/2)*a*c*d*f*x+(-b/a)^(1/2)*a*
d^2*e*x)*(d*x^2+c)^(1/2)*(b*x^2+a)^(1/2)/(-b/a)^(1/2)/c/d/(a*d-b*c)/(b*d*x^
4+a*d*x^2+b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.24

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx =$$

$$-\frac{(abd^2e-abcdf)\sqrt{bx^2+a}\sqrt{dx^2+c}x-(b^2cde-b^2c^2f+(b^2d^2e-b^2cdf)x^2)\sqrt{ac}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{a}{b}}{ab^2c^3d-a^2bc^2d^2+}$$

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] -((a*b*d^2*e - a*b*c*d*f)*sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*x - (b^2*c*d*e -
b^2*c^2*f + (b^2*d^2*e - b^2*c*d*f)*x^2)*sqrt(a*c)*sqrt(-b/a)*elliptic_e(ar
csin(x*sqrt(-b/a)), a*d/(b*c)) + ((a*b + b^2)*c*d*e + ((a*b + b^2)*d^2*e -
(b^2*c*d + a^2*d^2)*f)*x^2 - (b^2*c^2 + a^2*c*d)*f)*sqrt(a*c)*sqrt(-b/a)*el
liptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)))/(a*b^2*c^3*d - a^2*b*c^2*d^2 + (
a*b^2*c^2*d^2 - a^2*b*c*d^3)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x**2+e)/(d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c+dx^2)^{3/2}}\,dx=\int \frac{f\,x^2+e}{\sqrt{b\,x^2+a}\,(d\,x^2+c)^{3/2}}\,dx$$

```
[In] int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
[Out] int((e + f*x^2)/((a + b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

3.49 $\int \frac{e+fx^2}{\sqrt{a-bx^2}(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 247

$$\begin{aligned}\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx &= \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}}\\ &+\frac{\sqrt{a}\sqrt{b}(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{cd(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\\ &+\frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}\end{aligned}$$

```
[Out] (-c*f+d*e)*x*(-b*x^2+a)^(1/2)/c/(a*d+b*c)/(d*x^2+c)^(1/2)+(-c*f+d*e)*Ellipt
icE(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*b^(1/2)*(1-b*x^2/a)^(1/2)*(
d*x^2+c)^(1/2)/c/d/(a*d+b*c)/(-b*x^2+a)^(1/2)/(1+d*x^2/c)^(1/2)+f*EllipticF
(x*b^(1/2)/a^(1/2),(-a*d/b/c)^(1/2))*a^(1/2)*(1-b*x^2/a)^(1/2)*(1+d*x^2/c)^
(1/2)/d/b^(1/2)/(-b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used

= {541, 538, 438, 437, 435, 432, 430}

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{b}\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}(de-cf)E\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{cd\sqrt{a-bx^2}\sqrt{\frac{dx^2}{c}+1}(ad+bc)} + \frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{\frac{dx^2}{c}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}} + \frac{x\sqrt{a-bx^2}(de-cf)}{c\sqrt{c+dx^2}(ad+bc)}$$

```
[In] Int[(e + f*x^2)/(Sqrt[a - b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] ((d*e - c*f)*x*Sqrt[a - b*x^2])/(c*(b*c + a*d)*Sqrt[c + d*x^2]) + (Sqrt[a]*
Sqrt[b]*(d*e - c*f)*Sqrt[1 - (b*x^2)/a]*Sqrt[c + d*x^2]*EllipticE[ArcSin[(S
qrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(c*d*(b*c + a*d)*Sqrt[a - b*x^2]*Sqrt[
1 + (d*x^2)/c]) + (Sqrt[a]*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipt
icF[ArcSin[(Sqrt[b]*x)/Sqrt[a]], -((a*d)/(b*c))])/(Sqrt[b]*d*Sqrt[a - b*x^2
]*Sqrt[c + d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} - \frac{\int \frac{-c(be+af)-b(de-cf)x^2}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx}{c(bc+ad)} \\
&= \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} + \frac{f\int \frac{1}{\sqrt{a-bx^2}\sqrt{c+dx^2}}\,dx}{d} + \frac{(b(de-cf))\int \frac{\sqrt{c+dx^2}}{\sqrt{a-bx^2}}\,dx}{cd(bc+ad)} \\
&= \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} + \frac{\left(b(de-cf)\sqrt{1-\frac{bx^2}{a}}\right)\int \frac{\sqrt{c+dx^2}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{cd(bc+ad)\sqrt{a-bx^2}} \\
&\quad + \frac{\left(f\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{c+dx^2}} \\
&= \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} + \frac{\left(b(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}\right)\int \frac{\sqrt{1+\frac{dx^2}{c}}}{\sqrt{1-\frac{bx^2}{a}}}\,dx}{cd(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} \\
&\quad + \frac{\left(f\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}}\,dx}{d\sqrt{a-bx^2}\sqrt{c+dx^2}}
\end{aligned}$$

$$
= \frac{(de-cf)x\sqrt{a-bx^2}}{c(bc+ad)\sqrt{c+dx^2}} + \frac{\sqrt{a}\sqrt{b}(de-cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{c+dx^2}E\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{cd(bc+ad)\sqrt{a-bx^2}\sqrt{1+\frac{dx^2}{c}}} + \frac{\sqrt{a}f\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|-\frac{ad}{bc}\right)}{\sqrt{b}d\sqrt{a-bx^2}\sqrt{c+dx^2}}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.87 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.89

$$
\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{-\frac{b}{a}}d(de-cf)x(a-bx^2)+ibc(-de+cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\right.\right.}{\sqrt{-\frac{b}{a}}cd(bc+}
$$

```
[In] Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[-(b/a)]*d*(d*e - c*f)*x*(a - b*x^2) + I*b*c*(-(d*e) + c*f)*Sqrt[1 - (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/
(b*c))] - I*c*(b*c + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Ellipti
cF[I*ArcSinh[Sqrt[-(b/a)]*x], -((a*d)/(b*c))])/(Sqrt[-(b/a)]*c*d*(b*c + a*d
)*Sqrt[a - b*x^2]*Sqrt[c + d*x^2])
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.41

method	result
default	$\frac{\left(\sqrt{\frac{b}{a}}\,bcdf\,x^3-\sqrt{\frac{b}{a}}\,b\,d^2e\,x^3+\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)acdf+\sqrt{\frac{-b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-\frac{ad}{bc}}\right)b\,c^2f-\sqrt{\frac{-b\,x^2+a}{a}}\right.}{\sqrt{\frac{b}{a}}\,cd(ad+bc)(-bd\,x^4+}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)(d\,x^2+c)}\left(-\frac{\left(-bd\,x^2+ad\right)x(cf-de)}{dc(ad+bc)\sqrt{\left(x^2+\frac{c}{d}\right)\left(-bd\,x^2+ad\right)}}+\frac{\left(\frac{f}{d}-\frac{cf-de}{dc}+\frac{a(cf-de)}{c(ad+bc)}\right)\sqrt{1-\frac{b\,x^2}{a}}\,\sqrt{1+\frac{d\,x^2}{c}}\,F\left(x\sqrt{\frac{b}{a}},\sqrt{-1-\frac{ad-bc}{cb}}\right)}{\sqrt{\frac{b}{a}}\,\sqrt{-bd\,x^4+ad\,x^2-cb\,x^2+ac}}+\frac{b(cf-}{}\right.}{\sqrt{-b\,x^2+a}\,\sqrt{d\,x^2+c}}$

```
[In] int((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((b/a)^(1/2)*b*c*d*f*x^3-(b/a)^(1/2)*b*d^2*e*x^3+((-b*x^2+a)/a)^(1/2)*((d*x
^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*a*c*d*f+((-b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*
c^2*f-((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(b/a)^(1/2),(-a*
d/b/c)^(1/2))*b*c^2*f+((-b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*
(b/a)^(1/2),(-a*d/b/c)^(1/2))*b*c*d*e-(b/a)^(1/2)*a*c*d*f*x+(b/a)^(1/2)*a*d
```

```
^2*e*x)*(-b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(b/a)^(1/2)/c/d/(a*d+b*c)/(-b*d*x^
4+a*d*x^2-b*c*x^2+a*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.04

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \frac{(abd^2e-abcdf)\sqrt{-bx^2+a}\sqrt{dx^2+c}x+(b^2cde-b^2c^2f+(b^2d^2e-b^2cdf)x^2)\sqrt{}}{}$$

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] ((a*b*d^2*e - a*b*c*d*f)*sqrt(-b*x^2 + a)*sqrt(d*x^2 + c)*x + (b^2*c*d*e -
b^2*c^2*f + (b^2*d^2*e - b^2*c*d*f)*x^2)*sqrt(a*c)*sqrt(b/a)*elliptic_e(arc
sin(x*sqrt(b/a)), -a*d/(b*c)) + ((a*b - b^2)*c*d*e + ((a*b - b^2)*d^2*e + (
b^2*c*d + a^2*d^2)*f)*x^2 + (b^2*c^2 + a^2*c*d)*f)*sqrt(a*c)*sqrt(b/a)*elli
ptic_f(arcsin(x*sqrt(b/a)), -a*d/(b*c)))/(a*b^2*c^3*d + a^2*b*c^2*d^2 + (a*
b^2*c^2*d^2 + a^2*b*c*d^3)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x**2+e)/(d*x**2+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral((e + f*x**2)/(sqrt(a - b*x**2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)/(d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c+dx^2)^{3/2}}\,dx = \int \frac{f\,x^2+e}{\sqrt{a-b\,x^2}\,(d\,x^2+c)^{3/2}}\,dx$$

```
[In] int((e + f*x^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(3/2)),x)
[Out] int((e + f*x^2)/((a - b*x^2)^(1/2)*(c + d*x^2)^(3/2)), x)
```

3.50 $\int \frac{e+fx^2}{\sqrt{a+bx^2}(c-dx^2)^{3/2}} dx$

Optimal result

Integrand size = 31, antiderivative size = 237

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}(c-dx^2)^{3/2}} dx = \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}} - \frac{(de+cf)\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{e\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}$$

```
[Out] (c*f+d*e)*x*(b*x^2+a)^(1/2)/c/(a*d+b*c)/(-d*x^2+c)^(1/2)-(c*f+d*e)*Elliptic
E(x*d^(1/2)/c^(1/2),(-b*c/a/d)^(1/2))*(b*x^2+a)^(1/2)*(1-d*x^2/c)^(1/2)/(a*
d+b*c)/c^(1/2)/d^(1/2)/(1+b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)+e*EllipticF(x*d^(
1/2)/c^(1/2),(-b*c/a/d)^(1/2))*(1+b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)/c^(1/2)/
d^(1/2)/(b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used

= {541, 538, 438, 437, 435, 432, 430}

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = -\frac{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{bx^2}{a}+1}\sqrt{c-dx^2}(ad+bc)} + \frac{e\sqrt{\frac{bx^2}{a}+1}\sqrt{1-\frac{dx^2}{c}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}} + \frac{x\sqrt{a+bx^2}(cf+de)}{c\sqrt{c-dx^2}(ad+bc)}$$

```
[In] Int[(e + f*x^2)/(Sqrt[a + b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] ((d*e + c*f)*x*Sqrt[a + b*x^2])/(c*(b*c + a*d)*Sqrt[c - d*x^2]) - ((d*e + c
*f)*Sqrt[a + b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c
]], -((b*c)/(a*d))])/(Sqrt[c]*Sqrt[d]*(b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[
c - d*x^2]) + (e*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
Sqrt[d]*x)/Sqrt[c]], -((b*c)/(a*d))])/(Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2]*Sqrt
[c - d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}}+\frac{\int \frac{c(be-af)-b(de+cf)x^2}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx}{c(bc+ad)}\\
&= \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}}+\frac{e\int \frac{1}{\sqrt{a+bx^2}\sqrt{c-dx^2}}\,dx}{c}-\frac{(de+cf)\int \frac{\sqrt{a+bx^2}}{\sqrt{c-dx^2}}\,dx}{c(bc+ad)}\\
&= \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}}+\frac{\left(e\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{c\sqrt{c-dx^2}}\\
&\quad-\frac{\left((de+cf)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{a+bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{c(bc+ad)\sqrt{c-dx^2}}\\
&= \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}}-\frac{\left((de+cf)\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{c(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}}\\
&\quad+\frac{\left(e\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{c\sqrt{a+bx^2}\sqrt{c-dx^2}}
\end{aligned}$$

$$
= \frac{(de+cf)x\sqrt{a+bx^2}}{c(bc+ad)\sqrt{c-dx^2}} - \frac{(de+cf)\sqrt{a+bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{e\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}\sqrt{c-dx^2}}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.75 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.90

$$
\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{b}{a}}d(de+cf)x(a+bx^2) - ibc(de+cf)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}\right.\right.}{\sqrt{\frac{b}{a}}cd(bc+ad)\sqrt{}}
$$

```
[In] Integrate[(e + f*x^2)/(Sqrt[a + b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[b/a]*d*(d*e + c*f)*x*(a + b*x^2) - I*b*c*(d*e + c*f)*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], -((a*d)/(b*c))] +
 I*c*(b*c + a*d)*f*Sqrt[1 + (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[I*ArcS
inh[Sqrt[b/a]*x], -((a*d)/(b*c))])/(Sqrt[b/a]*c*d*(b*c + a*d)*Sqrt[a + b*x^
2]*Sqrt[c - d*x^2])
```

Maple [A] (verified)

Time = 4.51 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.41

method	result
default	$\frac{\left(\sqrt{\frac{d}{c}}\,bcf\,x^3+\sqrt{\frac{d}{c}}\,bde\,x^3+\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)ade+\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{\frac{b\,x^2+a}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{bc}{ad}}\right)bce-\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{b\,x}\right.}{\sqrt{\frac{d}{c}}\,c(ad+bc)(-bd\,x^4-ad\,x^2}$
elliptic	$\frac{\sqrt{(b\,x^2+a)(-d\,x^2+c)}\left(-\frac{\left(-bd\,x^2-ad\right)x(cf+de)}{dc(ad+bc)\sqrt{\left(x^2-\frac{c}{d}\right)\left(-bd\,x^2-ad\right)}}+\frac{\left(-\frac{f}{d}+\frac{cf+de}{dc}-\frac{a(cf+de)}{c(ad+bc)}\right)\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{b\,x^2}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad+bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{-bd\,x^4-ad\,x^2+cb\,x^2+ac}}+\frac{(c}{}\right.}{\sqrt{b\,x^2+a}\,\sqrt{-d\,x^2+c}}$

```
[In] int((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d/c)^(1/2)*b*c*f*x^3+(d/c)^(1/2)*b*d*e*x^3+((-d*x^2+c)/c)^(1/2)*((b*x^2+a
)/a)^(1/2)*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*a*d*e+((-d*x^2+c)/c)^(
1/2)*((b*x^2+a)/a)^(1/2)*EllipticF(x*(d/c)^(1/2),(-b*c/a/d)^(1/2))*b*c*e-((
-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(-b*c/a/d)^(
1/2))*a*c*f-((-d*x^2+c)/c)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2
),(-b*c/a/d)^(1/2))*a*d*e+(d/c)^(1/2)*a*c*f*x+(d/c)^(1/2)*a*d*e*x)*(b*x^2+a
```

```
)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/c/(a*d+b*c)/(-b*d*x^4-a*d*x^2+b*c*x^2+
a*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.07

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \frac{(acd^2e+ac^2df)\sqrt{bx^2+a}\sqrt{-dx^2+c}x-(acd^2e+ac^2df-(ad^3e+acd^2f)x^2)\sqrt{}}{}$$

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] ((a*c*d^2*e + a*c^2*d*f)*sqrt(b*x^2 + a)*sqrt(-d*x^2 + c)*x - (a*c*d^2*e +
a*c^2*d*f - (a*d^3*e + a*c*d^2*f)*x^2)*sqrt(a*c)*sqrt(d/c)*elliptic_e(arcsi
n(x*sqrt(d/c)), -b*c/(a*d)) - (((b*c^2*d + a*d^3)*e - (a*c^2*d - a*c*d^2)*f
)*x^2 - (b*c^3 + a*c*d^2)*e + (a*c^3 - a*c^2*d)*f)*sqrt(a*c)*sqrt(d/c)*elli
ptic_f(arcsin(x*sqrt(d/c)), -b*c/(a*d)))/(a*b*c^4*d + a^2*c^3*d^2 - (a*b*c^
3*d^2 + a^2*c^2*d^3)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x**2+e)/(-d*x**2+c)**(3/2)/(b*x**2+a)**(1/2),x)
```

```
[Out] Integral((e + f*x**2)/(sqrt(a + b*x**2)*(c - d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx = \int \frac{fx^2+e}{\sqrt{bx^2+a}(-dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx=\int \frac{fx^2+e}{\sqrt{bx^2+a}(-dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^2 + e)/(sqrt(b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a+bx^2}\,(c-dx^2)^{3/2}}\,dx=\int \frac{f\,x^2+e}{\sqrt{b\,x^2+a}\,{\left(c-d\,x^2\right)}^{3/2}}\,dx$$

```
[In] int((e + f*x^2)/((a + b*x^2)^(1/2)*(c - d*x^2)^(3/2)),x)
[Out] int((e + f*x^2)/((a + b*x^2)^(1/2)*(c - d*x^2)^(3/2)), x)
```

3.51 $\int \frac{e+fx^2}{\sqrt{a-bx^2}(c-dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 242

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx = -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}} + \frac{(de+cf)\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}} + \frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

```
[Out] -(c*f+d*e)*x*(-b*x^2+a)^(1/2)/c/(-a*d+b*c)/(-d*x^2+c)^(1/2)+(c*f+d*e)*Ellip
ticE(x*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*(-b*x^2+a)^(1/2)*(1-d*x^2/c)^(1/2)/
(-a*d+b*c)/c^(1/2)/d^(1/2)/(1-b*x^2/a)^(1/2)/(-d*x^2+c)^(1/2)+e*EllipticF(x
*d^(1/2)/c^(1/2),(b*c/a/d)^(1/2))*(1-b*x^2/a)^(1/2)*(1-d*x^2/c)^(1/2)/c^(1/
2)/d^(1/2)/(-b*x^2+a)^(1/2)/(-d*x^2+c)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used

= {541, 538, 438, 437, 435, 432, 430}

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx = \frac{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}(cf+de)E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}(bc-ad)} + \frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}} - \frac{x\sqrt{a-bx^2}(cf+de)}{c\sqrt{c-dx^2}(bc-ad)}$$

```
[In] Int[(e + f*x^2)/(Sqrt[a - b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] -(((d*e + c*f)*x*Sqrt[a - b*x^2])/(c*(b*c - a*d)*Sqrt[c - d*x^2])) + ((d*e
+ c*f)*Sqrt[a - b*x^2]*Sqrt[1 - (d*x^2)/c]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqr
t[c]], (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[1 - (b*x^2)/a]*Sqrt[
c - d*x^2]) + (e*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[ArcSin[(
Sqrt[d]*x)/Sqrt[c]], (b*c)/(a*d)])/(Sqrt[c]*Sqrt[d]*Sqrt[a - b*x^2]*Sqrt[c
- d*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}}-\frac{\int \frac{-c(be+af)+b(de+cf)x^2}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx}{c(bc-ad)} \\
&= -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}}+\frac{e\int \frac{1}{\sqrt{a-bx^2}\sqrt{c-dx^2}}\,dx}{c}+\frac{(de+cf)\int \frac{\sqrt{a-bx^2}}{\sqrt{c-dx^2}}\,dx}{c(bc-ad)} \\
&= -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}}+\frac{\left(e\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}}\,dx}{c\sqrt{c-dx^2}} \\
&\quad+\frac{\left((de+cf)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{a-bx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{c(bc-ad)\sqrt{c-dx^2}} \\
&= -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}}+\frac{\left((de+cf)\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{1-\frac{bx^2}{a}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{c(bc-ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}} \\
&\quad+\frac{\left(e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}}\,dx}{c\sqrt{a-bx^2}\sqrt{c-dx^2}}
\end{aligned}
$$

$$= -\frac{(de+cf)x\sqrt{a-bx^2}}{c(bc-ad)\sqrt{c-dx^2}} + \frac{(de+cf)\sqrt{a-bx^2}\sqrt{1-\frac{dx^2}{c}}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{1-\frac{bx^2}{a}}\sqrt{c-dx^2}}$$
$$+ \frac{e\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{a-bx^2}\sqrt{c-dx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.74 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.91

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx = \frac{\sqrt{-\frac{b}{a}}d(de+cf)x(a-bx^2) + ibc(de+cf)\sqrt{1-\frac{bx^2}{a}}\sqrt{1-\frac{dx^2}{c}}E\Big(i\text{arcsinh}\Big(\sqrt{}}{\sqrt{-\frac{b}{a}}cd(-bc+}$$

```
[In] Integrate[(e + f*x^2)/(Sqrt[a - b*x^2]*(c - d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[-(b/a)]*d*(d*e + c*f)*x*(a - b*x^2) + I*b*c*(d*e + c*f)*Sqrt[1 - (b*x
^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[-(b/a)]*x], (a*d)/(b*c)
] + I*c*(-(b*c) + a*d)*f*Sqrt[1 - (b*x^2)/a]*Sqrt[1 - (d*x^2)/c]*EllipticF[
I*ArcSinh[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*c*d*(-(b*c) + a*d)*S
qrt[a - b*x^2]*Sqrt[c - d*x^2])
```

Maple [A] (verified)

Time = 4.95 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.40

method	result
default	$\frac{\left(-\sqrt{\frac{d}{c}}\,bcf\,x^3-\sqrt{\frac{d}{c}}\,bde\,x^3+\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{\frac{-b\,x^2+a}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)ade-\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{\frac{-b\,x^2+a}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{\frac{bc}{ad}}\right)bce-\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{-}\right.}{\sqrt{\frac{d}{c}}\,c(ad-bc)(bd\,x^4-ad\,x}$
elliptic	$\frac{\sqrt{(-b\,x^2+a)(-d\,x^2+c)}\left(-\frac{\left(bd\,x^2-ad\right)x(cf+de)}{dc(ad-bc)\sqrt{\left(x^2-\frac{c}{d}\right)\left(bd\,x^2-ad\right)}}+\frac{\left(-\frac{f}{d}+\frac{cf+de}{dc}-\frac{a(cf+de)}{c(ad-bc)}\right)\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1-\frac{b\,x^2}{a}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{-ad-bc}{ad}}\right)}{\sqrt{\frac{d}{c}}\,\sqrt{bd\,x^4-ad\,x^2-cb\,x^2+ac}}+\frac{(c}{}\right.}{\sqrt{-b\,x^2+a}\,\sqrt{-d\,x^2+c}}$

```
[In] int((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(d/c)^(1/2)*b*c*f*x^3-(d/c)^(1/2)*b*d*e*x^3+((-d*x^2+c)/c)^(1/2)*((-b*x^2
+a)/a)^(1/2)*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*a*d*e-((-d*x^2+c)/c)^
(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticF(x*(d/c)^(1/2),(b*c/a/d)^(1/2))*b*c*e-(
(-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1/2),(b*c/a/d)^
(1/2))*a*c*f-((-d*x^2+c)/c)^(1/2)*((-b*x^2+a)/a)^(1/2)*EllipticE(x*(d/c)^(1
/2),(b*c/a/d)^(1/2))*a*d*e+(d/c)^(1/2)*a*c*f*x+(d/c)^(1/2)*a*d*e*x)*(-b*x^2
```

```
+a)^(1/2)*(-d*x^2+c)^(1/2)/(d/c)^(1/2)/c/(a*d-b*c)/(b*d*x^4-a*d*x^2-b*c*x^2
+a*c)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.06

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx =$$
$$-\frac{(acd^2e+ac^2df)\sqrt{-bx^2+a}\sqrt{-dx^2+c}x-(acd^2e+ac^2df-(ad^3e+acd^2f)x^2)\sqrt{ac}\sqrt{\frac{d}{c}}E(\arcsin\left(x\sqrt{\frac{d}{c}}\right) \mid \frac{b}{a}}{abc^4d-a^2c^3d^2-(a}$$

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -((a*c*d^2*e + a*c^2*d*f)*sqrt(-b*x^2 + a)*sqrt(-d*x^2 + c)*x - (a*c*d^2*e
+ a*c^2*d*f - (a*d^3*e + a*c*d^2*f)*x^2)*sqrt(a*c)*sqrt(d/c)*elliptic_e(arc
sin(x*sqrt(d/c)), b*c/(a*d)) + (((b*c^2*d - a*d^3)*e + (a*c^2*d - a*c*d^2)*
f)*x^2 - (b*c^3 - a*c*d^2)*e - (a*c^3 - a*c^2*d)*f)*sqrt(a*c)*sqrt(d/c)*ell
iptic_f(arcsin(x*sqrt(d/c)), b*c/(a*d)))/(a*b*c^4*d - a^2*c^3*d^2 - (a*b*c^
3*d^2 - a^2*c^2*d^3)*x^2)
```

Sympy [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx = \int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x**2+e)/(-d*x**2+c)**(3/2)/(-b*x**2+a)**(1/2),x)
```

```
[Out] Integral((e + f*x**2)/(sqrt(a - b*x**2)*(c - d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}(-dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx = \int \frac{fx^2+e}{\sqrt{-bx^2+a}(-dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)/(-d*x^2+c)^(3/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^2 + e)/(sqrt(-b*x^2 + a)*(-d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e+fx^2}{\sqrt{a-bx^2}\,(c-dx^2)^{3/2}}\,dx = \int \frac{f\,x^2+e}{\sqrt{a-b\,x^2}\,(c-d\,x^2)^{3/2}}\,dx$$

```
[In] int((e + f*x^2)/((a - b*x^2)^(1/2)*(c - d*x^2)^(3/2)),x)
[Out] int((e + f*x^2)/((a - b*x^2)^(1/2)*(c - d*x^2)^(3/2)), x)
```

3.52 $\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 191

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \frac{bx\sqrt{2+dx^2}}{d\sqrt{3+fx^2}} - \frac{\sqrt{2}b\sqrt{2+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} + \frac{a\sqrt{2+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}$$

```
[Out] b*x*(d*x^2+2)^(1/2)/d/(f*x^2+3)^(1/2)+1/2*a*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2+
9)^(1/2)*EllipticF(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1/2*(4-6*d/f)^(1/2))
*(d*x^2+2)^(1/2)*2^(1/2)/f^(1/2)/((d*x^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)^(1/2
)-b*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticE(x*f^(1/2)*3^(1/2)/(3*
f*x^2+9)^(1/2),1/2*(4-6*d/f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2)/d/f^(1/2)/((d*x
^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {545, 429, 506, 422}

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \frac{a\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}} - \frac{\sqrt{2}b\sqrt{dx^2+2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}} + \frac{bx\sqrt{dx^2+2}}{d\sqrt{fx^2+3}}$$

```
[In] Int[(a + b*x^2)/(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

```
[Out] (b*x*Sqrt[2 + d*x^2])/(d*Sqrt[3 + f*x^2]) - (Sqrt[2]*b*Sqrt[2 + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(d*Sqrt[f]*Sqrt[(2 + d
*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (a*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*Sqrt[f]*Sqrt[(2 + d*x^2)/(3
 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= a\int \frac{1}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx + b\int \frac{x^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx \\
&= \frac{bx\sqrt{2+dx^2}}{d\sqrt{3+fx^2}} + \frac{a\sqrt{2+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} - \frac{(3b)\int \frac{\sqrt{2+dx^2}}{(3+fx^2)^{3/2}}\,dx}{d} \\
&= \frac{bx\sqrt{2+dx^2}}{d\sqrt{3+fx^2}} - \frac{\sqrt{2}b\sqrt{2+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{d\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} + \frac{a\sqrt{2+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.42

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = -\frac{i\left(3bE\left(i\operatorname{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+(-3b+af)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)\right)}{\sqrt{3}\sqrt{d}f}$$

```
[In] Integrate[(a + b*x^2)/(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

```
[Out] ((-I)*(3*b*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (-3*b +
 a*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]))/(Sqrt[3]*Sqr
t[d]*f)
```

Maple [A] (verified)

Time = 3.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.55

method	result
default	$\frac{\left(F\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{d}{f}}}{2}\right)ad-2F\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{d}{f}}}{2}\right)b+2E\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3},\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\frac{d}{f}}}{2}\right)b\right)\sqrt{2}}{2d\sqrt{-f}}$
elliptic	$\frac{\sqrt{(f\,x^2+3)(d\,x^2+2)}\left(\frac{a\sqrt{3f\,x^2+9}\,\sqrt{2d\,x^2+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)}{2\sqrt{-3f}\,\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}}-\frac{b\sqrt{3f\,x^2+9}\,\sqrt{2d\,x^2+4}\left(F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)-E\left(\frac{x\sqrt{-3f}}{3},\right.\right.}{\sqrt{-3f}\,\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}\,d}\right.}{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}}$

```
[In] int((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*a*
d-2*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*b+2
*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*b)*2^(
1/2)/d/(-f)^(1/2)
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.65

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx =$$
$$-\frac{9\sqrt{3}\sqrt{df}bx\sqrt{-\frac{1}{f}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{f}}}{x}\right) \mid \frac{2f}{3d}) - \sqrt{3}(af^2+9b)\sqrt{df}x\sqrt{-\frac{1}{f}}F(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{f}}}{x}\right) \mid \frac{2f}{3d}) - 3}{3df^2x}$$

```
[In] integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(9*sqrt(3)*sqrt(d*f)*b*x*sqrt(-1/f)*elliptic_e(arcsin(sqrt(3)*sqrt(-1/
f)/x), 2/3*f/d) - sqrt(3)*(a*f^2 + 9*b)*sqrt(d*f)*x*sqrt(-1/f)*elliptic_f(a
rcsin(sqrt(3)*sqrt(-1/f)/x), 2/3*f/d) - 3*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)*b
*f)/(d*f^2*x)
```

Sympy [F]

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{a+bx^2}{\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

```
[In] integrate((b*x**2+a)/(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)/(sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{bx^2+a}{\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx=\int \frac{bx^2+a}{\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx=\int \frac{b\,x^2+a}{\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}}\,dx$$

```
[In] int((a + b*x^2)/((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)),x)
[Out] int((a + b*x^2)/((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)), x)
```

3.53 $\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 262

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = -\frac{(6bd-2bf-3adf)x\sqrt{2+dx^2}}{3df\sqrt{3+fx^2}}+\frac{bx\sqrt{2+dx^2}\sqrt{3+fx^2}}{3f}$$
$$+\frac{\sqrt{2}(6bd-2bf-3adf)\sqrt{2+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{3df^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}$$
$$-\frac{\sqrt{2}(b-af)\sqrt{2+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{f^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}$$

```
[Out] -1/3*(-3*a*d*f+6*b*d-2*b*f)*x*(d*x^2+2)^(1/2)/d/f/(f*x^2+3)^(1/2)+1/3*(-3*a
*d*f+6*b*d-2*b*f)*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticE(x*f^(1/
2)*3^(1/2)/(3*f*x^2+9)^(1/2),1/2*(4-6*d/f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2)/d
/f^(3/2)/((d*x^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)^(1/2)-(-a*f+b)*(1/(3*f*x^2+9
))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticF(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1/
2*(4-6*d/f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2)/f^(3/2)/((d*x^2+2)/(f*x^2+3))^(1
/2)/(f*x^2+3)^(1/2)+1/3*b*x*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {542, 545, 429, 506, 422}

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = -\frac{\sqrt{2}\sqrt{dx^2+2}(b-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{f^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}} + \frac{\sqrt{2}\sqrt{dx^2+2}(-3adf+6bd-2bf)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{3df^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}} - \frac{x\sqrt{dx^2+2}(-3adf+6bd-2bf)}{3df\sqrt{fx^2+3}} + \frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}$$

```
[In] Int[((a + b*x^2)*Sqrt[2 + d*x^2])/Sqrt[3 + f*x^2],x]
```

```
[Out] -1/3*((6*b*d - 2*b*f - 3*a*d*f)*x*Sqrt[2 + d*x^2])/(d*f*Sqrt[3 + f*x^2]) +
(b*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(3*f) + (Sqrt[2]*(6*b*d - 2*b*f - 3*a
*d*f)*Sqrt[2 + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f
)])/(3*d*f^(3/2)*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) - (Sqrt[2]*
(b - a*f)*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/
(2*f)])/(f^(3/2)*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{bx\sqrt{2+dx^2}\sqrt{3+fx^2}}{3f} + \frac{\int \frac{-6(b-af)+(-6bd+2bf+3adf)x^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx}{3f} \\
&= \frac{bx\sqrt{2+dx^2}\sqrt{3+fx^2}}{3f} - \frac{(2(b-af))\int \frac{1}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx}{f} \\
&\quad - \frac{(6bd-2bf-3adf)\int \frac{x^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx}{3f} \\
&= -\frac{(6bd-2bf-3adf)x\sqrt{2+dx^2}}{3df\sqrt{3+fx^2}} + \frac{bx\sqrt{2+dx^2}\sqrt{3+fx^2}}{3f} \\
&\quad - \frac{\sqrt{2}(b-af)\sqrt{2+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{f^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} \\
&\quad + \frac{(6bd-2bf-3adf)\int \frac{\sqrt{2+dx^2}}{(3+fx^2)^{3/2}}\,dx}{df} \\
&= -\frac{(6bd-2bf-3adf)x\sqrt{2+dx^2}}{3df\sqrt{3+fx^2}} + \frac{bx\sqrt{2+dx^2}\sqrt{3+fx^2}}{3f} \\
&\quad + \frac{\sqrt{2}(6bd-2bf-3adf)\sqrt{2+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{3df^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} \\
&\quad - \frac{\sqrt{2}(b-af)\sqrt{2+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{f^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.54

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx$$
$$= \frac{b\sqrt{d}fx\sqrt{2+dx^2}\sqrt{3+fx^2}+i\sqrt{3}(6bd-2bf-3adf)E\left(i\text{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+i\sqrt{3}(3d-2f)(-2b+af)\,\text{El}}{3\sqrt{d}f^2}$$

```
[In] Integrate[((a + b*x^2)*Sqrt[2 + d*x^2])/Sqrt[3 + f*x^2],x]
```

```
[Out] (b*Sqrt[d]*f*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2] + I*Sqrt[3]*(6*b*d - 2*b*f -
 3*a*d*f)*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + I*Sqrt[3
]*(3*d - 2*f)*(-2*b + a*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/
(3*d)])/(3*Sqrt[d]*f^2)
```

Maple [A] (verified)

Time = 4.93 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.08

method	result
elliptic	$\frac{\sqrt{(fx^2+3)(dx^2+2)}\left(\frac{bx\sqrt{dfx^4+3dx^2+2fx^2+6}}{3f}+\frac{\left(2a-\frac{2b}{f}\right)\sqrt{3fx^2+9}\sqrt{2dx^2+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)}{2\sqrt{-3f}\sqrt{dfx^4+3dx^2+2fx^2+6}}-\frac{\left(ad+2b-\frac{b(6d+4f)}{3f}\right)\sqrt{3fx^2+9}}{\cdots}\right.}{\sqrt{fx^2+3}\sqrt{dx^2+2}}$
risch	$\frac{bx\sqrt{dx^2+2}\sqrt{fx^2+3}}{3f}+\frac{\left(-\frac{3b\sqrt{3fx^2+9}\sqrt{2dx^2+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)}{\sqrt{-3f}\sqrt{dfx^4+3dx^2+2fx^2+6}}+\frac{3af\sqrt{3fx^2+9}\sqrt{2dx^2+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)}{\sqrt{-3f}\sqrt{dfx^4+3dx^2+2fx^2+6}}-\frac{(3a\cdots}{\cdots}\right.}{3f\sqrt{fx^2\cdots}}$
default	$\frac{\sqrt{dx^2+2}\sqrt{fx^2+3}\left(bd^2fx^5\sqrt{-f}+3\sqrt{2}\,E\left(\frac{x\sqrt{3}\sqrt{-f}}{3},\frac{\sqrt{2}\sqrt{3}\sqrt{\frac{d}{f}}}{2}\right)adf\sqrt{dx^2+2}\sqrt{fx^2+3}+3bd^2x^3\sqrt{-f}+2bdfx^3\sqrt{-f}-6\sqrt{2}\,E\left(\frac{x\sqrt{3}}{\cdots}\right.\right.}{\cdots}$

```
[In] int((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((f*x^2+3)*(d*x^2+2))^(1/2)/(f*x^2+3)^(1/2)/(d*x^2+2)^(1/2)*(1/3*b/f*x*(d*f
*x^4+3*d*x^2+2*f*x^2+6)^(1/2)+1/2*(2*a-2*b/f)/(-3*f)^(1/2)*(3*f*x^2+9)^(1/2
)*(2*d*x^2+4)^(1/2)/(d*f*x^4+3*d*x^2+2*f*x^2+6)^(1/2)*EllipticF(1/3*x*(-3*f
)^(1/2),1/2*(-4+2*(3*d+2*f)/f)^(1/2))-(a*d+2*b-1/3*b/f*(6*d+4*f))/(-3*f)^(1
/2)*(3*f*x^2+9)^(1/2)*(2*d*x^2+4)^(1/2)/(d*f*x^4+3*d*x^2+2*f*x^2+6)^(1/2)/d
```

```
*(EllipticF(1/3*x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/f)^(1/2))-EllipticE(1/3*
x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/f)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.69

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \frac{3\sqrt{3}(6\,bd-(3\,ad+2\,b)f)\sqrt{df}x\sqrt{-\frac{1}{f}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{f}}}{x}\right) \mid \frac{2\,f}{3\,d}) + \sqrt{3}(2\,af^3-2\,bf^2-18\,bd+3\,(3\,ad+2}{3\,df^3x}$$

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*(3*sqrt(3)*(6*b*d - (3*a*d + 2*b)*f)*sqrt(d*f)*x*sqrt(-1/f)*elliptic_e(
arcsin(sqrt(3)*sqrt(-1/f)/x), 2/3*f/d) + sqrt(3)*(2*a*f^3 - 2*b*f^2 - 18*b*
d + 3*(3*a*d + 2*b)*f)*sqrt(d*f)*x*sqrt(-1/f)*elliptic_f(arcsin(sqrt(3)*sqr
t(-1/f)/x), 2/3*f/d) + (b*d*f^2*x^2 - 6*b*d*f + (3*a*d + 2*b)*f^2)*sqrt(d*x
^2 + 2)*sqrt(f*x^2 + 3))/(d*f^3*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \int \frac{(a+bx^2)\sqrt{dx^2+2}}{\sqrt{fx^2+3}}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
```

```
[Out] Integral((a + b*x**2)*sqrt(d*x**2 + 2)/sqrt(f*x**2 + 3), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \int \frac{(bx^2+a)\sqrt{dx^2+2}}{\sqrt{fx^2+3}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3), x)
```

Giac [F]

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \int \frac{(bx^2+a)\sqrt{dx^2+2}}{\sqrt{fx^2+3}}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)/sqrt(f*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{2+dx^2}}{\sqrt{3+fx^2}}\,dx = \int \frac{(b\,x^2+a)\,\sqrt{d\,x^2+2}}{\sqrt{f\,x^2+3}}\,dx$$

```
[In] int(((a + b*x^2)*(d*x^2 + 2)^(1/2))/(f*x^2 + 3)^(1/2),x)
[Out] int(((a + b*x^2)*(d*x^2 + 2)^(1/2))/(f*x^2 + 3)^(1/2), x)
```

3.54 $\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 356

$$\begin{aligned}
\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx
&= \frac{(5adf(3d+2f)-2b(9d^2-6df+4f^2))\,x\sqrt{2+dx^2}}{15d^2f\sqrt{3+fx^2}} \\
&\quad + \frac{(3bd-4bf+5adf)x\sqrt{2+dx^2}\sqrt{3+fx^2}}{15df} + \frac{bx(2+dx^2)^{3/2}\sqrt{3+fx^2}}{5d} \\
&\quad - \frac{\sqrt{2}(5adf(3d+2f)-2b(9d^2-6df+4f^2))\sqrt{2+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{15d^2f^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} \\
&\quad - \frac{\sqrt{2}(3bd+2bf-10adf)\sqrt{2+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{5df^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}
\end{aligned}$$

```
[Out] 1/15*(5*a*d*f*(3*d+2*f)-2*b*(9*d^2-6*d*f+4*f^2))*x*(d*x^2+2)^(1/2)/d^2/f/(f
*x^2+3)^(1/2)-1/15*(5*a*d*f*(3*d+2*f)-2*b*(9*d^2-6*d*f+4*f^2))*(1/(3*f*x^2+
9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticE(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1
/2*(4-6*d/f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2)/d^2/f^(3/2)/((d*x^2+2)/(f*x^2+3
))^(1/2)/(f*x^2+3)^(1/2)-1/5*(-10*a*d*f+3*b*d+2*b*f)*(1/(3*f*x^2+9))^(1/2)*
(3*f*x^2+9)^(1/2)*EllipticF(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1/2*(4-6*d/
f)^(1/2))*2^(1/2)*(d*x^2+2)^(1/2)/d/f^(3/2)/((d*x^2+2)/(f*x^2+3))^(1/2)/(f*
x^2+3)^(1/2)+1/5*b*x*(d*x^2+2)^(3/2)*(f*x^2+3)^(1/2)/d+1/15*(5*a*d*f+3*b*d-
4*b*f)*x*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {542, 545, 429, 506, 422}

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx$$
$$= -\frac{\sqrt{2}\sqrt{dx^2+2}(5adf(3d+2f)-2b(9d^2-6df+4f^2))\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{15d^2f^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}$$
$$-\frac{\sqrt{2}\sqrt{dx^2+2}(-10adf+3bd+2bf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{5df^{3/2}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}}$$
$$+\frac{x\sqrt{dx^2+2}(5adf(3d+2f)-2b(9d^2-6df+4f^2))}{15d^2f\sqrt{fx^2+3}}$$
$$+\frac{x\sqrt{dx^2+2}\sqrt{fx^2+3}(5adf+3bd-4bf)}{15df}+\frac{bx(dx^2+2)^{3/2}\sqrt{fx^2+3}}{5d}$$

```
[In] Int[(a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2],x]
```

```
[Out] ((5*a*d*f*(3*d + 2*f) - 2*b*(9*d^2 - 6*d*f + 4*f^2))*x*Sqrt[2 + d*x^2])/(15
*d^2*f*Sqrt[3 + f*x^2]) + ((3*b*d - 4*b*f + 5*a*d*f)*x*Sqrt[2 + d*x^2]*Sqrt
[3 + f*x^2])/(15*d*f) + (b*x*(2 + d*x^2)^(3/2)*Sqrt[3 + f*x^2])/(5*d) - (Sq
rt[2]*(5*a*d*f*(3*d + 2*f) - 2*b*(9*d^2 - 6*d*f + 4*f^2))*Sqrt[2 + d*x^2]*E
llipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(15*d^2*f^(3/2)*Sqr
t[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) - (Sqrt[2]*(3*b*d + 2*b*f - 10*
a*d*f)*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*
f)])/(5*d*f^(3/2)*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{bx(2+dx^2)^{3/2}\sqrt{3+fx^2}}{5d} + \frac{\int \frac{\sqrt{2+dx^2}\left(-3(2b-5ad)+(3bd-4bf+5adf)x^2\right)}{\sqrt{3+fx^2}}\,dx}{5d} \\
&= \frac{(3bd-4bf+5adf)x\sqrt{2+dx^2}\sqrt{3+fx^2}}{15df} + \frac{bx(2+dx^2)^{3/2}\sqrt{3+fx^2}}{5d} \\
&\quad + \frac{\int \frac{-6(3bd+2bf-10adf)+\left(5adf(3d+2f)-2b\left(9d^2-6df+4f^2\right)\right)x^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx}{15df} \\
&= \frac{(3bd-4bf+5adf)x\sqrt{2+dx^2}\sqrt{3+fx^2}}{15df} + \frac{bx(2+dx^2)^{3/2}\sqrt{3+fx^2}}{5d} \\
&\quad - \frac{(2(3bd+2bf-10adf))\int \frac{1}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx}{5df} \\
&\quad + \frac{(5adf(3d+2f)-2b(9d^2-6df+4f^2))\int \frac{x^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx}{15df}
\end{aligned}$$

$$= \frac{(5adf(3d+2f)-2b(9d^2-6df+4f^2))\,x\sqrt{2+dx^2}}{15d^2f\sqrt{3+fx^2}} + \frac{(3bd-4bf+5adf)x\sqrt{2+dx^2}\sqrt{3+fx^2}}{15df} + \frac{bx(2+dx^2)^{3/2}\sqrt{3+fx^2}}{5d} - \frac{\sqrt{2}(3bd+2bf-10adf)\sqrt{2+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{5df^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} - \frac{(5adf(3d+2f)-2b(9d^2-6df+4f^2))\int\frac{\sqrt{2+dx^2}}{(3+fx^2)^{3/2}}dx}{5d^2f}$$

$$= \frac{(5adf(3d+2f)-2b(9d^2-6df+4f^2))\,x\sqrt{2+dx^2}}{15d^2f\sqrt{3+fx^2}} + \frac{(3bd-4bf+5adf)x\sqrt{2+dx^2}\sqrt{3+fx^2}}{15df} + \frac{bx(2+dx^2)^{3/2}\sqrt{3+fx^2}}{5d} - \frac{\sqrt{2}(5adf(3d+2f)-2b(9d^2-6df+4f^2))\sqrt{2+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{15d^2f^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} - \frac{\sqrt{2}(3bd+2bf-10adf)\sqrt{2+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{5df^{3/2}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.22 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.52

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx$$
$$= \frac{\sqrt{d}fx\sqrt{2+dx^2}\sqrt{3+fx^2}(2bf+5adf+3bd(1+fx^2))+i\sqrt{3}(-5adf(3d+2f)+2b(9d^2-6df+4f^2))\,E\Big(}{15d^{3/2}f^2}$$

```
[In] Integrate[(a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2],x]
```

```
[Out] (Sqrt[d]*f*x*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]*(2*b*f + 5*a*d*f + 3*b*d*(1 +
f*x^2)) + I*Sqrt[3]*(-5*a*d*f*(3*d + 2*f) + 2*b*(9*d^2 - 6*d*f + 4*f^2))*El
lipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + I*Sqrt[3]*(3*d - 2*f
)*(-6*b*d + 2*b*f + 5*a*d*f)*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f
)/(3*d)])/(15*d^(3/2)*f^2)
```

Maple [A] (verified)

Time = 4.85 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.10

method	result
elliptic	$\frac{\sqrt{(f\,x^2+3)(d\,x^2+2)}\left(\frac{b\,x^3\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}}{5}+\frac{\left(adf+3bd+2bf-\frac{b(12d+8f)}{5}\right)x\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}}{3df}+\frac{\left(6a-\frac{2\left(adf+3bd+2bf-\frac{b(12d+}{5}\right.}{df}\right.}{2\sqrt{}}\right.}{}$
risch	$\frac{x(3bdf\,x^2+5adf+3bd+2bf)\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}}{15df}+\frac{\left(-\frac{9bd\sqrt{3f\,x^2+9}\,\sqrt{2d\,x^2+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)}{\sqrt{-3f}\,\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}}-\frac{6bf\sqrt{3f\,x^2+9}\,\sqrt{2d\,x^2+4}\,F\left(\right.}{\sqrt{-3f}\,\sqrt{df\,x^4+3}}\right.}{}$
default	$\frac{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}\left(3b\,d^3f^2x^7\sqrt{-f}+5a\,d^3f^2x^5\sqrt{-f}+12b\,d^3f\,x^5\sqrt{-f}+8b\,d^2f^2x^5\sqrt{-f}+15a\,d^3f\,x^3\sqrt{-f}+10a\,d^2f^2x^3\sqrt{-f}+15\sqrt{2}\right.}{}$

```
[In] int((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((f*x^2+3)*(d*x^2+2))^(1/2)/(f*x^2+3)^(1/2)/(d*x^2+2)^(1/2)*(1/5*b*x^3*(d*f
*x^4+3*d*x^2+2*f*x^2+6)^(1/2)+1/3*(a*d*f+3*b*d+2*b*f-1/5*b*(12*d+8*f))/d/f*
x*(d*f*x^4+3*d*x^2+2*f*x^2+6)^(1/2)+1/2*(6*a-2*(a*d*f+3*b*d+2*b*f-1/5*b*(12
*d+8*f))/d/f)/(-3*f)^(1/2)*(3*f*x^2+9)^(1/2)*(2*d*x^2+4)^(1/2)/(d*f*x^4+3*d
*x^2+2*f*x^2+6)^(1/2)*EllipticF(1/3*x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/f)^(
1/2))-(3*a*d+2*a*f+12/5*b-1/3*(a*d*f+3*b*d+2*b*f-1/5*b*(12*d+8*f))/d/f*(6*d
+4*f))/(-3*f)^(1/2)*(3*f*x^2+9)^(1/2)*(2*d*x^2+4)^(1/2)/(d*f*x^4+3*d*x^2+2*
f*x^2+6)^(1/2)/d*(EllipticF(1/3*x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/f)^(1/2)
)-EllipticE(1/3*x*(-3*f)^(1/2),1/2*(-4+2*(3*d+2*f)/f)^(1/2))))
```

Fricas [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.77

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx$$

$$=\frac{3\sqrt{3}(18\,bd^2-2\,(5\,ad-4\,b)f^2-3\,(5\,ad^2+4\,bd)f)\sqrt{df}x\sqrt{-\frac{1}{f}}E(\arcsin\left(\frac{\sqrt{3}\sqrt{-\frac{1}{f}}}{x}\right)\mid\frac{2f}{3d})+\sqrt{3}(4\,(5\,ad-b}{}$$

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*sqrt(3)*(18*b*d^2 - 2*(5*a*d - 4*b)*f^2 - 3*(5*a*d^2 + 4*b*d)*f)*sq
rt(d*f)*x*sqrt(-1/f)*elliptic_e(arcsin(sqrt(3)*sqrt(-1/f)/x), 2/3*f/d) + sq
rt(3)*(4*(5*a*d - b)*f^3 - 54*b*d^2 + 6*((5*a - b)*d - 4*b)*f^2 + 9*(5*a*d^
2 + 4*b*d)*f)*sqrt(d*f)*x*sqrt(-1/f)*elliptic_f(arcsin(sqrt(3)*sqrt(-1/f)/x
), 2/3*f/d) + (3*b*d^2*f^3*x^4 - 18*b*d^2*f + 2*(5*a*d - 4*b)*f^3 + 3*(5*a*
d^2 + 4*b*d)*f^2 + (3*b*d^2*f^2 + (5*a*d^2 + 2*b*d)*f^3)*x^2)*sqrt(d*x^2 +
2)*sqrt(f*x^2 + 3))/(d^2*f^3*x)
```

Sympy [F]

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx=\int (a+bx^2)\sqrt{dx^2+2}\sqrt{fx^2+3}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x**2+2)**(1/2)*(f*x**2+3)**(1/2),x)
[Out] Integral((a + b*x**2)*sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3), x)
```

Maxima [F]

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx=\int (bx^2+a)\sqrt{dx^2+2}\sqrt{fx^2+3}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3), x)
```

Giac [F]

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx=\int (bx^2+a)\sqrt{dx^2+2}\sqrt{fx^2+3}\,dx$$

```
[In] integrate((b*x^2+a)*(d*x^2+2)^(1/2)*(f*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}\,dx=\int (b\,x^2+a)\,\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}\,dx$$

```
[In] int((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2),x)
[Out] int((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2), x)
```

3.55 $\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx$

Optimal result

Integrand size = 87, antiderivative size = 113

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[Out] -1/2*EllipticE(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2),((b-(-4*a*c+b
^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b+(-4*a*c+b^2)^(1/2))*(b-(-4*a*c
+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.023, Rules used = {21, 435}

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx = -\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(\sqrt{b^2-4ac}+b\right)E\left(\arcsin\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}$$

```
[In] Int[(-b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(-b - Sqrt[b^2 -
 4*a*c])]*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*(b + Sqrt[b^2 - 4*a*c])*EllipticE[ArcSin[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]], (b - Sqrt[b^2 - 4*a*c])/(b +
 Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \left(-b-\sqrt{b^2-4ac}\right)\int \frac{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx\\ &= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\left(b+\sqrt{b^2-4ac}\right)E\left(\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\begin{aligned}&\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx\\ &= -2i\sqrt{2}a\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}E\left(i\text{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{-b+\sqrt{b^2-4ac}}}x\right)|\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)\end{aligned}$$

```
[In] Integrate[(-b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(-b - Sqrt
[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]),x]
```

```
[Out] (-2*I)*Sqrt[2]*a*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[
2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b
^2 - 4*a*c])]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2537 vs. $2(92) = 184$.

Time = 3.72 (sec) , antiderivative size = 2538, normalized size of antiderivative = 22.46

method	result	size
elliptic	Expression too large to display	2538

```
[In] int((2*c*x^2-(-4*a*c+b^2)^(1/2)-b)/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)))^(1/2
)/(1+2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*((-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(2*c*x
^2+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)*(-(2*c*x^2+(-4*a*c+b^2)^(1/
2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)/((-2*c*x^2+(-4*a*c+b^2)^(1
/2)+b)/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/((2*c*x^2+(-4*a*c+b^2)^(1/2)-b)/(-b+(-
4*a*c+b^2)^(1/2)))^(1/2)/(-2*((-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(2*c*x^2+(-4*
a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)*c*x^2-4*(-(2*c*x^2+(-4*a*c+b^2)^(1
/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*a*c+(-(2*c*x^2+(-4*a*c+b^
2)^(1/2)-b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*b^2+((-2*c*x^2+(-4*a
*c+b^2)^(1/2)+b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)*b)*(
1/2*(4*a*c-b^2)/(-2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2+16*a^2*b*c^2
-4*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2))^(
1/2)*(4+2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2+16*a^2*b*c^2-4*a*b^3*c
)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2)*x^2)^(1/2)*(
4-2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2-16*a^2*b*c^2+4*a*b^3*c)/(-b+
(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2)*x^2)^(1/2)/(-4*a*c
+b^2-8*c^2/(-b+(-4*a*c+b^2)^(1/2))*x^2*a+2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2*b^
2-8*c^2*x^2/(-b-(-4*a*c+b^2)^(1/2))*a+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2))*b^2-1
6*c^3*x^4/(-b-(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))*a+4*c^2*x^4/(-b-(
-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))*b^2)^(1/2)*EllipticF(1/2*x*(-2*(
(-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2+16*a^2*b*c^2-4*a*b^3*c)/(-b+(-4*a
*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2))^(1/2),1/2*(-4-2*(-8*c^
2/(-b+(-4*a*c+b^2)^(1/2))*a+2*c/(-b+(-4*a*c+b^2)^(1/2))*b^2-8*c^2/(-b-(-4*a
*c+b^2)^(1/2))*a+2*c/(-b-(-4*a*c+b^2)^(1/2))*b^2)*((-4*a*c+b^2)^(5/2)-(-4*a
*c+b^2)^(3/2)*b^2-16*a^2*b*c^2+4*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*
c+b^2)^(1/2))/a/(4*a*c-b^2)/(-16*c^3/(-b-(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^
2)^(1/2))*a+4*c^2/(-b-(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))*b^2))^(1/
2))-1/2*b/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*
c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4
*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(
-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(1+2*c/(-b+(-4*a*c+
b^2)^(1/2))*x^2+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2))+4*c^2*x^4/(-b-(-4*a*c+b^2)^
(1/2))/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)*EllipticF(1/2*x*(-2*((-4*a*c+b^2)^(3/
2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^
```

```
(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(-b+(-4*a*c+b^2)^(1/2))+2*c/(-b-(-4*a*c+b^2
)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^
2)^(1/2))/a/c^2*(-b-(-4*a*c+b^2)^(1/2)))^(1/2))-2*c/(-2*((-4*a*c+b^2)^(3/2)
-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(
-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^
(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)/(1+2*c/(-b+(-4*a*c+b^2)^(1/2))*x^2+2*c*x^2/(-b-(-4*a
*c+b^2)^(1/2))+4*c^2*x^4/(-b-(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2)))^(
1/2)/(2*c/(-b+(-4*a*c+b^2)^(1/2))+2*c/(-b-(-4*a*c+b^2)^(1/2))-(-4*a*c+b^2)^
(1/2)/a)*(EllipticF(1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*
a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(
2*c/(-b+(-4*a*c+b^2)^(1/2))+2*c/(-b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2
)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/a/c^2*(-b-(-4*a*c+
b^2)^(1/2)))^(1/2))-EllipticE(1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1
/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/
4*(-16-2*(2*c/(-b+(-4*a*c+b^2)^(1/2))+2*c/(-b-(-4*a*c+b^2)^(1/2)))*((-4*a*c
+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/a/c^2*(-
b-(-4*a*c+b^2)^(1/2)))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(92) = 184$.

Time = 0.11 (sec) , antiderivative size = 369, normalized size of antiderivative = 3.27

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx$$

$$= \frac{2\sqrt{\frac{1}{2}}\left(acx\sqrt{\frac{b^2-4\,ac}{c^2}}+abx\right)\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}+b}{c}}\sqrt{\frac{c}{a}}E\left(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}+b}{c}}}{x}\right)\middle|-\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}-b^2+2\,ac}{2\,ac}\right)+\sqrt{\frac{1}{2}}\left(\sqrt{b\cdots}\right.}{}$$

```
[In] integrate((-b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(1/2)*(a*c*x*sqrt((b^2 - 4*a*c)/c^2) + a*b*x)*sqrt((c*sqrt((b^2
- 4*a*c)/c^2) + b)/c)*sqrt(c/a)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b
^2 - 4*a*c)/c^2) + b)/c)/x), -1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) - b^2 + 2*a*
c)/(a*c)) + sqrt(1/2)*(sqrt(b^2 - 4*a*c)*b*x - (2*a*b - b^2)*x - ((2*a + b)
*c*x + sqrt(b^2 - 4*a*c)*c*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt((c*sqrt((b^2 -
4*a*c)/c^2) + b)/c)*sqrt(c/a)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2
 - 4*a*c)/c^2) + b)/c)/x), -1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) - b^2 + 2*a*c)
/(a*c)) + 2*a*c*sqrt(-(b*x^2 + sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a)*sqrt(-(b*x^2
 - sqrt(b^2 - 4*a*c)*x^2 - 2*a)/a))/(c*x)
```

Sympy [F]

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{-b+2cx^2-\sqrt{-4ac+b^2}}{\sqrt{\frac{-b+2cx^2-\sqrt{-4ac+b^2}}{-b-\sqrt{-4ac+b^2}}}\sqrt{\frac{-b+2cx^2+\sqrt{-4ac+b^2}}{-b+\sqrt{-4ac+b^2}}}}\,dx$$

```
[In] integrate((-b+2*c*x**2-(-4*a*c+b**2)**(1/2))/(1+2*c*x**2/(-b-(-4*a*c+b**2)*
*(1/2)))**(1/2)/(1+2*c*x**2/(-b+(-4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral((-b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(sqrt((-b + 2*c*x**2 - sqrt(
-4*a*c + b**2))/(-b - sqrt(-4*a*c + b**2)))*sqrt((-b + 2*c*x**2 + sqrt(-4*a
*c + b**2))/(-b + sqrt(-4*a*c + b**2)))), x)
```

Maxima [F]

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{2\,cx^2-b-\sqrt{b^2-4\,ac}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}\,dx$$

```
[In] integrate((-b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(sqrt(-2*c*x^2/(b + sqrt(b^2 -
4*a*c)) + 1)*sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

Giac [F]

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx = \int \frac{2\,cx^2-b-\sqrt{b^2-4\,ac}}{\sqrt{-\frac{2\,cx^2}{b+\sqrt{b^2-4\,ac}}+1}\sqrt{-\frac{2\,cx^2}{b-\sqrt{b^2-4\,ac}}+1}}\,dx$$

```
[In] integrate((-b+2*c*x^2-(-4*a*c+b^2)^(1/2))/(1+2*c*x^2/(-b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(1+2*c*x^2/(-b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*x^2 - b - sqrt(b^2 - 4*a*c))/(sqrt(-2*c*x^2/(b + sqrt(b^2 -
4*a*c)) + 1)*sqrt(-2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{-b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{-b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{-b+\sqrt{b^2-4ac}}}}\,dx = \int -\frac{b-2\,c\,x^2+\sqrt{b^2-4\,a\,c}}{\sqrt{1-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}}\,\sqrt{1-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}}}\,dx$$

```
[In] int(-(b - 2*c*x^2 + (b^2 - 4*a*c)^(1/2))/((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)
^(1/2)))^(1/2)*(1 - (2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)))^(1/2)),x)
```

```
[Out] int(-(b - 2*c*x^2 + (b^2 - 4*a*c)^(1/2))/((1 - (2*c*x^2)/(b - (b^2 - 4*a*c)
^(1/2)))^(1/2)*(1 - (2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)))^(1/2)), x)
```

3.56 $\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$

Optimal result

Integrand size = 81, antiderivative size = 526

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx = \frac{\left(b-\sqrt{b^2-4ac}\right)x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} - \frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right),-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

```
[Out] x*(b-(-4*a*c+b^2)^(1/2))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^
2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*(1/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^
(1/2)*EllipticE(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(1+2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))^(1/2),(-2*(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2)))
^(1/2))*(b-(-4*a*c+b^2)^(1/2))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(b+
(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/((1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
))/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)+1/2*(1/(1+2*c*x^2/(b+(-4*a*c+b
^2)^(1/2))))^(1/2)*EllipticF(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),(-2*(-4*a*c+b^2)^(1/2)/(b-(-4*a*c
+b^2)^(1/2)))^(1/2))*(b-(-4*a*c+b^2)^(1/2))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2
)))^(1/2)*(b+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/((1+2*c*x^2/(b-(-4*a
*c+b^2)^(1/2)))/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))))^(1/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {21, 433, 429, 506, 422}

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$$
$$=\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right),-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$
$$-\frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}E\left(\arctan\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$
$$+\frac{x\left(b-\sqrt{b^2-4ac}\right)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}}{\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

```
[In] Int[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]),x]
```

```
[Out] ((b - Sqrt[b^2 - 4*a*c])*x*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/Sqr
t[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])] - ((b - Sqrt[b^2 - 4*a*c])*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a
*c])/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]) + ((b - Sqrt[b^2 - 4*a*c])*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[ArcTan[(Sqrt
[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]], (-2*Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])])/(Sqrt[2]*Sqrt[c]*Sqrt[(1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c]))/(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))]*Sqrt[1 + (2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c])])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \left(b-\sqrt{b^2-4ac}\right)\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx \\
&= (2c)\int \frac{x^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx \\
&\quad+\left(b-\sqrt{b^2-4ac}\right)\int \frac{1}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx
\end{aligned}$$

$$
\begin{aligned}
&= \frac{\left(b-\sqrt{b^2-4ac}\right)x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} \\
&+ \frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} \\
&+ \left(-b+\sqrt{b^2-4ac}\right)\int\frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}dx \\
&= \frac{\left(b-\sqrt{b^2-4ac}\right)x\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}}{\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} \\
&- \frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}E\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} \\
&+ \frac{\left(b-\sqrt{b^2-4ac}\right)\sqrt{b+\sqrt{b^2-4ac}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}F\left(\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)|-\frac{2\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\frac{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.19

$$
\begin{aligned}
&\int\frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}dx \\
&= -2i\sqrt{2}a\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}E\left(i\operatorname{arcsinh}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)
\end{aligned}
$$

```
[In] Integrate[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(Sqrt[1 + (2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]),x]
```

```
[Out] (-2*I)*Sqrt[2]*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2
]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2
 - 4*a*c])]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2476 vs. 2(499) = 998.

Time = 1.37 (sec) , antiderivative size = 2477, normalized size of antiderivative = 4.71

method	result	size
elliptic	Expression too large to display	2477

```
[In] int((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)
/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(-(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(2*c
*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(-2*c
*x^2+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)/((-2*c*x^2+(-4*a*c+b^2)^(
1/2)-b)/(-b+(-4*a*c+b^2)^(1/2)))^(1/2)/((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/(b+(
-4*a*c+b^2)^(1/2)))^(1/2)/(2*((2*c*x^2+(-4*a*c+b^2)^(1/2)+b)*(-2*c*x^2+(-4*
a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)*c*x^2+4*(-(-2*c*x^2+(-4*a*c+b^2)^(
1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*a*c-(-(-2*c*x^2+(-4*a*c+b
^2)^(1/2)-b)*(2*c*x^2+(-4*a*c+b^2)^(1/2)+b)/a/c)^(1/2)*b^2+((2*c*x^2+(-4*a*
c+b^2)^(1/2)+b)*(-2*c*x^2+(-4*a*c+b^2)^(1/2)-b)*(4*a*c-b^2)/a/c)^(1/2)*b)*(
1/2*b/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^
2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4*a*c
+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x
^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a
*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b
^2)^(1/2))+2*c*x^2/(b-(-4*a*c+b^2)^(1/2))+4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(
-4*a*c+b^2)^(1/2))*x^4)^(1/2)*EllipticF(1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a
*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/4*(-16-2*(2*c/(b+(-4*a*c+b^2)^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2)))*
((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2+4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))
/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2))-2*c/(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b
^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1
/2)*(4+2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*c+b^2
)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4-2*((-4*a*c+b^2)^(3/2)-(-4*
a*c+b^2)^(1/2)*b^2+4*a*b*c)/(b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2))+2*c*x^2/(b-(-4*a*c+b^2)^(1/2
))+4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4)^(1/2)/(2*c/(b+(
-4*a*c+b^2)^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2))-(-4*a*c+b^2)^(1/2)/a)*(Ellipt
icF(1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(b+(-4*a*
c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(b+(-4*a*c+b
^2)^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/
2)*b^2+4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)))^(1/2)
)-EllipticE(1/2*x*(-2*((-4*a*c+b^2)^(3/2)-(-4*a*c+b^2)^(1/2)*b^2-4*a*b*c)/(
b+(-4*a*c+b^2)^(1/2))/(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/4*(-16-2*(2*c/(b+(
-4*a*c+b^2)^(1/2))+2*c/(b-(-4*a*c+b^2)^(1/2)))*((-4*a*c+b^2)^(3/2)-(-4*a*c+
```

```
b^2)^(1/2)*b^2+4*a*b*c)/(-b+(-4*a*c+b^2)^(1/2))/a/c^2*(b-(-4*a*c+b^2)^(1/2)
))^(1/2)))+1/2*(4*a*c-b^2)/(-2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2-1
6*a^2*b*c^2+4*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*
a*c-b^2))^(1/2)*(4+2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2-16*a^2*b*c^
2+4*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2)*x
^2)^(1/2)*(4-2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2+16*a^2*b*c^2-4*a*
b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2)*x^2)^(1
/2)/(-4*a*c+b^2-8*c^2*x^2/(b+(-4*a*c+b^2)^(1/2))*a+2*c*x^2/(b+(-4*a*c+b^2)^
(1/2))*b^2-8*c^2*x^2/(b-(-4*a*c+b^2)^(1/2))*a+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
)*b^2-16*c^3/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4*a+4*c^2/(b-(
-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*x^4*b^2)^(1/2)*EllipticF(1/2*x*(-
2*((-4*a*c+b^2)^(5/2)-(-4*a*c+b^2)^(3/2)*b^2-16*a^2*b*c^2+4*a*b^3*c)/(-b+(-
4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))/a/(4*a*c-b^2))^(1/2),1/2*(-4-2*(-8
*c^2/(b+(-4*a*c+b^2)^(1/2))*a+2*c/(b+(-4*a*c+b^2)^(1/2))*b^2-8*c^2/(b-(-4*a
*c+b^2)^(1/2))*a+2*c/(b-(-4*a*c+b^2)^(1/2))*b^2)*((-4*a*c+b^2)^(5/2)-(-4*a*
c+b^2)^(3/2)*b^2+16*a^2*b*c^2-4*a*b^3*c)/(-b+(-4*a*c+b^2)^(1/2))/(b+(-4*a*c
+b^2)^(1/2))/a/(4*a*c-b^2)/(-16*c^3/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^
(1/2))*a+4*c^2/(b-(-4*a*c+b^2)^(1/2))/(b+(-4*a*c+b^2)^(1/2))*b^2))^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.71

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx$$

$$= \frac{2\sqrt{\frac{1}{2}}\left(acx\sqrt{\frac{b^2-4\,ac}{c^2}}-abx\right)\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}\sqrt{\frac{c}{a}}E(\arcsin\left(\frac{\sqrt{\frac{1}{2}}\sqrt{\frac{c\sqrt{\frac{b^2-4\,ac}{c^2}}-b}{c}}}{x}\right)\mid\frac{bc\sqrt{\frac{b^2-4\,ac}{c^2}}+b^2-2\,ac}{2\,ac})-\sqrt{\frac{1}{2}}\left(\sqrt{b^2}\right.}{}$$

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*sqrt(1/2)*(a*c*x*sqrt((b^2 - 4*a*c)/c^2) - a*b*x)*sqrt((c*sqrt((b^2
- 4*a*c)/c^2) - b)/c)*sqrt(c/a)*elliptic_e(arcsin(sqrt(1/2)*sqrt((c*sqrt((b
^2 - 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c
)/(a*c)) - sqrt(1/2)*(sqrt(b^2 - 4*a*c)*b*x - (2*a*b + b^2)*x + ((2*a - b)*
c*x + sqrt(b^2 - 4*a*c)*c*x)*sqrt((b^2 - 4*a*c)/c^2))*sqrt((c*sqrt((b^2 - 4
*a*c)/c^2) - b)/c)*sqrt(c/a)*elliptic_f(arcsin(sqrt(1/2)*sqrt((c*sqrt((b^2
- 4*a*c)/c^2) - b)/c)/x), 1/2*(b*c*sqrt((b^2 - 4*a*c)/c^2) + b^2 - 2*a*c)/(
a*c)) + 2*a*c*sqrt((b*x^2 + sqrt(b^2 - 4*a*c)*x^2 + 2*a)/a)*sqrt((b*x^2 - s
qrt(b^2 - 4*a*c)*x^2 + 2*a)/a))/(c*x)
```

Sympy [F]

$$\int \frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{\sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}}\sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}}}\,dx = \int \frac{b + 2cx^2 - \sqrt{-4ac + b^2}}{\sqrt{\frac{b + 2cx^2 - \sqrt{-4ac + b^2}}{b - \sqrt{-4ac + b^2}}}\sqrt{\frac{b + 2cx^2 + \sqrt{-4ac + b^2}}{b + \sqrt{-4ac + b^2}}}}\,dx$$

```
[In] integrate((2*c*x**2-(-4*a*c+b**2)**(1/2)+b)/(1+2*c*x**2/(b-(-4*a*c+b**2)**(
1/2)))**(1/2)/(1+2*c*x**2/(b+(-4*a*c+b**2)**(1/2)))**(1/2),x)
```

```
[Out] Integral((b + 2*c*x**2 - sqrt(-4*a*c + b**2))/(sqrt((b + 2*c*x**2 - sqrt(-4
*a*c + b**2))/(b - sqrt(-4*a*c + b**2)))*sqrt((b + 2*c*x**2 + sqrt(-4*a*c +
 b**2))/(b + sqrt(-4*a*c + b**2)))), x)
```

Maxima [F]

$$\int \frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{\sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}}\sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}}}\,dx = \int \frac{2\,cx^2 + b - \sqrt{b^2 - 4\,ac}}{\sqrt{\frac{2\,cx^2}{b + \sqrt{b^2 - 4\,ac}} + 1}\sqrt{\frac{2\,cx^2}{b - \sqrt{b^2 - 4\,ac}} + 1}}\,dx$$

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(sqrt(2*c*x^2/(b + sqrt(b^2 - 4
*a*c)) + 1)*sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

Giac [F]

$$\int \frac{b - \sqrt{b^2 - 4ac} + 2cx^2}{\sqrt{1 + \frac{2cx^2}{b - \sqrt{b^2 - 4ac}}}\sqrt{1 + \frac{2cx^2}{b + \sqrt{b^2 - 4ac}}}}\,dx = \int \frac{2\,cx^2 + b - \sqrt{b^2 - 4\,ac}}{\sqrt{\frac{2\,cx^2}{b + \sqrt{b^2 - 4\,ac}} + 1}\sqrt{\frac{2\,cx^2}{b - \sqrt{b^2 - 4\,ac}} + 1}}\,dx$$

```
[In] integrate((2*c*x^2-(-4*a*c+b^2)^(1/2)+b)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((2*c*x^2 + b - sqrt(b^2 - 4*a*c))/(sqrt(2*c*x^2/(b + sqrt(b^2 - 4
*a*c)) + 1)*sqrt(2*c*x^2/(b - sqrt(b^2 - 4*a*c)) + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{b-\sqrt{b^2-4ac}+2cx^2}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}\,dx=\int \frac{b+2\,c\,x^2-\sqrt{b^2-4\,a\,c}}{\sqrt{\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}+1}\,\sqrt{\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}+1}}\,dx$$

```
[In] int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/((((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2
)) + 1)^(1/2)*((2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)),x)
```

```
[Out] int((b + 2*c*x^2 - (b^2 - 4*a*c)^(1/2))/((((2*c*x^2)/(b - (b^2 - 4*a*c)^(1/2
)) + 1)^(1/2)*((2*c*x^2)/(b + (b^2 - 4*a*c)^(1/2)) + 1)^(1/2)), x)
```

3.57 $\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 128

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx = \frac{bx\sqrt{c+dx^2}}{2f} - \frac{(2bde-bcf-2adf)\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}f^2} + \frac{(be-af)\sqrt{de-cf}\operatorname{arctanh}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{e}f^2}$$

```
[Out] -1/2*(-2*a*d*f-b*c*f+2*b*d*e)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))/f^2/d^(1/2
)+(-a*f+b*e)*arctanh(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1/2))*(-c*f+d*e)
^(1/2)/f^2/e^(1/2)+1/2*b*x*(d*x^2+c)^(1/2)/f
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {542, 537, 223, 212, 385, 214}

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)(-2adf-bcf+2bde)}{2\sqrt{d}f^2} + \frac{(be-af)\sqrt{de-cf}\operatorname{arctanh}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{e}f^2} + \frac{bx\sqrt{c+dx^2}}{2f}$$

```
[In] Int[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

```
[Out] (b*x*Sqrt[c + d*x^2])/(2*f) - ((2*b*d*e - b*c*f - 2*a*d*f)*ArcTanh[(Sqrt[d]
*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*f^2) + ((b*e - a*f)*Sqrt[d*e - c*f]*ArcTan
h[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])])/(Sqrt[e]*f^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{bx\sqrt{c+dx^2}}{2f} + \frac{\int \frac{-c(be-2af)+(-2bde+bcf+2adf)x^2}{\sqrt{c+dx^2}(e+fx^2)}\,dx}{2f}\\ &= \frac{bx\sqrt{c+dx^2}}{2f} + \frac{((be-af)(de-cf))\int \frac{1}{\sqrt{c+dx^2}(e+fx^2)}\,dx}{f^2} - \frac{(2bde-bcf-2adf)\int \frac{1}{\sqrt{c+dx^2}}\,dx}{2f^2}\end{aligned}$$

$$= \frac{bx\sqrt{c+dx^2}}{2f} + \frac{((be-af)(de-cf))\text{Subst}\left(\int \frac{1}{e-(de-cf)x^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{f^2} - \frac{(2bde-bcf-2adf)\text{Subst}\left(\int \frac{1}{1-dx^2}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2f^2}$$

$$= \frac{bx\sqrt{c+dx^2}}{2f} - \frac{(2bde-bcf-2adf)\tanh^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}f^2} + \frac{(be-af)\sqrt{de-cf}\tanh^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)}{\sqrt{e}f^2}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx$$

$$= \frac{bfx\sqrt{c+dx^2} + \frac{2(be-af)\sqrt{-de+cf}\arctan\left(\frac{-fx\sqrt{c+dx^2}+\sqrt{d}(e+fx^2)}{\sqrt{e}\sqrt{-de+cf}}\right)}{\sqrt{e}} + \frac{(2bde-bcf-2adf)\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)}{\sqrt{d}}}{2f^2}$$

```
[In] Integrate[((a + b*x^2)*Sqrt[c + d*x^2])/(e + f*x^2),x]
```

```
[Out] (b*f*x*Sqrt[c + d*x^2] + (2*(b*e - a*f)*Sqrt[-(d*e) + c*f]*ArcTan[(-(f*x*Sq
rt[c + d*x^2]) + Sqrt[d]*(e + f*x^2))/(Sqrt[e]*Sqrt[-(d*e) + c*f])])/Sqrt[e
] + ((2*b*d*e - b*c*f - 2*a*d*f)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]])/Sqrt[
d])/(2*f^2)
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{-\sqrt{d x^2+c}\, b f x-\frac{(2adf+bcf-2bde)\operatorname{arctanh}\left(\frac{\sqrt{d x^2+c}}{x\sqrt{d}}\right)}{\sqrt{d}}+\frac{2(cf-de)(af-be)\arctan\left(\frac{e\sqrt{d x^2+c}}{x\sqrt{(cf-de)e}}\right)}{\sqrt{(cf-de)e}}}{2f^2}$
risch	$\frac{bx\sqrt{dx^2+c}}{2f}+\frac{\frac{(2adf+bcf-2bde)\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{f\sqrt{d}}-\frac{\left(acf^2-adef-bcef+bde^2\right)\ln\left(\frac{\frac{2cf-2de}{f}+\frac{2d\sqrt{-ef}\left(x-\frac{\sqrt{-ef}}{f}\right)}{f}+2\sqrt{\frac{cf-de}{f}}\cdots}{x-\cdots}\right)}{\sqrt{-ef}\,f\sqrt{\frac{cf-de}{f}}\cdots}}{\cdots}$
default	$\frac{b\left(\frac{x\sqrt{dx^2+c}}{2}+\frac{c\ln\left(x\sqrt{d}+\sqrt{dx^2+c}\right)}{2\sqrt{d}}\right)}{f}+\frac{(af-be)\left(\sqrt{d\left(x-\frac{\sqrt{-ef}}{f}\right)^2+\frac{2d\sqrt{-ef}\left(x-\frac{\sqrt{-ef}}{f}\right)}{f}+\frac{cf-de}{f}}+\frac{\sqrt{d}\sqrt{-ef}\ln\left(\frac{\frac{d\sqrt{-ef}}{f}+d\cdots}{\cdots}\right)}{\cdots}\cdots\right)}{\cdots}$

```
[In] int((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/f^2*(-(d*x^2+c)^(1/2)*b*f*x-(2*a*d*f+b*c*f-2*b*d*e)/d^(1/2)*arctanh((d
*x^2+c)^(1/2)/x/d^(1/2))+2*(c*f-d*e)*(a*f-b*e)/((c*f-d*e)*e)^(1/2)*arctan(e
*(d*x^2+c)^(1/2)/x/((c*f-d*e)*e)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.90 (sec) , antiderivative size = 777, normalized size of antiderivative = 6.07

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx = \left[\frac{2\sqrt{dx^2+c}bdfx-(2bde-(bc+2ad)f)\sqrt{d}\log\left(-2dx^2-2\sqrt{dx^2+c}\sqrt{d}x-c\right)-(bde-adf)\sqrt{\frac{de-cf}{e}}\log\cdots}{4df^2}\cdots\right.$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(d*x^2 + c)*b*d*f*x - (2*b*d*e - (b*c + 2*a*d)*f)*sqrt(d)*log(-
2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (b*d*e - a*d*f)*sqrt((d*e - c*
```

```
f)/e)*log(((8*d^2*e^2 - 8*c*d*e*f + c^2*f^2)*x^4 + c^2*e^2 + 2*(4*c*d*e^2 -
 3*c^2*e*f)*x^2 - 4*(c*e^2*x + (2*d*e^2 - c*e*f)*x^3)*sqrt(d*x^2 + c)*sqrt(
(d*e - c*f)/e))/(f^2*x^4 + 2*e*f*x^2 + e^2)))/(d*f^2), 1/4*(2*sqrt(d*x^2 +
c)*b*d*f*x + 2*(2*b*d*e - (b*c + 2*a*d)*f)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(
d*x^2 + c)) - (b*d*e - a*d*f)*sqrt((d*e - c*f)/e)*log(((8*d^2*e^2 - 8*c*d*e
*f + c^2*f^2)*x^4 + c^2*e^2 + 2*(4*c*d*e^2 - 3*c^2*e*f)*x^2 - 4*(c*e^2*x +
(2*d*e^2 - c*e*f)*x^3)*sqrt(d*x^2 + c)*sqrt((d*e - c*f)/e))/(f^2*x^4 + 2*e*
f*x^2 + e^2)))/(d*f^2), 1/4*(2*sqrt(d*x^2 + c)*b*d*f*x - 2*(b*d*e - a*d*f)*
sqrt(-(d*e - c*f)/e)*arctan(1/2*((2*d*e - c*f)*x^2 + c*e)*sqrt(d*x^2 + c)*s
qrt(-(d*e - c*f)/e)/((d^2*e - c*d*f)*x^3 + (c*d*e - c^2*f)*x)) - (2*b*d*e -
 (b*c + 2*a*d)*f)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c))/
(d*f^2), 1/2*(sqrt(d*x^2 + c)*b*d*f*x + (2*b*d*e - (b*c + 2*a*d)*f)*sqrt(-d
)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (b*d*e - a*d*f)*sqrt(-(d*e - c*f)/e)
*arctan(1/2*((2*d*e - c*f)*x^2 + c*e)*sqrt(d*x^2 + c)*sqrt(-(d*e - c*f)/e)/
((d^2*e - c*d*f)*x^3 + (c*d*e - c^2*f)*x)))/(d*f^2)]
```

Sympy [F]

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx$$

```
[In] integrate((b*x**2+a)*(d*x**2+c)**(1/2)/(f*x**2+e),x)
[Out] Integral((a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx = \text{Exception raised: ValueError}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e>0)', see `assume?` for more detai
ls)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*x^2+a)*(d*x^2+c)^(1/2)/(f*x^2+e),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\sqrt{c+dx^2}}{e+fx^2}\,dx = \int \frac{(b\,x^2+a)\,\sqrt{d\,x^2+c}}{f\,x^2+e}\,dx$$

```
[In] int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2),x)
```

```
[Out] int(((a + b*x^2)*(c + d*x^2)^(1/2))/(e + f*x^2), x)
```

3.58 $\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 304

$$\begin{aligned}\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx = &-\frac{b^2(bc-ad)x\sqrt{e+fx^2}}{2d^2f}-\frac{3b^2(be-2af)x\sqrt{e+fx^2}}{8df^2}\\&+\frac{b^2x(a+bx^2)\sqrt{e+fx^2}}{4df}-\frac{(bc-ad)^3\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^3\sqrt{de-cf}}\\&+\frac{b(bc-ad)^2\operatorname{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^3\sqrt{f}}\\&+\frac{b(bc-ad)(be-2af)\operatorname{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2d^2f^{3/2}}\\&+\frac{b(3b^2e^2-8abef+8a^2f^2)\operatorname{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{8df^{5/2}}\end{aligned}$$

```
[Out] 1/2*b*(-a*d+b*c)*(-2*a*f+b*e)*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d^2/f^(3/2
)+1/8*b*(8*a^2*f^2-8*a*b*e*f+3*b^2*e^2)*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/
d/f^(5/2)+b*(-a*d+b*c)^2*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d^3/f^(1/2)-(-a
*d+b*c)^3*arctan(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/d^3/c^(1/2)/(-
c*f+d*e)^(1/2)-1/2*b^2*(-a*d+b*c)*x*(f*x^2+e)^(1/2)/d^2/f-3/8*b^2*(-2*a*f+b
*e)*x*(f*x^2+e)^(1/2)/d/f^2+1/4*b^2*x*(b*x^2+a)*(f*x^2+e)^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {559, 427, 396, 223, 212, 537, 385, 211}

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{b(8a^2f^2-8abef+3b^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{8df^{5/2}} - \frac{(bc-ad)^3\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^3\sqrt{de-cf}} + \frac{b(bc-ad)^2\operatorname{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^3\sqrt{f}} + \frac{b(bc-ad)(be-2af)\operatorname{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2d^2f^{3/2}} - \frac{b^2x\sqrt{e+fx^2}(bc-ad)}{2d^2f} - \frac{3b^2x\sqrt{e+fx^2}(be-2af)}{8df^2} + \frac{b^2x(a+bx^2)\sqrt{e+fx^2}}{4df}$$

```
[In] Int[(a + b*x^2)^3/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -1/2*(b^2*(b*c - a*d)*x*Sqrt[e + f*x^2])/(d^2*f) - (3*b^2*(b*e - 2*a*f)*x*S
qrt[e + f*x^2])/(8*d*f^2) + (b^2*x*(a + b*x^2)*Sqrt[e + f*x^2])/(4*d*f) - (
(b*c - a*d)^3*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[
c]*d^3*Sqrt[d*e - c*f]) + (b*(b*c - a*d)^2*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x
^2]])/(d^3*Sqrt[f]) + (b*(b*c - a*d)*(b*e - 2*a*f)*ArcTanh[(Sqrt[f]*x)/Sqrt
[e + f*x^2]])/(2*d^2*f^(3/2)) + (b*(3*b^2*e^2 - 8*a*b*e*f + 8*a^2*f^2)*ArcT
anh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(8*d*f^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 559

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\int \frac{(a+bx^2)^2}{\sqrt{e+fx^2}}\,dx}{d} + \frac{(-bc+ad)\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{d}\\ &= \frac{b^2x(a+bx^2)\sqrt{e+fx^2}}{4df} - \frac{(b(bc-ad))\int \frac{a+bx^2}{\sqrt{e+fx^2}}\,dx}{d^2}\\ &\quad + \frac{(bc-ad)^2\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{d^2} + \frac{b\int \frac{-a(be-4af)-3b(be-2af)x^2}{\sqrt{e+fx^2}}\,dx}{4df}\end{aligned}$$

$$= -\frac{b^2(bc-ad)x\sqrt{e+fx^2}}{2d^2f} - \frac{3b^2(be-2af)x\sqrt{e+fx^2}}{8df^2} + \frac{b^2x(a+bx^2)\sqrt{e+fx^2}}{4df} + \frac{(b(bc-ad)^2)\int \frac{1}{\sqrt{e+fx^2}}\,dx}{d^3} - \frac{(bc-ad)^3\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{d^3} + \frac{(b(bc-ad)(be-2af))\int \frac{1}{\sqrt{e+fx^2}}\,dx}{2d^2f} + \frac{(b(3b^2e^2-8abef+8a^2f^2))\int \frac{1}{\sqrt{e+fx^2}}\,dx}{8df^2}$$

$$= -\frac{b^2(bc-ad)x\sqrt{e+fx^2}}{2d^2f} - \frac{3b^2(be-2af)x\sqrt{e+fx^2}}{8df^2} + \frac{b^2x(a+bx^2)\sqrt{e+fx^2}}{4df} + \frac{(b(bc-ad)^2)\,\text{Subst}\left(\int \frac{1}{1-fx^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{d^3} - \frac{(bc-ad)^3\text{Subst}\left(\int \frac{1}{c-(-de+cf)x^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{d^3} + \frac{(b(bc-ad)(be-2af))\text{Subst}\left(\int \frac{1}{1-fx^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{2d^2f} + \frac{(b(3b^2e^2-8abef+8a^2f^2))\,\text{Subst}\left(\int \frac{1}{1-fx^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{8df^2}$$

$$= -\frac{b^2(bc-ad)x\sqrt{e+fx^2}}{2d^2f} - \frac{3b^2(be-2af)x\sqrt{e+fx^2}}{8df^2} + \frac{b^2x(a+bx^2)\sqrt{e+fx^2}}{4df} - \frac{(bc-ad)^3\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^3\sqrt{de-cf}} + \frac{b(bc-ad)^2\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^3\sqrt{f}} + \frac{b(bc-ad)(be-2af)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2d^2f^{3/2}} + \frac{b(3b^2e^2-8abef+8a^2f^2)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{8df^{5/2}}$$

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$

$$= \frac{\frac{b^2dx\sqrt{e+fx^2}(12adf+b(-3de-4cf+2dfx^2))}{f^2} + \frac{8(bc-ad)^3\arctan\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}x-\sqrt{e+fx^2}\right)}{\sqrt{c}\sqrt{de-cf}}\right)}{\sqrt{c}\sqrt{de-cf}} - \frac{b(24a^2d^2f^2-12abdf(de+2cf)+b^2(3d^2e^2+4}{f^{5/2}}}{8d^3}$$

```
[In] Integrate[(a + b*x^2)^3/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((b^2*d*x*Sqrt[e + f*x^2]*(12*a*d*f + b*(-3*d*e - 4*c*f + 2*d*f*x^2)))/f^2
+ (8*(b*c - a*d)^3*ArcTan[(c*Sqrt[f] + d*x*(Sqrt[f]*x - Sqrt[e + f*x^2]))/(
Sqrt[c]*Sqrt[d*e - c*f])])/(Sqrt[c]*Sqrt[d*e - c*f]) - (b*(24*a^2*d^2*f^2 -
 12*a*b*d*f*(d*e + 2*c*f) + b^2*(3*d^2*e^2 + 4*c*d*e*f + 8*c^2*f^2))*Log[-(
Sqrt[f]*x) + Sqrt[e + f*x^2]])/f^(5/2))/(8*d^3)
```

Maple [A] (verified)

Time = 3.57 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{f^{\frac{9}{2}}(ad-bc)^3 \operatorname{arctanh}\left(\frac{c\sqrt{f x^2+e}}{x\sqrt{(cf-de)c}}\right)+\frac{3\left(2\left(\frac{b^2c^2f^2}{3}-\left(af-\frac{be}{6}\right)bfdc+d^2\left(a^2f^2-\frac{1}{2}abfe+\frac{1}{8}b^2e^2\right)\right)f^2 \operatorname{arctanh}\left(\frac{\sqrt{f x^2+e}}{x\sqrt{f}}\right)+b\sqrt{f x^2+e}\cdots}{2}}{\sqrt{(cf-de)c}\, d^3 f^{\frac{9}{2}}}$
risch	$\frac{b^2x(2bdf x^2+12adf-4bcf-3bde)\sqrt{f x^2+e}}{8f^2d^2}+\frac{\frac{b\left(24a^2d^2f^2-24abcd f^2-12ab d^2ef+8b^2c^2f^2+4b^2cdef+3b^2d^2e^2\right)\ln\left(\sqrt{f}\,x+\sqrt{f x^2+e}\cdots\right.}{d\sqrt{f}}}{\cdots}$
default	$\frac{b\left(\frac{b^2c^2\ln\left(\sqrt{f}\,x+\sqrt{f x^2+e}\right)}{\sqrt{f}}+b^2d^2\left(\frac{x^3\sqrt{f x^2+e}}{4f}-\frac{3e\left(\frac{x\sqrt{f x^2+e}}{2f}-\frac{e\ln\left(\sqrt{f}\,x+\sqrt{f x^2+e}\right)}{2f^{\frac{3}{2}}}\right)}{4f}\right)+\frac{3a^2d^2\ln\left(\sqrt{f}\,x+\sqrt{f x^2+e}\right)}{\sqrt{f}}-\frac{3abcd\ln\cdots}{\cdots}\right.}{d^3}$

```
[In] int((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((c*f-d*e)*c)^(1/2)*(f^(9/2)*(a*d-b*c)^3*arctanh(c*(f*x^2+e)^(1/2)/x/((c*
f-d*e)*c)^(1/2))+3/2*(2*(1/3*b^2*c^2*f^2-(a*f-1/6*b*e)*b*f*d*c+d^2*(a^2*f^2
-1/2*a*b*f*e+1/8*b^2*e^2))*f^2*arctanh((f*x^2+e)^(1/2)/x/f^(1/2))+b*(f*x^2+
e)^(1/2)*(-1/3*b*c*f+((1/6*b*x^2+a)*f-1/4*b*e)*d)*f^(5/2)*x*d)*b*((c*f-d*e)
*c)^(1/2))/d^3/f^(9/2)
```

Fricas [A] (verification not implemented)

none

Time = 3.87 (sec) , antiderivative size = 1718, normalized size of antiderivative = 5.65

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\, dx = \text{Too large to display}$$

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c*d*e +
c^2*f)*f^3*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*
e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3 - c*e*x)*sqrt(-c*d*e + c^2*f)*s
qrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2)) + (3*b^3*c*d^3*e^3 + (b^3*c^2*
d^2 - 12*a*b^2*c*d^3)*e^2*f + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d^
3)*e*f^2 - 8*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*c^2*d^2)*f^3)*sqrt(f)*log(-
2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x - e) + 2*(2*(b^3*c*d^3*e*f^2 - b^3*c^
2*d^2*f^3)*x^3 - (3*b^3*c*d^3*e^2*f + (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*e*f^2
- 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2)*f^3)*x)*sqrt(f*x^2 + e))/(c*d^4*e*f^3 - c
^2*d^3*f^4), -1/16*(8*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*s
qrt(c*d*e - c^2*f)*f^3*arctan(1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*c*f)*x^2 -
c*e)*sqrt(f*x^2 + e)/((c*d*e*f - c^2*f^2)*x^3 + (c*d*e^2 - c^2*e*f)*x)) - (
3*b^3*c*d^3*e^3 + (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*e^2*f + 4*(b^3*c^3*d - 3*a
*b^2*c^2*d^2 + 6*a^2*b*c*d^3)*e*f^2 - 8*(b^3*c^4 - 3*a*b^2*c^3*d + 3*a^2*b*
c^2*d^2)*f^3)*sqrt(f)*log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x - e) - 2*(
2*(b^3*c*d^3*e*f^2 - b^3*c^2*d^2*f^3)*x^3 - (3*b^3*c*d^3*e^2*f + (b^3*c^2*d
^2 - 12*a*b^2*c*d^3)*e*f^2 - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2)*f^3)*x)*sqrt(f
*x^2 + e))/(c*d^4*e*f^3 - c^2*d^3*f^4), 1/8*(2*(b^3*c^3 - 3*a*b^2*c^2*d + 3
*a^2*b*c*d^2 - a^3*d^3)*sqrt(-c*d*e + c^2*f)*f^3*log(((d^2*e^2 - 8*c*d*e*f
+ 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*
f)*x^3 - c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2
+ c^2)) - (3*b^3*c*d^3*e^3 + (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*e^2*f + 4*(b^3*
c^3*d - 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d^3)*e*f^2 - 8*(b^3*c^4 - 3*a*b^2*c^3*d
 + 3*a^2*b*c^2*d^2)*f^3)*sqrt(-f)*arctan(sqrt(-f)*x/sqrt(f*x^2 + e)) + (2*(
b^3*c*d^3*e*f^2 - b^3*c^2*d^2*f^3)*x^3 - (3*b^3*c*d^3*e^2*f + (b^3*c^2*d^2
- 12*a*b^2*c*d^3)*e*f^2 - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2)*f^3)*x)*sqrt(f*x^
2 + e))/(c*d^4*e*f^3 - c^2*d^3*f^4), -1/8*(4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a
^2*b*c*d^2 - a^3*d^3)*sqrt(c*d*e - c^2*f)*f^3*arctan(1/2*sqrt(c*d*e - c^2*f
)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2 + e)/((c*d*e*f - c^2*f^2)*x^3 + (c*d
*e^2 - c^2*e*f)*x)) + (3*b^3*c*d^3*e^3 + (b^3*c^2*d^2 - 12*a*b^2*c*d^3)*e^2
*f + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 6*a^2*b*c*d^3)*e*f^2 - 8*(b^3*c^4 - 3
*a*b^2*c^3*d + 3*a^2*b*c^2*d^2)*f^3)*sqrt(-f)*arctan(sqrt(-f)*x/sqrt(f*x^2
+ e)) - (2*(b^3*c*d^3*e*f^2 - b^3*c^2*d^2*f^3)*x^3 - (3*b^3*c*d^3*e^2*f + (
b^3*c^2*d^2 - 12*a*b^2*c*d^3)*e*f^2 - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2)*f^3)*
x)*sqrt(f*x^2 + e))/(c*d^4*e*f^3 - c^2*d^3*f^4)]
```

Sympy [F]

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((b*x**2+a)**3/(d*x**2+c)/(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)**3/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^3}{(dx^2+c)\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^3/((d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*x^2+a)^3/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^3}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^3}{(d\,x^2+c)\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int((a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)^(1/2)),x)
[Out] int((a + b*x^2)^3/((c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.59 $\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 166

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{e+fx^2}}{2df} + \frac{(bc-ad)^2\arctan\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^2\sqrt{de-cf}} - \frac{b(bc-ad)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^2\sqrt{f}} - \frac{b(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2df^{3/2}}$$

```
[Out] -1/2*b*(-2*a*f+b*e)*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d/f^(3/2)-b*(-a*d+b*
c)*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d^2/f^(1/2)+(-a*d+b*c)^2*arctan(x*(-c
*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/d^2/c^(1/2)/(-c*f+d*e)^(1/2)+1/2*b^2
*x*(f*x^2+e)^(1/2)/d/f
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {559, 396, 223, 212, 537, 385, 211}

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{(bc-ad)^2\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^2\sqrt{de-cf}} - \frac{b(bc-ad)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^2\sqrt{f}} - \frac{b(be-2af)\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2df^{3/2}} + \frac{b^2x\sqrt{e+fx^2}}{2df}$$

```
[In] Int[(a + b*x^2)^2/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (b^2*x*Sqrt[e + f*x^2])/(2*d*f) + ((b*c - a*d)^2*ArcTan[(Sqrt[d*e - c*f]*x)
/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*d^2*Sqrt[d*e - c*f]) - (b*(b*c - a*d)
*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(d^2*Sqrt[f]) - (b*(b*e - 2*a*f)*Arc
Tanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(2*d*f^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 559

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b\int \frac{a+bx^2}{\sqrt{e+fx^2}}\,dx}{d}+\frac{(-bc+ad)\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{d}\\
&= \frac{b^2x\sqrt{e+fx^2}}{2df}-\frac{(b(bc-ad))\int \frac{1}{\sqrt{e+fx^2}}\,dx}{d^2}\\
&\quad+\frac{(bc-ad)^2\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{d^2}-\frac{(b(be-2af))\int \frac{1}{\sqrt{e+fx^2}}\,dx}{2df}\\
&= \frac{b^2x\sqrt{e+fx^2}}{2df}-\frac{(b(bc-ad))\text{Subst}\left(\int \frac{1}{1-fx^2}\,dx,x,\frac{x}{\sqrt{e+fx^2}}\right)}{d^2}\\
&\quad+\frac{(bc-ad)^2\text{Subst}\left(\int \frac{1}{c-(-de+cf)x^2}\,dx,x,\frac{x}{\sqrt{e+fx^2}}\right)}{d^2}\\
&\quad-\frac{(b(be-2af))\text{Subst}\left(\int \frac{1}{1-fx^2}\,dx,x,\frac{x}{\sqrt{e+fx^2}}\right)}{2df}\\
&= \frac{b^2x\sqrt{e+fx^2}}{2df}+\frac{(bc-ad)^2\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d^2\sqrt{de-cf}}\\
&\quad-\frac{b(bc-ad)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d^2\sqrt{f}}-\frac{b(be-2af)\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{2df^{3/2}}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.91

$$
\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{\frac{b^2dx\sqrt{e+fx^2}}{f}-\frac{2(bc-ad)^2\arctan\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}x-\sqrt{e+fx^2}\right)}{\sqrt{c}\sqrt{de-cf}}\right)}{\sqrt{c}\sqrt{de-cf}}+\frac{b(bde+2bcf-4adf)\log\left(-\sqrt{f}x+\sqrt{e+fx^2}\right)}{f^{3/2}}}{2d^2}
$$

```
[In] Integrate[(a + b*x^2)^2/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((b^2*d*x*Sqrt[e + f*x^2])/f - (2*(b*c - a*d)^2*ArcTan[(c*Sqrt[f] + d*x*(Sq
rt[f]*x - Sqrt[e + f*x^2]))/(Sqrt[c]*Sqrt[d*e - c*f])])/(Sqrt[c]*Sqrt[d*e -
 c*f]) + (b*(b*d*e + 2*b*c*f - 4*a*d*f)*Log[-(Sqrt[f]*x) + Sqrt[e + f*x^2]]
)/f^(3/2))/(2*d^2)
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{f^{\frac{5}{2}}(ad-bc)^2\operatorname{arctanh}\left(\frac{c\sqrt{f x^2+e}}{x\sqrt{(cf-de)c}}\right)+\frac{\sqrt{(cf-de)c}\,b\left(\left((4ad-2bc)f^2-ebdf\right)\operatorname{arctanh}\left(\frac{\sqrt{f x^2+e}}{x\sqrt{f}}\right)+bd\sqrt{f x^2+e}\,x\,f^{\frac{3}{2}}\right)}{2}}{\sqrt{(cf-de)c}\,d^2 f^{\frac{5}{2}}}$
risch	$\frac{b^2x\sqrt{f x^2+e}}{2df}+\frac{\frac{b(4adf-2bcf-bde)\ln\left(\sqrt{f}\,x+\sqrt{f x^2+e}\right)}{d\sqrt{f}}-\frac{f\left(a^2d^2-2abcd+b^2c^2\right)\ln\left(\frac{-\frac{2(cf-de)}{d}+\frac{2f\sqrt{-cd}\left(x-\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{-\frac{cf-de}{d}}}{x-}\right)}{\sqrt{-cd}\,d\sqrt{-\frac{cf-de}{d}}}}{}$
default	$\frac{b\left(bd\left(\frac{x\sqrt{f x^2+e}}{2f}-\frac{e\ln\left(\sqrt{f}\,x+\sqrt{f x^2+e}\right)}{2f^{\frac{3}{2}}}\right)+\frac{2ad\ln\left(\sqrt{f}\,x+\sqrt{f x^2+e}\right)}{\sqrt{f}}-\frac{bc\ln\left(\sqrt{f}\,x+\sqrt{f x^2+e}\right)}{\sqrt{f}}\right)}{d^2}-\frac{\left(-a^2d^2+2abcd-b^2c^2\right)\ln\left(\right.}{}$

```
[In] int((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/((c*f-d*e)*c)^(1/2)*(f^(5/2)*(a*d-b*c)^2*arctanh(c*(f*x^2+e)^(1/2)/x/((c*
f-d*e)*c)^(1/2))+1/2*((c*f-d*e)*c)^(1/2)*b*(((4*a*d-2*b*c)*f^2-e*b*d*f)*arc
tanh((f*x^2+e)^(1/2)/x/f^(1/2))+b*d*(f*x^2+e)^(1/2)*x*f^(3/2)))/d^2/f^(5/2)
```

Fricas [A] (verification not implemented)

none

Time = 1.11 (sec) , antiderivative size = 1111, normalized size of antiderivative = 6.69

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$
$$= \left[-\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-cde+c^2f}\,f^2\log\left(\frac{(d^2e^2-8\,cdef+8\,c^2f^2)x^4+c^2e^2-2\,(3\,cde^2-4\,c^2ef)x^2-4\,((de-2\,cf)x^3-cex)\sqrt{-c}}{d^2x^4+2\,cdx^2+c^2}\right.}{}\right.$$
$$-\frac{(b^2c^2-2\,abcd+a^2d^2)\sqrt{-cde+c^2f}\,f^2\log\left(\frac{(d^2e^2-8\,cdef+8\,c^2f^2)x^4+c^2e^2-2\,(3\,cde^2-4\,c^2ef)x^2-4\,((de-2\,cf)x^3-cex)\sqrt{-cd}}{d^2x^4+2\,cdx^2+c^2}\right.}{}$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(-c*d*e + c^2*f)*f^2*log(((d^2*e
^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 -
```

```
4*((d*e - 2*c*f)*x^3 - c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x
^4 + 2*c*d*x^2 + c^2)) - 2*(b^2*c*d^2*e*f - b^2*c^2*d*f^2)*sqrt(f*x^2 + e)*
x + (b^2*c*d^2*e^2 + (b^2*c^2*d - 4*a*b*c*d^2)*e*f - 2*(b^2*c^3 - 2*a*b*c^2
*d)*f^2)*sqrt(f)*log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x - e))/(c*d^3*e*
f^2 - c^2*d^2*f^3), 1/4*(2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d*e - c^2
*f)*f^2*arctan(1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2
 + e)/((c*d*e*f - c^2*f^2)*x^3 + (c*d*e^2 - c^2*e*f)*x)) + 2*(b^2*c*d^2*e*f
 - b^2*c^2*d*f^2)*sqrt(f*x^2 + e)*x - (b^2*c*d^2*e^2 + (b^2*c^2*d - 4*a*b*c
*d^2)*e*f - 2*(b^2*c^3 - 2*a*b*c^2*d)*f^2)*sqrt(f)*log(-2*f*x^2 - 2*sqrt(f*
x^2 + e)*sqrt(f)*x - e))/(c*d^3*e*f^2 - c^2*d^2*f^3), -1/4*((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*sqrt(-c*d*e + c^2*f)*f^2*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2
*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3
- c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2))
 - 2*(b^2*c*d^2*e*f - b^2*c^2*d*f^2)*sqrt(f*x^2 + e)*x - 2*(b^2*c*d^2*e^2 +
 (b^2*c^2*d - 4*a*b*c*d^2)*e*f - 2*(b^2*c^3 - 2*a*b*c^2*d)*f^2)*sqrt(-f)*ar
ctan(sqrt(-f)*x/sqrt(f*x^2 + e)))/(c*d^3*e*f^2 - c^2*d^2*f^3), 1/2*((b^2*c^
2 - 2*a*b*c*d + a^2*d^2)*sqrt(c*d*e - c^2*f)*f^2*arctan(1/2*sqrt(c*d*e - c^
2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2 + e)/((c*d*e*f - c^2*f^2)*x^3 + (
c*d*e^2 - c^2*e*f)*x)) + (b^2*c*d^2*e*f - b^2*c^2*d*f^2)*sqrt(f*x^2 + e)*x
+ (b^2*c*d^2*e^2 + (b^2*c^2*d - 4*a*b*c*d^2)*e*f - 2*(b^2*c^3 - 2*a*b*c^2*d
)*f^2)*sqrt(-f)*arctan(sqrt(-f)*x/sqrt(f*x^2 + e)))/(c*d^3*e*f^2 - c^2*d^2*
f^3)]
```

Sympy [F]

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((b*x**2+a)**2/(d*x**2+c)/(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)**2/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^2}{(dx^2+c)\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)^2/((d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(b\,x^2+a)^2}{(d\,x^2+c)\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int((a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^2/((c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.60 $\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 91

$$\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = -\frac{(bc-ad)\arctan\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d\sqrt{de-cf}}+\frac{b\,\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}$$

```
[Out] b*arctanh(x*f^(1/2)/(f*x^2+e)^(1/2))/d/f^(1/2)-(-a*d+b*c)*arctan(x*(-c*f+d*
e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/d/c^(1/2)/(-c*f+d*e)^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {537, 223, 212, 385, 211}

$$\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{b\,\text{arctanh}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}-\frac{(bc-ad)\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d\sqrt{de-cf}}$$

```
[In] Int[(a + b*x^2)/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -(((b*c - a*d)*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt
[c]*d*Sqrt[d*e - c*f])) + (b*ArcTanh[(Sqrt[f]*x)/Sqrt[e + f*x^2]])/(d*Sqrt[
f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b\int \frac{1}{\sqrt{e+fx^2}}\,dx}{d}+\frac{(-bc+ad)\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{d}\\
&= \frac{b\text{Subst}\left(\int \frac{1}{1-fx^2}\,dx,x,\frac{x}{\sqrt{e+fx^2}}\right)}{d}+\frac{(-bc+ad)\text{Subst}\left(\int \frac{1}{c-(-de+cf)x^2}\,dx,x,\frac{x}{\sqrt{e+fx^2}}\right)}{d}\\
&= -\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}d\sqrt{de-cf}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e+fx^2}}\right)}{d\sqrt{f}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22

$$\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx=\frac{\frac{(bc-ad)\arctan\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}x-\sqrt{e+fx^2}\right)}{\sqrt{c}\sqrt{de-cf}}\right)}{\sqrt{c}\sqrt{de-cf}}-\frac{b\log\left(-\sqrt{f}x+\sqrt{e+fx^2}\right)}{\sqrt{f}}}{d}$$

```
[In] Integrate[(a + b*x^2)/((c + d*x^2)*Sqrt[e + f*x^2]),x]
[Out] (((b*c - a*d)*ArcTan[(c*Sqrt[f] + d*x*(Sqrt[f]*x - Sqrt[e + f*x^2]))/(Sqrt[
c]*Sqrt[d*e - c*f])])/(Sqrt[c]*Sqrt[d*e - c*f]) - (b*Log[-(Sqrt[f]*x) + Sqr
t[e + f*x^2]])/Sqrt[f])/d
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\frac{(ad-bc)\operatorname{arctanh}\left(\frac{c\sqrt{f\,x^2+e}}{x\sqrt{(cf-de)c}}\right)}{\sqrt{(cf-de)c}}+\frac{b\operatorname{arctanh}\left(\frac{\sqrt{f\,x^2+e}}{x\sqrt{f}}\right)}{\sqrt{f}}}{d}$
default	$\frac{b\ln\left(\sqrt{f}\,x+\sqrt{f\,x^2+e}\right)}{d\sqrt{f}}-\frac{(-ad+bc)\ln\left(\frac{-\frac{2(cf-de)}{d}-\frac{2f\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{-\frac{cf-de}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2 f-\frac{2f\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}}}{x+\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\,d\sqrt{-\frac{cf-de}{d}}}$

```
[In] int((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*((a*d-b*c)/((c*f-d*e)*c)^(1/2)*arctanh(c*(f*x^2+e)^(1/2)/x/((c*f-d*e)*c
)^(1/2))+b/f^(1/2)*arctanh((f*x^2+e)^(1/2)/x/f^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(75) = 150$.

Time = 0.79 (sec) , antiderivative size = 737, normalized size of antiderivative = 8.10

$$\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$
$$= \left[\frac{\sqrt{-cde+c^2f}(bc-ad)f\log\left(\frac{(d^2e^2-8\,cdef+8\,c^2f^2)x^4+c^2e^2-2\left(3\,cde^2-4\,c^2ef\right)x^2-4\left((de-2\,cf)x^3-cex\right)\sqrt{-cde+c^2f}\sqrt{fx^2+e}}{d^2x^4+2\,cdx^2+c^2}\right)}{4\left(cd^2ef-c^2df^2\right)}\right.$$
$$-\frac{\sqrt{cde-c^2f}(bc-ad)f\arctan\left(\frac{\sqrt{cde-c^2f}\left((de-2\,cf)x^2-ce\right)\sqrt{fx^2+e}}{2\left((cdef-c^2f^2)x^3+(cde^2-c^2ef)x\right)}\right)-(bcde-bc^2f)\sqrt{f}\log\left(-2\,fx^2-2\sqrt{fx\right.}}{2\left(cd^2ef-c^2df^2\right)}$$
$$-\frac{\sqrt{cde-c^2f}(bc-ad)f\arctan\left(\frac{\sqrt{cde-c^2f}\left((de-2\,cf)x^2-ce\right)\sqrt{fx^2+e}}{2\left((cdef-c^2f^2)x^3+(cde^2-c^2ef)x\right)}\right)+2\left(bcde-bc^2f\right)\sqrt{-f}\arctan\left(\frac{\sqrt{-f}x}{\sqrt{fx^2+e}}\right)}{2\left(cd^2ef-c^2df^2\right)}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-c*d*e + c^2*f)*(b*c - a*d)*f*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2*
f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3 -
 c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2))
+ 2*(b*c*d*e - b*c^2*f)*sqrt(f)*log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x
- e))/(c*d^2*e*f - c^2*d*f^2), -1/2*(sqrt(c*d*e - c^2*f)*(b*c - a*d)*f*arct
an(1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2 + e)/((c*d*
e*f - c^2*f^2)*x^3 + (c*d*e^2 - c^2*e*f)*x)) - (b*c*d*e - b*c^2*f)*sqrt(f)*
```

```
log(-2*f*x^2 - 2*sqrt(f*x^2 + e)*sqrt(f)*x - e))/(c*d^2*e*f - c^2*d*f^2), 1
/4*(sqrt(-c*d*e + c^2*f)*(b*c - a*d)*f*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^
2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3 - c
*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2)) -
4*(b*c*d*e - b*c^2*f)*sqrt(-f)*arctan(sqrt(-f)*x/sqrt(f*x^2 + e)))/(c*d^2*e
*f - c^2*d*f^2), -1/2*(sqrt(c*d*e - c^2*f)*(b*c - a*d)*f*arctan(1/2*sqrt(c*
d*e - c^2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2 + e)/((c*d*e*f - c^2*f^2)
*x^3 + (c*d*e^2 - c^2*e*f)*x)) + 2*(b*c*d*e - b*c^2*f)*sqrt(-f)*arctan(sqrt
(-f)*x/sqrt(f*x^2 + e)))/(c*d^2*e*f - c^2*d*f^2)]
```

Sympy [F]

$$\int \frac{a + bx^2}{(c + dx^2)\sqrt{e + fx^2}}\, dx = \int \frac{a + bx^2}{(c + dx^2)\sqrt{e + fx^2}}\, dx$$

```
[In] integrate((b*x**2+a)/(d*x**2+c)/(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{a + bx^2}{(c + dx^2)\sqrt{e + fx^2}}\, dx = \int \frac{bx^2 + a}{(dx^2 + c)\sqrt{fx^2 + e}}\, dx$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((b*x^2 + a)/((d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{a + bx^2}{(c + dx^2)\sqrt{e + fx^2}}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater Err
or: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{b\,x^2+a}{(d\,x^2+c)\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int((a + b*x^2)/((c + d*x^2)*(e + f*x^2)^(1/2)),x)
[Out] int((a + b*x^2)/((c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.61 $\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}$$

```
[Out] arctan(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/c^(1/2)/(-c*f+d*e)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {385, 211}

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}$$

```
[In] Int[1/((c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])]/(Sqrt[c]*Sqrt[d*e - c
*f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
```

```
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \text{Subst}\left(\int \frac{1}{c-(-de+cf)x^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}\sqrt{de-cf}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx = -\frac{\arctan\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}x-\sqrt{e+fx^2}\right)}{\sqrt{c}\sqrt{de-cf}}\right)}{\sqrt{c}\sqrt{de-cf}}$$

```
[In] Integrate[1/((c + d*x^2)*Sqrt[e + f*x^2]),x]
[Out] -(ArcTan[(c*Sqrt[f] + d*x*(Sqrt[f]*x - Sqrt[e + f*x^2]))/(Sqrt[c]*Sqrt[d*e
- c*f])]/(Sqrt[c]*Sqrt[d*e - c*f]))
```

Maple [A] (verified)

Time = 3.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\frac{c\sqrt{f\,x^2+e}}{x\sqrt{(cf-de)c}}\right)}{\sqrt{(cf-de)c}}$
default	$\frac{\ln\left(\frac{-\frac{2(cf-de)}{d}-\frac{2f\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}+2\sqrt{-\frac{cf-de}{d}}\sqrt{\left(x+\frac{\sqrt{-cd}}{d}\right)^2 f-\frac{2f\sqrt{-cd}\left(x+\frac{\sqrt{-cd}}{d}\right)}{d}-\frac{cf-de}{d}}}{x+\frac{\sqrt{-cd}}{d}}\right)}{2\sqrt{-cd}\sqrt{-\frac{cf-de}{d}}} - \ln\left(-\frac{2(cf-de)}{d}+2f\sqrt{-\right.$

```
[In] int(1/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/((c*f-d*e)*c)^(1/2)*arctanh(c*(f*x^2+e)^(1/2)/x/((c*f-d*e)*c)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(39) = 78$.

Time = 0.30 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.92

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \left[-\frac{\sqrt{-cde+c^2f}\log\left(\frac{(d^2e^2-8\,cdef+8\,c^2f^2)x^4+c^2e^2-2\,(3\,cde^2-4\,c^2ef)x^2-4\,((de-2\,cf)x^3-cex)\sqrt{-cde+c^2f}\sqrt{fx^2+e}}{d^2x^4+2\,cdx^2+c^2}\right)}{4\,(cde-c^2f)}, \frac{\arctan\left(\cdots\right.}{\cdots}\right.$$

```
[In] integrate(1/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-c*d*e + c^2*f)*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2
*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3 - c*e*x)*sqrt(-
c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2))/(c*d*e - c^2*f
), 1/2*arctan(1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2
+ e)/((c*d*e*f - c^2*f^2)*x^3 + (c*d*e^2 - c^2*e*f)*x))/sqrt(c*d*e - c^2*f)
]
```

Sympy [F]

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx$$

```
[In] integrate(1/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{1}{(dx^2+c)\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx = -\frac{\sqrt{f}\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2d-de+2\,cf}{2\sqrt{cdef-c^2f^2}}\right)}{\sqrt{cdef-c^2f^2}}$$

```
[In] integrate(1/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] -sqrt(f)*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d - d*e + 2*c*f)/sqrt(
c*d*e*f - c^2*f^2))/sqrt(c*d*e*f - c^2*f^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{x\sqrt{d\,e-c\,f}}{\sqrt{c}\sqrt{f\,x^2+e}}\right)}{\sqrt{-c\,(c\,f-d\,e)}} & \text{if } 0<d\,e-c\,f \\ \frac{\ln\left(\frac{\sqrt{c\,(f\,x^2+e)}+x\,\sqrt{c\,f-d\,e}}{\sqrt{c\,(f\,x^2+e)}-x\,\sqrt{c\,f-d\,e}}\right)}{2\sqrt{c\,(c\,f-d\,e)}} & \text{if } d\,e-c\,f<0 \\ \int \frac{1}{(d\,x^2+c)\,\sqrt{f\,x^2+e}}\,dx & \text{if } d\,e-c\,f\notin\mathbb{R}\vee c\,f=d\,e \end{cases}$$

```
[In] int(1/((c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] piecewise(0 < - c*f + d*e, atan((x*(- c*f + d*e)^(1/2))/(c^(1/2)*(e + f*x^2
)^(1/2)))/(-c*(c*f - d*e))^(1/2), - c*f + d*e < 0, log(((c*(e + f*x^2))^(1/
2) + x*(c*f - d*e)^(1/2))/((c*(e + f*x^2))^(1/2) - x*(c*f - d*e)^(1/2)))/(2
*(c*(c*f - d*e))^(1/2)), ~in(- c*f + d*e, 'real') | c*f == d*e, int(1/((c +
 d*x^2)*(e + f*x^2)^(1/2)), x))
```

3.62 $\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 122

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{b\arctan\left(\frac{\sqrt{be-af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right)}{\sqrt{a}(bc-ad)\sqrt{be-af}} - \frac{d\arctan\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)\sqrt{de-cf}}$$

```
[Out] b*arctan(x*(-a*f+b*e)^(1/2)/a^(1/2)/(f*x^2+e)^(1/2))/(-a*d+b*c)/a^(1/2)/(-a
*f+b*e)^(1/2)-d*arctan(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/(-a*d+b*
c)/c^(1/2)/(-c*f+d*e)^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {546, 385, 211}

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{b\arctan\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)}{\sqrt{a}(bc-ad)\sqrt{be-af}} - \frac{d\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)\sqrt{de-cf}}$$

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (b*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])])/(Sqrt[a]*(b*c - a
*d)*Sqrt[b*e - a*f]) - (d*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^
2])])/(Sqrt[c]*(b*c - a*d)*Sqrt[d*e - c*f])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 546

```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)^2
]), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/((a + b*x^2)*Sqrt[e + f*x^2]), x
], x] - Dist[d/(b*c - a*d), Int[1/((c + d*x^2)*Sqrt[e + f*x^2]), x], x] /;
FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\int \frac{1}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{bc-ad} - \frac{d\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{bc-ad} \\ &= \frac{b\text{Subst}\left(\int \frac{1}{a-(-be+af)x^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{bc-ad} - \frac{d\text{Subst}\left(\int \frac{1}{c-(-de+cf)x^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{bc-ad} \\ &= \frac{b\tan^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right)}{\sqrt{a}(bc-ad)\sqrt{be-af}} - \frac{d\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)\sqrt{de-cf}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.25

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{-\frac{b\arctan\left(\frac{a\sqrt{f}+bx\left(\sqrt{f}x-\sqrt{e+fx^2}\right)}{\sqrt{a}\sqrt{be-af}}\right)}{\sqrt{a}\sqrt{be-af}} + \frac{d\arctan\left(\frac{c\sqrt{f}+dx\left(\sqrt{f}x-\sqrt{e+fx^2}\right)}{\sqrt{c}\sqrt{de-cf}}\right)}{\sqrt{c}\sqrt{de-cf}}}{bc-ad}$$

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (-((b*ArcTan[(a*Sqrt[f] + b*x*(Sqrt[f]*x - Sqrt[e + f*x^2]))/(Sqrt[a]*Sqrt[
b*e - a*f])])/(Sqrt[a]*Sqrt[b*e - a*f])) + (d*ArcTan[(c*Sqrt[f] + d*x*(Sqrt
[f]*x - Sqrt[e + f*x^2]))/(Sqrt[c]*Sqrt[d*e - c*f])])/(Sqrt[c]*Sqrt[d*e - c
*f]))/(b*c - a*d)
```

Maple [A] (verified)

Time = 3.43 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$\frac{d\,\mathrm{arctanh}\left(\frac{c\sqrt{f\,x^{2}+e}}{x\sqrt{(cf-de)c}}\right)\sqrt{(af-be)a}-b\,\mathrm{arctanh}\left(\frac{\sqrt{f\,x^{2}+e}\,a}{x\sqrt{(af-be)a}}\right)\sqrt{(cf-de)c}}{(ad-bc)\sqrt{(cf-de)c}\,\sqrt{(af-be)a}}$
default	$-\frac{b^{2}d\ln\left(\frac{-\frac{2(af-be)}{b}+\frac{2f\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}+2\sqrt{-\frac{af-be}{b}}\sqrt{\left(x-\frac{\sqrt{-ab}}{b}\right)^{2}f+\frac{2f\sqrt{-ab}\left(x-\frac{\sqrt{-ab}}{b}\right)}{b}-\frac{af-be}{b}}}{x-\frac{\sqrt{-ab}}{b}}\right)}{2\sqrt{-ab}\left(b\sqrt{-cd}+d\sqrt{-ab}\right)\left(d\sqrt{-ab}-b\sqrt{-cd}\right)\sqrt{-\frac{af-be}{b}}}+\frac{b^{2}d\ln\left(\frac{-\frac{2(af-b}{b}}{}\right.}{}$

[In] int(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)

[Out] (d*arctanh(c*(f*x^2+e)^(1/2)/x/((c*f-d*e)*c)^(1/2))*((a*f-b*e)*a)^(1/2)-b*arctanh((f*x^2+e)^(1/2)/x*a/((a*f-b*e)*a)^(1/2))*((c*f-d*e)*c)^(1/2))/(a*d-b*c)/((c*f-d*e)*c)^(1/2)/((a*f-b*e)*a)^(1/2)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(102) = 204.

Time = 61.43 (sec) , antiderivative size = 1305, normalized size of antiderivative = 10.70

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = \text{Too large to display}$$

[In] integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")

[Out] [1/4*((b*c*d*e - b*c^2*f)*sqrt(-a*b*e + a^2*f)*log(((b^2*e^2 - 8*a*b*e*f + 8*a^2*f^2)*x^4 + a^2*e^2 - 2*(3*a*b*e^2 - 4*a^2*e*f)*x^2 + 4*((b*e - 2*a*f)*x^3 - a*e*x)*sqrt(-a*b*e + a^2*f)*sqrt(f*x^2 + e))/(b^2*x^4 + 2*a*b*x^2 + a^2)) + (a*b*d*e - a^2*d*f)*sqrt(-c*d*e + c^2*f)*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3 - c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((a*b^2*c^2*d - a^2*b*c*d^2)*e^2 - (a*b^2*c^3 - a^3*c*d^2)*e*f + (a^2*b*c^3 - a^3*c^2*d)*f^2), 1/4*(2*(b*c*d*e - b*c^2*f)*sqrt(a*b*e - a^2*f)*arctan(1/2*sqrt(a*b*e - a^2*f)*((b*e - 2*a*f)*x^2 - a*e)*sqrt(f*x^2 + e)/((a*b*e*f - a^2*f^2)*x^3 + (a*b*e^2 - a^2*e*f)*x)) + (a*b*d*e - a^2*d*f)*sqrt(-c*d*e + c^2*f)*log(((d^2*e^2 - 8*c*d*e*f + 8*c^2*f^2)*x^4 + c^2*e^2 - 2*(3*c*d*e^2 - 4*c^2*e*f)*x^2 - 4*((d*e - 2*c*f)*x^3 - c*e*x)*sqrt(-c*d*e + c^2*f)*sqrt(f*x^2 + e))/(d^2*x^4 + 2*c*d*x^2 + c^2)))/((a*b^2*c^2*d - a^2*b*c*d^2)*e^2 - (a*b^2*c^3 - a^3*c*d^2)*e*f + (a^2*b*c^3 - a^3*c^2*d)*f^2), -1/4*(2*(a*b*d*e - a^2*d*f)*sqrt(c*d*e - c^2*f)*arctan(1/2*sqrt(c*d*e - c^2*f

```
)*((d*e - 2*c*f)*x^2 - c*e)*sqrt(f*x^2 + e)/((c*d*e*f - c^2*f^2)*x^3 + (c*d
*e^2 - c^2*e*f)*x)) - (b*c*d*e - b*c^2*f)*sqrt(-a*b*e + a^2*f)*log(((b^2*e^
2 - 8*a*b*e*f + 8*a^2*f^2)*x^4 + a^2*e^2 - 2*(3*a*b*e^2 - 4*a^2*e*f)*x^2 +
4*((b*e - 2*a*f)*x^3 - a*e*x)*sqrt(-a*b*e + a^2*f)*sqrt(f*x^2 + e))/(b^2*x^
4 + 2*a*b*x^2 + a^2)))/((a*b^2*c^2*d - a^2*b*c*d^2)*e^2 - (a*b^2*c^3 - a^3*
c*d^2)*e*f + (a^2*b*c^3 - a^3*c^2*d)*f^2), 1/2*((b*c*d*e - b*c^2*f)*sqrt(a*
b*e - a^2*f)*arctan(1/2*sqrt(a*b*e - a^2*f)*((b*e - 2*a*f)*x^2 - a*e)*sqrt(
f*x^2 + e)/((a*b*e*f - a^2*f^2)*x^3 + (a*b*e^2 - a^2*e*f)*x)) - (a*b*d*e -
a^2*d*f)*sqrt(c*d*e - c^2*f)*arctan(1/2*sqrt(c*d*e - c^2*f)*((d*e - 2*c*f)*
x^2 - c*e)*sqrt(f*x^2 + e)/((c*d*e*f - c^2*f^2)*x^3 + (c*d*e^2 - c^2*e*f)*x
)))/((a*b^2*c^2*d - a^2*b*c*d^2)*e^2 - (a*b^2*c^3 - a^3*c*d^2)*e*f + (a^2*b
*c^3 - a^3*c^2*d)*f^2)]
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx=\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)/(f*x**2+e)**(1/2),x)
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx=\int \frac{1}{(bx^2+a)(dx^2+c)\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [A] (verification not implemented)

none

Time = 0.34 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.35

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = -f^{\frac{3}{2}}\left(\frac{b\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2 b-be+2\,af}{2\,\sqrt{abef-a^2f^2}}\right)}{\sqrt{abef-a^2f^2}(bcf-adf)}-\frac{d\arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2 d-de+2\,cf}{2\,\sqrt{cdef-c^2f^2}}\right)}{\sqrt{cdef-c^2f^2}(bcf-adf)}\right)$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
```

```
[Out] -f^(3/2)*(b*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*b - b*e + 2*a*f)/sq
rt(a*b*e*f - a^2*f^2))/(sqrt(a*b*e*f - a^2*f^2)*(b*c*f - a*d*f)) - d*arctan
(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d - d*e + 2*c*f)/sqrt(c*d*e*f - c^2*f
^2))/(sqrt(c*d*e*f - c^2*f^2)*(b*c*f - a*d*f)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)\,(d\,x^2+c)\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int(1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)*(c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.63 $\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 203

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{b^2x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)(a+bx^2)} + \frac{b(b^2ce-3abde-2abcf+4a^2df)\arctan\left(\frac{\sqrt{be-af}\,x}{\sqrt{a}\sqrt{e+fx^2}}\right)}{2a^{3/2}(bc-ad)^2(be-af)^{3/2}} + \frac{d^2\arctan\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}$$

```
[Out] 1/2*b*(4*a^2*d*f-2*a*b*c*f-3*a*b*d*e+b^2*c*e)*arctan(x*(-a*f+b*e)^(1/2)/a^(1/2)/(f*x^2+e)^(1/2))/a^(3/2)/(-a*d+b*c)^2/(-a*f+b*e)^(3/2)+d^2*arctan(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2))/(-a*d+b*c)^2/c^(1/2)/(-c*f+d*e)^(1/2)+1/2*b^2*x*(f*x^2+e)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {560, 385, 211, 541, 12}

$$\int \frac{1}{(a+bx^2)^2(c+dx^2)\sqrt{e+fx^2}}\,dx = \frac{b\arctan\left(\frac{x\sqrt{be-af}}{\sqrt{a}\sqrt{e+fx^2}}\right)(4a^2df-2abcf-3abde+b^2ce)}{2a^{3/2}(bc-ad)^2(be-af)^{3/2}} + \frac{d^2\arctan\left(\frac{x\sqrt{de-cf}}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}} + \frac{b^2x\sqrt{e+fx^2}}{2a(a+bx^2)(bc-ad)(be-af)}$$

```
[In] Int[1/((a + b*x^2)^2*(c + d*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (b^2*x*Sqrt[e + f*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*(a + b*x^2)) + (b*(b^2
*c*e - 3*a*b*d*e - 2*a*b*c*f + 4*a^2*d*f)*ArcTan[(Sqrt[b*e - a*f]*x)/(Sqrt[
a]*Sqrt[e + f*x^2])])/(2*a^(3/2)*(b*c - a*d)^2*(b*e - a*f)^(3/2)) + (d^2*Ar
cTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*(b*c - a*d)^2
*Sqrt[d*e - c*f])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\text{integral} = -\frac{b\int \frac{-bc+2ad+bdx^2}{(a+bx^2)^2\sqrt{e+fx^2}}\,dx}{(bc-ad)^2}+\frac{d^2\int \frac{1}{(c+dx^2)\sqrt{e+fx^2}}\,dx}{(bc-ad)^2}$$

$$= \frac{b^2 x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)} + \frac{d^2 \text{Subst}\left(\int \frac{1}{c-(-de+cf)x^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{(bc-ad)^2} + \frac{b\int \frac{b^2ce-3abde-2abcf+4a^2df}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{2a(bc-ad)^2(be-af)}$$

$$= \frac{b^2 x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)} + \frac{d^2 \tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}} + \frac{(b(b^2ce-3abde-2abcf+4a^2df))\int \frac{1}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{2a(bc-ad)^2(be-af)}$$

$$= \frac{b^2 x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)} + \frac{d^2 \tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}} + \frac{(b(b^2ce-3abde-2abcf+4a^2df))\,\text{Subst}\left(\int \frac{1}{a-(-be+af)x^2}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{2a(bc-ad)^2(be-af)}$$

$$= \frac{b^2 x\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)} + \frac{b(b^2ce-3abde-2abcf+4a^2df)\tan^{-1}\left(\frac{\sqrt{be-af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)}{2a^{3/2}(bc-ad)^2(be-af)^{3/2}} + \frac{d^2 \tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}$$

Mathematica [A] (verified)

Time = 15.33 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(a+bx^2\right)^2\left(c+dx^2\right)\sqrt{e+fx^2}}\,dx = \frac{1}{2}\left(\frac{b^2 x\sqrt{e+fx^2}}{a(-bc+ad)(-be+af)\left(a+bx^2\right)} + \frac{b(b^2ce+4a^2df-ab(3de+2cf))\arctan\left(\frac{\sqrt{be-af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)}{a^{3/2}(bc-ad)^2(be-af)^{3/2}} + \frac{2d^2\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)}{\sqrt{c}(bc-ad)^2\sqrt{de-cf}}\right)$$

```
[In] Integrate[1/((a + b*x^2)^2*(c + d*x^2)*Sqrt[e + f*x^2]),x]
[Out] ((b^2*x*Sqrt[e + f*x^2])/(a*(-(b*c) + a*d)*(-(b*e) + a*f)*(a + b*x^2)) + (b
*(b^2*c*e + 4*a^2*d*f - a*b*(3*d*e + 2*c*f))*ArcTan[(Sqrt[b*e - a*f]*x)/(Sq
rt[a]*Sqrt[e + f*x^2])])/(a^(3/2)*(b*c - a*d)^2*(b*e - a*f)^(3/2)) + (2*d^2
*ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])])/(Sqrt[c]*(b*c - a*d
)^2*Sqrt[d*e - c*f]))/2
```

Maple [A] (verified)

Time = 3.56 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.83

method	result	size
pseudoelliptic	$\frac{\frac{d^{2} \operatorname{arctanh}\left(\frac{c \sqrt{f\,x^{2}+e}}{x \sqrt{(c f-d e) c}}\right)}{\sqrt{(c f-d e) c}}+\frac{b \left(\frac{b (a d-b c) \sqrt{f\,x^{2}+e}\,x}{b\,x^{2}+a}-\frac{\left(4 a^{2} d f-2 a c f b-3 a b d e+b^{2} c e\right) \operatorname{arctanh}\left(\frac{\sqrt{f\,x^{2}+e}\,a}{x \sqrt{(a f-b e) a}}\right)}{\sqrt{(a f-b e) a}}\right)}{2 (a f-b e) a}}{(a d-b c)^{2}}$	169
default	Expression too large to display	1400

```
[In] int(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a*d-b*c)^2*(d^2/((c*f-d*e)*c)^(1/2)*arctanh(c*(f*x^2+e)^(1/2)/x/((c*f-d*
e)*c)^(1/2))+1/2*b/(a*f-b*e)/a*(b*(a*d-b*c)*(f*x^2+e)^(1/2)*x/(b*x^2+a)-(4*
a^2*d*f-2*a*b*c*f-3*a*b*d*e+b^2*c*e)/((a*f-b*e)*a)^(1/2)*arctanh((f*x^2+e)^
(1/2)/x*a/((a*f-b*e)*a)^(1/2))))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2) \sqrt{e+fx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2) \sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^2 (c+dx^2) \sqrt{e+fx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)**2*(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2) \sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^2 (dx^2+c) \sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate(1/((b*x^2 + a)^2*(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(177) = 354$.

Time = 4.55 (sec) , antiderivative size = 458, normalized size of antiderivative = 2.26

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2) \sqrt{e+fx^2}}\,dx = -\frac{1}{2}\left(\frac{2\,d^2 \arctan\left(\frac{\left(\sqrt{f}x-\sqrt{fx^2+e}\right)^2 d-de+2\,cf}{2\,\sqrt{cdef-c^2f^2}}\right)}{(b^2c^2f^2-2\,abcdf^2+a^2d^2f^2)\sqrt{cdef-c^2f^2}} + \frac{(b^3ce-3\,ab^2de-2\,ab^2cf+4\,a^2bdf)\arctan\left(\right.}{(ab^3c^2ef^2-2\,a^2b^2cdef^2+a^3bd^2ef^2-a^2b^2c^2f^3+2\,a\right.}$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] -1/2*(2*d^2*arctan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*d - d*e + 2*c*f)/sq
rt(c*d*e*f - c^2*f^2))/((b^2*c^2*f^2 - 2*a*b*c*d*f^2 + a^2*d^2*f^2)*sqrt(c*
d*e*f - c^2*f^2)) + (b^3*c*e - 3*a*b^2*d*e - 2*a*b^2*c*f + 4*a^2*b*d*f)*arc
tan(1/2*((sqrt(f)*x - sqrt(f*x^2 + e))^2*b - b*e + 2*a*f)/sqrt(a*b*e*f - a^
2*f^2))/((a*b^3*c^2*e*f^2 - 2*a^2*b^2*c*d*e*f^2 + a^3*b*d^2*e*f^2 - a^2*b^2
*c^2*f^3 + 2*a^3*b*c*d*f^3 - a^4*d^2*f^3)*sqrt(a*b*e*f - a^2*f^2)) + 2*((sq
rt(f)*x - sqrt(f*x^2 + e))^2*b^2*e - 2*(sqrt(f)*x - sqrt(f*x^2 + e))^2*a*b*
f - b^2*e^2)/((a*b^2*c*e*f^2 - a^2*b*d*e*f^2 - a^2*b*c*f^3 + a^3*d*f^3)*((s
qrt(f)*x - sqrt(f*x^2 + e))^4*b - 2*(sqrt(f)*x - sqrt(f*x^2 + e))^2*b*e + 4
*(sqrt(f)*x - sqrt(f*x^2 + e))^2*a*f + b*e^2)))*f^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2 (c+dx^2) \sqrt{e+fx^2}} \, dx = \int \frac{1}{\left(b\,x^2+a\right)^2 \,\left(d\,x^2+c\right)\,\sqrt{f\,x^2+e}} \, dx$$

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)*(e + f*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^2*(c + d*x^2)*(e + f*x^2)^(1/2)), x)
```

3.64 $\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 608

$$
\begin{aligned}
\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx &= \frac{d\left(7ce-\frac{2de^2}{f}+\frac{3c^2f}{d}\right)x\sqrt{c+dx^2}}{15b\sqrt{e+fx^2}} \\
&+ \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^3\sqrt{e+fx^2}} + \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} \\
&- \frac{2d(de-3cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15bf} + \frac{d^2x\sqrt{c+dx^2}\left(e+fx^2\right)^{3/2}}{5bf} \\
&- \frac{\sqrt{e}\left(15a^2d^2f^2-5abdf(de+7cf)+b^2(-2d^2e^2+12cdef+23c^2f^2)\right)\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15b^3f^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{de^{3/2}\left(-40abcdf+15a^2d^2f+b^2c(-de+34cf)\right)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15b^3cf^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{(bc-ad)^3e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ab^3c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

```
[Out] 1/5*d^2*x*(f*x^2+e)^(3/2)*(d*x^2+c)^(1/2)/b/f+1/15*d*(7*c*e-2*d*e^2/f+3*c^2
*f/d)*x*(d*x^2+c)^(1/2)/b/(f*x^2+e)^(1/2)+1/3*(-a*d+b*c)*(-3*a*d*f+4*b*c*f+
b*d*e)*x*(d*x^2+c)^(1/2)/b^3/(f*x^2+e)^(1/2)+1/15*d*e^(3/2)*(-40*a*b*c*d*f+
15*a^2*d^2*f+b^2*c*(34*c*f-d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*El
lipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1
/2)/b^3/c/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/15*(15*
a^2*d^2*f^2-5*a*b*d*f*(7*c*f+d*e)+b^2*(23*c^2*f^2+12*c*d*e*f-2*d^2*e^2))*(1
/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/
```

```
e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b^3/f^(3/2)/(e*(d*x^2+c
)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+(-a*d+b*c)^3*e^(3/2)*(1/(1+f*x^2/e))^(
1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e
/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/b^3/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x
^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*(-a*d+b*c)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(
1/2)/b^2-2/15*d*(-3*c*f+d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/f
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 776, normalized size of antiderivative = 1.28, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {559, 427, 542, 545, 429, 506, 422, 557, 553}

$$\begin{aligned}
\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = {} & \frac{de^{3/2}\sqrt{c+dx^2}(5bc-3ad)(bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3b^3c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} \\
& + \frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)^3\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ab^3c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} \\
& - \frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(-3adf+4bcf+bde)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} \\
& + \frac{x\sqrt{c+dx^2}(bc-ad)(-3adf+4bcf+bde)}{3b^3\sqrt{e+fx^2}} + \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)}{3b^2} \\
& + \frac{\sqrt{e}\sqrt{c+dx^2}(-3c^2f^2-7cdef+2d^2e^2)\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15bf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} \\
& - \frac{de^{3/2}\sqrt{c+dx^2}(de-9cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15bf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} \\
& + \frac{dx\sqrt{c+dx^2}\left(\frac{3c^2f}{d}+7ce-\frac{2de^2}{f}\right)}{15b\sqrt{e+fx^2}} + \frac{d^2x\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5bf} \\
& - \frac{2dx\sqrt{c+dx^2}\sqrt{e+fx^2}(de-3cf)}{15bf}
\end{aligned}$$

```
[In] Int[((c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] (d*(7*c*e - (2*d*e^2)/f + (3*c^2*f)/d)*x*Sqrt[c + d*x^2])/(15*b*Sqrt[e + f*
x^2]) + ((b*c - a*d)*(b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^3*
Sqrt[e + f*x^2]) + (d*(b*c - a*d)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b^2
) - (2*d*(d*e - 3*c*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*b*f) + (d^2*x
```

```
*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(5*b*f) - ((b*c - a*d)*Sqrt[e]*(b*d*e +
 4*b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]],
1 - (d*e)/(c*f)])/(3*b^3*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2]) + (Sqrt[e]*(2*d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2)*Sqrt[c + d*x^2]
*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b*f^(3/2)*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(5*b*c - 3*a*d)*(b
*c - a*d)*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
- (d*e)/(c*f)])/(3*b^3*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt
[e + f*x^2]) - (d*e^(3/2)*(d*e - 9*c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(S
qrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c
*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)^3*e^(3/2)*Sqrt[c + d*x^2]*El
lipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a
*b^3*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 557

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[(2*b*c - a*d + b*d*x^2)*(Sqrt
[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rule 559

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\int (c+dx^2)^{3/2}\sqrt{e+fx^2}\,dx}{b} + \frac{(bc-ad)\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx}{b} \\
&= \frac{d^2x\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5bf} + \frac{(d(bc-ad))\int \frac{(2bc-ad+bdx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b^3} \\
&\quad + \frac{(bc-ad)^3\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b^3} + \frac{d\int \frac{\sqrt{e+fx^2}(-c(de-5cf)-2d(de-3cf)x^2)}{\sqrt{c+dx^2}}\,dx}{5bf}
\end{aligned}$$

$$= \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} - \frac{2d(de-3cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15bf} + \frac{d^2x\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5bf} + \frac{(bc-ad)^3e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^3c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(bc-ad)\int\frac{d(5bc-3ad)e+d(bde+4bcf-3adf)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3b^3} + \frac{\int\frac{-cde(de-9cf)-d(2d^2e^2-7cdef-3c^2f^2)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15bf}$$

$$= \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} - \frac{2d(de-3cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15bf} + \frac{d^2x\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5bf} + \frac{(bc-ad)^3e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^3c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(d(5bc-3ad)(bc-ad)e)\int\frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3b^3} - \frac{(cde(de-9cf))\int\frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15bf} + \frac{(d(bc-ad)(bde+4bcf-3adf))\int\frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3b^3} - \frac{(d(2d^2e^2-7cdef-3c^2f^2))\int\frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15bf}$$

$$= \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^3\sqrt{e+fx^2}}$$
$$- \frac{(2d^2e^2-7cdef-3c^2f^2)\,x\sqrt{c+dx^2}}{15bf\sqrt{e+fx^2}} + \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2}$$
$$- \frac{2d(de-3cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15bf} + \frac{d^2x\sqrt{c+dx^2}(e+fx^2)^{3/2}}{5bf}$$
$$+ \frac{d(5bc-3ad)(bc-ad)e^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{de^{3/2}(de-9cf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15bf^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{(bc-ad)^3e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^3c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{((bc-ad)e(bde+4bcf-3adf))\int\frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{3b^3}$$
$$+ \frac{(e(2d^2e^2-7cdef-3c^2f^2))\int\frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{15bf}$$

$$
\begin{aligned}
&= \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^3\sqrt{e+fx^2}} \\
&\quad - \frac{(2d^2e^2-7cdef-3c^2f^2)\,x\sqrt{c+dx^2}}{15bf\sqrt{e+fx^2}} + \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} \\
&\quad - \frac{2d(de-3cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15bf} + \frac{d^2x\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}{5bf} \\
&\quad - \frac{(bc-ad)\sqrt{e}(bde+4bcf-3adf)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{\sqrt{e}(2d^2e^2-7cdef-3c^2f^2)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15bf^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{d(5bc-3ad)(bc-ad)e^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{de^{3/2}(de-9cf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15bf^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{(bc-ad)^3e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^3c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.60 (sec) , antiderivative size = 456, normalized size of antiderivative = 0.75

$$
\int \frac{(c+dx^2)^{5/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \frac{-iabde(15a^2d^2f^2-5abdf(de+7cf)+b^2(-2d^2e^2+12cdef+23c^2f^2))\sqrt{1-}}{}
$$

```
[In] Integrate[((c + d*x^2)^(5/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] ((-I)*a*b*d*e*(15*a^2*d^2*f^2 - 5*a*b*d*f*(d*e + 7*c*f) + b^2*(-2*d^2*e^2 +
 12*c*d*e*f + 23*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elliptic
E[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(45*a^2*b*c*d^2*f^3 - 15*a^3*d
^3*f^3 + 5*a*b^2*d*f*(d^2*e^2 - c*d*e*f - 9*c^2*f^2) + b^3*(2*d^3*e^3 - 13*
c*d^2*e^2*f + 11*c^2*d*e*f^2 + 15*c^3*f^3))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f
*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + f*(a*b^2*d*Sqrt[d
/c]*x*(c + d*x^2)*(e + f*x^2)*(11*b*c*f - 5*a*d*f + b*d*(e + 3*f*x^2)) - (1
5*I)*(b*c - a*d)^3*f*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*El
```

```
lipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(15*a*b^4*Sqrt
[d/c]*f^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 8.06 (sec) , antiderivative size = 657, normalized size of antiderivative = 1.08

method	result
risch	$-\frac{xd\left(-3bdf\,x^{2}+5adf-11bcf-bde\right)\sqrt{d\,x^{2}+c}\,\sqrt{f\,x^{2}+e}}{15f\,b^{2}}+\frac{\left(-\frac{d\left(15a^{2}d^{2}f^{2}-35abcd\,f^{2}-5ab\,d^{2}ef+23b^{2}c^{2}f^{2}+12b^{2}cdef-2b^{2}d^{2}e^{2}\right)e\sqrt{1+\frac{d\,x^{2}}{c}}}{b\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+cf\,x^{2}}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

```
[In] int((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*x*d*(-3*b*d*f*x^2+5*a*d*f-11*b*c*f-b*d*e)*(d*x^2+c)^(1/2)*(f*x^2+e)^(
1/2)/f/b^2+1/15/f/b^2*(-1/b*d*(15*a^2*d^2*f^2-35*a*b*c*d*f^2-5*a*b*d^2*e*f+
23*b^2*c^2*f^2+12*b^2*c*d*e*f-2*b^2*d^2*e^2)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/
2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d
/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/
e/d)^(1/2)))-(15*a^3*d^3*f^2-45*a^2*b*c*d^2*f^2-15*a^2*b*d^3*e*f+45*a*b^2*c
^2*d*f^2+40*a*b^2*c*d^2*e*f-15*b^3*c^3*f^2-34*b^3*c^2*d*e*f+b^3*c*d^2*e^2)/
b^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x
^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+15*(a^4*d^
3*f-3*a^3*b*c*d^2*f-a^3*b*d^3*e+3*a^2*b^2*c^2*d*f+3*a^2*b^2*c*d^2*e-a*b^3*c
^3*f-3*a*b^3*c^2*d*e+b^4*c^3*e)*f/b^2/a/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f
*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2)
,b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2)))*((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^
(1/2)/(f*x^2+e)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\left(c+dx^{2}\right)^{5/2}\sqrt{e+fx^{2}}}{a+bx^{2}}\,dx=\text{Timed out}$$

```
[In] integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(c + dx^2)^{5/2} \sqrt{e + fx^2}}{a + bx^2} dx = \int \frac{(c + dx^2)^{\frac{5}{2}} \sqrt{e + fx^2}}{a + bx^2} dx$$

```
[In] integrate((d*x**2+c)**(5/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
[Out] Integral((c + d*x**2)**(5/2)*sqrt(e + f*x**2)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c + dx^2)^{5/2} \sqrt{e + fx^2}}{a + bx^2} dx = \int \frac{(dx^2 + c)^{\frac{5}{2}} \sqrt{fx^2 + e}}{bx^2 + a} dx$$

```
[In] integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c + dx^2)^{5/2} \sqrt{e + fx^2}}{a + bx^2} dx = \int \frac{(dx^2 + c)^{\frac{5}{2}} \sqrt{fx^2 + e}}{bx^2 + a} dx$$

```
[In] integrate((d*x^2+c)^(5/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c + dx^2)^{5/2} \sqrt{e + fx^2}}{a + bx^2} dx = \int \frac{(d\,x^2 + c)^{5/2}\,\sqrt{f\,x^2 + e}}{b\,x^2 + a} dx$$

```
[In] int(((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2))/(a + b*x^2),x)
[Out] int(((c + d*x^2)^(5/2)*(e + f*x^2)^(1/2))/(a + b*x^2), x)
```

3.65 $\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 400

$$\begin{aligned}\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx &= \frac{(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^2\sqrt{e+fx^2}} \\ &+ \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} \\ &- \frac{\sqrt{e}(bde+4bcf-3adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{d(5bc-3ad)e^{3/2}\sqrt{c+dx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3b^2c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{(bc-ad)^2e^{3/2}\sqrt{c+dx^2}\operatorname{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ab^2c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

```
[Out] 1/3*(-3*a*d*f+4*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)/b^2/(f*x^2+e)^(1/2)+1/3*d*(-
3*a*d+5*b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^
(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/b^2/c/f^
(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+(-a*d+b*c)^2*e^(3/2)*
(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(1+f*x
^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/b^2/c/f^(1/2)/(e
*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(-3*a*d*f+4*b*c*f+b*d*e)*
(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^
2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b^2/f^(1/2)/(e*(d*x^2
+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1
/2)/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {557, 553, 542, 545, 429, 506, 422}

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \frac{de^{3/2}\sqrt{c+dx^2}(5bc-3ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3b^2c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)^2\operatorname{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ab^2c\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{\sqrt{e}\sqrt{c+dx^2}(-3adf+4bcf+bde)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{x\sqrt{c+dx^2}(-3adf+4bcf+bde)}{3b^2\sqrt{e+fx^2}}+\frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b}$$

```
[In] Int[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] ((b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^2*Sqrt[e + f*x^2]) + (
d*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b) - (Sqrt[e]*(b*d*e + 4*b*c*f - 3*
a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*
f)])/(3*b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
+ (d*(5*b*c - 3*a*d)*e^(3/2)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(3*b^2*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*
x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi
[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*b^2*c*S
qrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 557

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[(2*b*c - a*d + b*d*x^2)*(Sqrt
[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rubi steps

$$\text{integral} = \frac{d\int \frac{(2bc-ad+bdx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b^2} + \frac{(bc-ad)^2\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b^2}$$

$$= \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} + \frac{(bc-ad)^2 e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^2 c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{\int \frac{d(5bc-3ad)e+d(bde+4bcf-3adf)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3b^2}$$

$$= \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} + \frac{(bc-ad)^2 e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^2 c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{(d(5bc-3ad)e)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3b^2} + \frac{(d(bde+4bcf-3adf))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3b^2}$$

$$= \frac{(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^2\sqrt{e+fx^2}} + \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b}$$
$$+ \frac{d(5bc-3ad)e^{3/2}\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2 c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{(bc-ad)^2 e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^2 c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{(e(bde+4bcf-3adf))\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{3b^2}$$

$$= \frac{(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b^2\sqrt{e+fx^2}} + \frac{dx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b}$$
$$- \frac{\sqrt{e}(bde+4bcf-3adf)\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{d(5bc-3ad)e^{3/2}\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2 c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{(bc-ad)^2 e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ab^2 c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.98 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \frac{-iabde(bde+4bcf-3adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)\mid\frac{cf}{de}\right)-ia(-}{}$$

[In] Integrate[((c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(a + b*x^2),x]

[Out] ((-I)*a*b*d*e*(b*d*e + 4*b*c*f - 3*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x ^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-6*a*b*c*d*f^2 + 3*a^2*d^2*f^2 + b^2*(-(d^2*e^2) + c*d*e*f + 3*c^2*f^2))*Sqrt[1 + (d*x^2) /c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + f* (a*b^2*d*Sqrt[d/c]*x*(c + d*x^2)*(e + f*x^2) - (3*I)*(b*c - a*d)^2*(b*e - a *f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSi nh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*a*b^3*Sqrt[d/c]*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])

Maple [A] (verified)

Time = 8.40 (sec) , antiderivative size = 845, normalized size of antiderivative = 2.11

method	result
risch	$\frac{dx\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{3b}-\frac{\left(\frac{\left(3a^3d^2f-6a^2bcdf-3a^2b\,d^2e+3a\,b^2c^2f+6a\,b^2cde-3b^3c^2e\right)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)}{b^2a\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{-\frac{3a^2d^2f\sqrt{}}{}}{}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

[In] int((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/3*d*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b-1/3/b*((3*a^3*d^2*f-6*a^2*b*c*d*f -3*a^2*b*d^2*e+3*a*b^2*c^2*f+6*a*b^2*c*d*e-3*b^3*c^2*e)/b^2/a/(-d/c)^(1/2)* (1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Ell ipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))+1/b^2*(-3*a^2*d^2 *f/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^ 2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-3*b^2*c^2*f /(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+ c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+3*a*b*d^2*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*

```
e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-5*b^2*d*c*e/(-d
/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+6*a*b*c*d*f/(-d/c
)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(3*a*b*d^2*f-4*b^2*
c*d*f-b^2*d^2*e)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^
4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)
^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)))))*((d*x^2+c)*(f
*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}{a+bx^2}\,dx$$

```
[In] integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
[Out] Integral((c + d*x**2)**(3/2)*sqrt(e + f*x**2)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{(d\,x^2+c)^{3/2}\sqrt{f\,x^2+e}}{b\,x^2+a}\,dx$$

```
[In] int(((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2))/(a + b*x^2),x)
[Out] int(((c + d*x^2)^(3/2)*(e + f*x^2)^(1/2))/(a + b*x^2), x)
```

3.66 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 321

$$\begin{aligned}
&\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2} dx \\
&= \frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} - \frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{de^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{bc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{(bc-ad)e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

```
[Out] f*x*(d*x^2+c)^(1/2)/b/(f*x^2+e)^(1/2)+d*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*
x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2
))*(d*x^2+c)^(1/2)/b/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1
/2)+(-a*d+b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x
*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(
1/2)/a/b/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(1/(1+f*
x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/
2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/b/(e*(d*x^2+c)/c/(f*x
^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {549, 433, 429, 506, 422, 553}

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \frac{e^{3/2}\sqrt{c+dx^2}(bc-ad)\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{de^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{bc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}}$$

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] (f*x*Sqrt[c + d*x^2])/(b*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2
]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*Sqrt[(e*(c +
d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*e^(3/2)*Sqrt[c + d*x^2]*Elli
pticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*c*Sqrt[f]*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)*e^(3/2)*Sqrt[c
 + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(a*b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x
^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
```

```
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 549

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Dist[d/b, Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
Dist[(b*c - a*d)/b, Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\text{integral} = \frac{d\int \frac{\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b} + \frac{(bc-ad)\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b}$$

$$= \frac{(bc-ad)e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(de)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{b} + \frac{(df)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{b}$$

$$= \frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} + \frac{de^{3/2}\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(bc-ad)e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{(ef)\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{b}$$

$$= \frac{fx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} - \frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{de^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(bc-ad)e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.72 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx = -\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(abdeE\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+(bc-ad)\left(af\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right)+(b\right.\right.}{ab^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*d*e*EllipticE[I*ArcSinh[
Sqrt[d/c]*x], (c*f)/(d*e)] + (b*c - a*d)*(a*f*EllipticF[I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)])))/(a*b^2*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.06

method	result
default	$\frac{\left(-F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2df+F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abcf+E\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abde+\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)a^2df-\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)abc\right.}{(df\,x^4+cf\,x^2+de\,x^2+ce)b^2\sqrt{-\frac{d}{c}}\,a}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(-\frac{\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)adf}{b^2\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)cf}{b\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{de\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1}}{b\sqrt{-\frac{d}{c}}}\right.}{}$

```
[In] int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] (-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d*f+EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*a*b*c*f+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d*
e+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*d*f-Elli
pticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*f-EllipticPi
(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*d*e+EllipticPi(x*(-d
/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c*e)*(d*x^2+c)^(1/2)*(f*x^
2+e)^(1/2)*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2
+c*e)/b^2/(-d/c)^(1/2)/a
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a),x)
[Out] Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{bx^2+a}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{b\,x^2+a}\,dx$$

```
[In] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2),x)
[Out] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2), x)
```

3.67 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 102

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1/2
)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/f^(1/2
)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {553}

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[In] Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqr
t[e]], 1 - (d*e)/(c*f)])/(a*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]
*Sqrt[e + f*x^2])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
```

```
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\text{integral} = \frac{e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = -\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(af\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right)+(be-af)\,\text{EllipticPi}\left(\frac{bc}{ad},i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right)\right)}{ab\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[In] Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*f*EllipticF[I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)]))/(a*b*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.87

method	result
default	$\frac{\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)af-\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)af+\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)be\right)\sqrt{\frac{f\,x^{2}+e}{e}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{d\,x^{2}+c}\,\sqrt{f\,x^{2}+e}}{ba\sqrt{-\frac{d}{c}}\,(df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce)}$
elliptic	$\frac{\sqrt{(d\,x^{2}+c)(f\,x^{2}+e)}\left(\frac{f\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{f\,x^{2}}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{b\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce}}-\frac{\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{f\,x^{2}}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)f}{b\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce}}+\frac{\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{f\,x^{2}}{e}}\,\Pi\left(\right.}{a\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+c}}\right.}{\sqrt{d\,x^{2}+c}\,\sqrt{f\,x^{2}+e}}$

```
[In] int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*f-EllipticPi(x*(-d/c)^(1/2),b*
c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*f+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/
e)^(1/2)/(-d/c)^(1/2))*b*e)/b*((f*x^2+e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(d*x^
2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(-d/c)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

```
[In] integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(1/2),x)
[Out] Integral(sqrt(e + f*x**2)/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{\left(b\,x^2+a\right)\,\sqrt{d\,x^2+c}}\,dx$$

```
[In] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
[Out] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

3.68 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 209

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{\sqrt{d}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac(bc-ad)\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] b*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1
/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/(-a*
d+b*c)/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(1/(1+d*x^2/
c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(
1-c*f/d/e)^(1/2))*d^(1/2)*(f*x^2+e)^(1/2)/(-a*d+b*c)/c^(1/2)/(d*x^2+c)^(1/2
)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {555, 553, 422}

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{be^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{\sqrt{d}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[In] Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[d]*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)
/(d*e)])/(Sqrt[c]*(b*c - a*d)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c +
d*x^2))])) + (b*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[
(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)*Sqrt[f]*Sqrt[(e*(c
 + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 555

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{b\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{bc-ad} - \frac{d\int \frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx}{bc-ad}\\ &= -\frac{\sqrt{d}\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{be^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.00 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.66

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(ad\sqrt{\frac{d}{c}}ex+ad\sqrt{\frac{d}{c}}fx^3+iade\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)\mid\frac{cf}{de}\right.\right.}{}$$

[In] Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(3/2)),x]

[Out] (Sqrt[d/c]*(a*d*Sqrt[d/c]*e*x + a*d*Sqrt[d/c]*f*x^3 + I*a*d*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*a*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*c*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*d*(-(b*c) + a*d)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])

Maple [A] (verified)

Time = 4.35 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.87

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\,adf\,x^3-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)acf+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ade-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,E\left(x\sqrt{-\cdots}\right.\right.}{ca\sqrt{-\frac{d}{c}}\,(ad-bc)\cdots}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{(df\,x^2+de)x}{c(ad-bc)\sqrt{\left(x^2+\frac{c}{d}\right)\left(df\,x^2+de\right)}}-\frac{\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)f}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}\,(ad-bc)}+\frac{de\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\cdots\right.}{(ad-bc)c\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+\cdots}}\cdots\right.}{\cdots}$

[In] int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] ((-d/c)^(1/2)*a*d*f*x^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*c*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c*e+(-d/c)^(1/2)*a*d*e*x)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/c/a/(-d/c)^(1/2)/(a*d-b*c)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(3/2),x)
[Out] Integral(sqrt(e + f*x**2)/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\left(c+dx^2\right)^{3/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{(b\,x^2+a)\,\left(d\,x^2+c\right)^{3/2}}\,dx$$

```
[In] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
[Out] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.69 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 32, antiderivative size = 401

$$
\begin{aligned}
\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx &= -\frac{dx\sqrt{e+fx^2}}{3c(bc-ad)(c+dx^2)^{3/2}} \\
&- \frac{\sqrt{d}(bc(5de-4cf)-ad(2de-cf))\sqrt{e+fx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}(bc-ad)^2(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&+ \frac{de^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2(bc-ad)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{b^2e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

```
[Out] b^2*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^
(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/(-
a*d+b*c)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*e^
(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(
1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-a*d+b*c)/
(-c*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*d*x*(f*x^2+e
)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)^(3/2)-1/3*(b*c*(-4*c*f+5*d*e)-a*d*(-c*f+2*d*
e))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+
d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*d^(1/2)*(f*x^2+e)^(1/2)/c^(3/2)/(-a*d+b*c
)^2/(-c*f+d*e)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {560, 553, 540, 539, 429, 422}

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \frac{b^2e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{\sqrt{d}\sqrt{e+fx^2}(bc(5de-4cf)-ad(2de-cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{c+dx^2}(bc-ad)^2(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{de^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{dx\sqrt{e+fx^2}}{3c\left(c+dx^2\right)^{3/2}(bc-ad)}$$

```
[In] Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/3*(d*x*Sqrt[e + f*x^2])/(c*(b*c - a*d)*(c + d*x^2)^(3/2)) - (Sqrt[d]*(b*
c*(5*d*e - 4*c*f) - a*d*(2*d*e - c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*(b*c - a*d)^2*(d*e - c*f)*S
qrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + (d*e^(3/2)*Sqrt[f]*
Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3
*c^2*(b*c - a*d)*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2]) + (b^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[
(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*Sqrt[(e*
(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
```

```
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2 \int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{(bc-ad)^2} - \frac{d \int \frac{(2bc-ad+bdx^2)\sqrt{e+fx^2}}{(c+dx^2)^{5/2}}\,dx}{(bc-ad)^2} \\
&= -\frac{dx\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{b^2 e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{\int \frac{-d(5bc-2ad)e-d(4bc-ad)fx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx}{3c(bc-ad)^2}
\end{aligned}$$

$$= -\frac{dx\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{b^2e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(def)\int\frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{3c(bc-ad)(de-cf)} - \frac{(d(bc(5de-4cf)-ad(2de-cf)))\int\frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}dx}{3c(bc-ad)^2(de-cf)}$$

$$= -\frac{dx\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{\sqrt{d}(bc(5de-4cf)-ad(2de-cf))\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}(bc-ad)^2(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{de^{3/2}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(bc-ad)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{b^2e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.25 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.06

$$\int\frac{\sqrt{e+fx^2}}{(a+bx^2)\left(c+dx^2\right)^{5/2}}\,dx = \frac{ac\left(\frac{d}{c}\right)^{3/2}x(e+fx^2)\left(bc(6cde-5c^2f+5d^2ex^2-4cdfx^2)+ad(-3cde+2c^2f\right.}{}$$

```
[In] Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (a*c*(d/c)^(3/2)*x*(e + f*x^2)*(b*c*(6*c*d*e - 5*c^2*f + 5*d^2*e*x^2 - 4*c*
d*f*x^2) + a*d*(-3*c*d*e + 2*c^2*f - 2*d^2*e*x^2 + c*d*f*x^2)) - I*a*d*e*(a
*d*(2*d*e - c*f) + b*c*(-5*d*e + 4*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sq
rt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-(d
*e) + c*f)*(2*a*d^2*e + b*c*(-5*d*e + 3*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/
c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*
I)*b*c^2*(b*e - a*f)*(-(d*e) + c*f)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/
(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2*(-(d*e) + c*f)*(c + d*x^2)^(3/2)*Sqrt[e +
f*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1365 vs. 2(466) = 932.

Time = 5.28 (sec) , antiderivative size = 1366, normalized size of antiderivative = 3.41

method	result	size
elliptic	Expression too large to display	1366
default	Expression too large to display	2067

```
[In] int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/3/d/c*x/(a*d
-b*c)*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2+1/3*(d*f*x^2+d*e)/c^2
/(c*f-d*e)*x*(a*c*d*f-2*a*d^2*e-4*b*c^2*f+5*b*c*d*e)/(a*d-b*c)^2/((x^2+c/d)
*(d*f*x^2+d*e))^(1/2)-1/3*d^2/(c*f-d*e)/c/(a*d-b*c)^2*e/(-d/c)^(1/2)*(1+d*x
^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a*f+4/3*d/(c*f-d*e)/(a*d-b*c)^2*e
/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+
c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*b*f-1/3/(-d/c
)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/c/(a*d-b*c)^2*a*d*f
+2/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*
x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/c^2/(a*d-
b*c)^2*a*e*d^2-5/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^
4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1
/2))/c/(a*d-b*c)^2*b*d*e+2/3*d^3/(c*f-d*e)/c^2/(a*d-b*c)^2*e^2/(-d/c)^(1/2)
*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*El
lipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a-5/3*d^2/(c*f-d*e)/c/(a*d
-b*c)^2*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x
^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*b+
1/(a*d-b*c)^2/a*b^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x
^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2
)/(-d/c)^(1/2))*e+1/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f
*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)
^(1/2))*f/(a*d-b*c)/c+4/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/
(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/
e/d)^(1/2))/(a*d-b*c)^2*b*f-1/(a*d-b*c)^2*b/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*
(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(
1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*f)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(5/2),x)
[Out] Integral(sqrt(e + f*x**2)/((a + b*x**2)*(c + d*x**2)**(5/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{5/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{5/2}}\,dx=\int \frac{\sqrt{f\,x^2+e}}{\left(b\,x^2+a\right)\,{\left(d\,x^2+c\right)}^{5/2}}\,dx$$

```
[In] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
[Out] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.70 $\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 630

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = -\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)(c+dx^2)^{5/2}}$$
$$-\frac{d(bc(9de-8cf)-ad(4de-3cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(de-cf)(c+dx^2)^{3/2}}-\frac{b^2\sqrt{d}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)^3\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+\frac{\sqrt{d}(ad(8d^2e^2-13cdef+3c^2f^2)-2bc(9d^2e^2-14cdef+4c^2f^2))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+\frac{de^{3/2}\sqrt{f}(bc(9de-11cf)-2ad(2de-3cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{b^3e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac(bc-ad)^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] b^3*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^
(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/(-
a*d+b*c)^3/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/15*d*e
^(3/2)*(b*c*(-11*c*f+9*d*e)-2*a*d*(-3*c*f+2*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+
f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1
/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^3/(-a*d+b*c)^2/(-c*f+d*e)^2/(e*(d*x^2+c)/c/(
f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/5*d*x*(f*x^2+e)^(1/2)/c/(-a*d+b*c)/(d*x^2
+c)^(5/2)-1/15*d*(b*c*(-8*c*f+9*d*e)-a*d*(-3*c*f+4*d*e))*x*(f*x^2+e)^(1/2)/
c^2/(-a*d+b*c)^2/(-c*f+d*e)/(d*x^2+c)^(3/2)+1/15*(a*d*(3*c^2*f^2-13*c*d*e*f
```

```
+8*d^2*e^2)-2*b*c*(4*c^2*f^2-14*c*d*e*f+9*d^2*e^2))*(1/(1+d*x^2/c))^(1/2)*(
1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^
(1/2))*d^(1/2)*(f*x^2+e)^(1/2)/c^(5/2)/(-a*d+b*c)^2/(-c*f+d*e)^2/(d*x^2+c)^
(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)-b^2*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)
^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*d^(
1/2)*(f*x^2+e)^(1/2)/(-a*d+b*c)^3/c^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d
*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {560, 555, 553, 422, 540, 541, 539, 429}

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \frac{b^3e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{b^2\sqrt{d}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+\frac{de^{3/2}\sqrt{f}\sqrt{c+dx^2}(bc(9de-11cf)-2ad(2de-3cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{\sqrt{d}\sqrt{e+fx^2}(ad(3c^2f^2-13cdef+8d^2e^2)-2bc(4c^2f^2-14cdef+9d^2e^2))\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{c+dx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{dx\sqrt{e+fx^2}(bc(9de-8cf)-ad(4de-3cf))}{15c^2(c+dx^2)^{3/2}(bc-ad)^2(de-cf)}-\frac{dx\sqrt{e+fx^2}}{5c(c+dx^2)^{5/2}(bc-ad)}$$

```
[In] Int[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

```
[Out] -1/5*(d*x*Sqrt[e + f*x^2])/(c*(b*c - a*d)*(c + d*x^2)^(5/2)) - (d*(b*c*(9*d
*e - 8*c*f) - a*d*(4*d*e - 3*c*f))*x*Sqrt[e + f*x^2])/(15*c^2*(b*c - a*d)^2
*(d*e - c*f)*(c + d*x^2)^(3/2)) - (b^2*Sqrt[d]*Sqrt[e + f*x^2]*EllipticE[Ar
cTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)^3*Sqrt[c
+ d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + (Sqrt[d]*(a*d*(8*d^2*e^2
- 13*c*d*e*f + 3*c^2*f^2) - 2*b*c*(9*d^2*e^2 - 14*c*d*e*f + 4*c^2*f^2))*Sqr
t[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c
^(5/2)*(b*c - a*d)^2*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*
(c + d*x^2))]) + (d*e^(3/2)*Sqrt[f]*(b*c*(9*d*e - 11*c*f) - 2*a*d*(2*d*e -
3*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c
*f)])/(15*c^3*(b*c - a*d)^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^
2))]*Sqrt[e + f*x^2]) + (b^3*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(
```

```
a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^3*Sqr
t[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
```

```
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 555

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b^2 \int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx}{(bc-ad)^2} - \frac{d \int \frac{(2bc-ad+bdx^2)\sqrt{e+fx^2}}{(c+dx^2)^{7/2}}\,dx}{(bc-ad)^2} \\
&= -\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} + \frac{b^3 \int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{(bc-ad)^3} \\
&\quad - \frac{(b^2 d) \int \frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx}{(bc-ad)^3} + \frac{\int \frac{-d(9bc-4ad)e-d(8bc-3ad)fx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx}{5c(bc-ad)^2} \\
&= -\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} - \frac{d(bc(9de-8cf)-ad(4de-3cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(de-cf)\left(c+dx^2\right)^{3/2}} \\
&\quad - \frac{b^2\sqrt{d}\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)^3\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&\quad + \frac{b^3e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{\int \frac{de(bc(18de-19cf)-ad(8de-9cf))+df(bc(9de-8cf)-ad(4de-3cf))x^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx}{15c^2(bc-ad)^2(de-cf)}
\end{aligned}
$$

$$= -\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} - \frac{d(bc(9de-8cf)-ad(4de-3cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(de-cf)\left(c+dx^2\right)^{3/2}}$$
$$- \frac{b^2\sqrt{d}\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)^3\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+ \frac{b^3e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{(def(bc(9de-11cf)-2ad(2de-3cf)))\int\frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{15c^2(bc-ad)^2(de-cf)^2}$$
$$+ \frac{(d(ad(8d^2e^2-13cdef+3c^2f^2)-2bc(9d^2e^2-14cdef+4c^2f^2)))\int\frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}dx}{15c^2(bc-ad)^2(de-cf)^2}$$

$$= -\frac{dx\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} - \frac{d(bc(9de-8cf)-ad(4de-3cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(de-cf)\left(c+dx^2\right)^{3/2}}$$
$$- \frac{b^2\sqrt{d}\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)^3\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+ \frac{\sqrt{d}(ad(8d^2e^2-13cdef+3c^2f^2)-2bc(9d^2e^2-14cdef+4c^2f^2))\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\right.}{15c^{5/2}(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+ \frac{de^{3/2}\sqrt{f}(bc(9de-11cf)-2ad(2de-3cf))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{b^3e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.80 (sec) , antiderivative size = 584, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\left(c+dx^2\right)^{7/2}}\,dx = \frac{-ad\sqrt{\frac{d}{c}}x(e+fx^2)\left(3c^2(bc-ad)^2(de-cf)^2+c(bc-ad)(-de+cf)(ad(4de\right.}{}$$

```
[In] Integrate[Sqrt[e + f*x^2]/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
[Out] (-(a*d*Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 + c*(b*c
- a*d)*(-(d*e) + c*f)*(a*d*(4*d*e - 3*c*f) + b*c*(-9*d*e + 8*c*f))*(c + d*x
```

```
^2) + (a*b*c*d*(-26*d^2*e^2 + 41*c*d*e*f - 11*c^2*f^2) + a^2*d^2*(8*d^2*e^2
 - 13*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(33*d^2*e^2 - 58*c*d*e*f + 23*c^2*f^2)
)*(c + d*x^2)^2)) - I*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]
*(a*d*e*(a*b*c*d*(-26*d^2*e^2 + 41*c*d*e*f - 11*c^2*f^2) + a^2*d^2*(8*d^2*e
^2 - 13*c*d*e*f + 3*c^2*f^2) + b^2*c^2*(33*d^2*e^2 - 58*c*d*e*f + 23*c^2*f^
2))*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (d*e - c*f)*(-(a*(2*a*
b*c*d^2*e*(13*d*e - 14*c*f) + a^2*d^3*e*(-8*d*e + 9*c*f) + b^2*c^2*(-33*d^2
*e^2 + 49*c*d*e*f - 15*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d
*e)]) + 15*b^2*c^3*(b*e - a*f)*(-(d*e) + c*f)*EllipticPi[(b*c)/(a*d), I*Arc
Sinh[Sqrt[d/c]*x], (c*f)/(d*e)])))/(15*a*c^3*Sqrt[d/c]*(b*c - a*d)^3*(d*e -
 c*f)^2*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3137 vs. $2(714) = 1428$.

Time = 6.96 (sec) , antiderivative size = 3138, normalized size of antiderivative = 4.98

method	result	size
elliptic	Expression too large to display	3138
default	Expression too large to display	6245

```
[In] int((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-4/15/(-d/c)^(
1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2
)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*f/(c*f-d*e)/c^2/(a*d-b
*c)^2*a*e*d^2+3/5/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4
+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/
2))*f/(c*f-d*e)/c/(a*d-b*c)^2*b*d*e+13/15/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1
+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2
),(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*e)/c^2/(a*d-b*c)^3*a^2*d^3*e*f+26/15/(-d
/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*e)/c^2/(a*
d-b*c)^3*a*b*d^3*e^2-8/15/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/
(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/
e/d)^(1/2))*f^2/(c*f-d*e)/(a*d-b*c)^2*b-b^3/(a*d-b*c)^3/a/(-d/c)^(1/2)*(1+d
*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Ellipti
cPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*e+1/15*(d*f*x^2+d*e)/
c^3/(c*f-d*e)^2*x*(3*a^2*c^2*d^2*f^2-13*a^2*c*d^3*e*f+8*a^2*d^4*e^2-11*a*b*
c^3*d*f^2+41*a*b*c^2*d^2*e*f-26*a*b*c*d^3*e^2+23*b^2*c^4*f^2-58*b^2*c^3*d*e
*f+33*b^2*c^2*d^2*e^2)/(a*d-b*c)^3/((x^2+c/d)*(d*f*x^2+d*e))^(1/2)+1/5/d^2/
c*x/(a*d-b*c)*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^3-23/15/(-d/c)^
(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/
2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*e)*c/(a*d-b*c)
```

```
^3*b^2*f^2-41/15/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+
c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2
))/(c*f-d*e)/c/(a*d-b*c)^3*a*b*d^2*e*f+11/15*d^2/(c*f-d*e)^2/(a*d-b*c)^3*e/
(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c
*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a*b*f^2-41/15*
d^3/(c*f-d*e)^2/c/(a*d-b*c)^3*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e
)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c
*f+d*e)/e/d)^(1/2))*a*b*f+b^2/(a*d-b*c)^3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1
+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/
2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*f+1/15*(3*a*c*d*f-4*a*d^2*e-8*b*c^2*f
+9*b*c*d*e)/d/(c*f-d*e)/c^2/(a*d-b*c)^2*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/
2)/(x^2+c/d)^2+1/5/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^
4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1
/2))*f^2/(c*f-d*e)/c/(a*d-b*c)^2*a*d-1/5/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+
f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2)
,(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*e)/c/(a*d-b*c)^3*a^2*d^2*f^2-8/15/(-d/c)^
(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/
2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*e)/c^3/(a*d-b*
c)^3*a^2*d^4*e^2+11/15/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*
f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d
)^(1/2))/(c*f-d*e)/(a*d-b*c)^3*a*b*d*f^2+58/15/(-d/c)^(1/2)*(1+d*x^2/c)^(1/
2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)
^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*e)/(a*d-b*c)^3*b^2*d*e*f-11/5/(-d/c
)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*e)/c/(a*d-b*
c)^3*b^2*d^2*e^2-1/5*d^3/(c*f-d*e)^2/c/(a*d-b*c)^3*e/(-d/c)^(1/2)*(1+d*x^2/
c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*
(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a^2*f^2+13/15*d^4/(c*f-d*e)^2/c^2/(a
*d-b*c)^3*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f
*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*
a^2*f+26/15*d^4/(c*f-d*e)^2/c^2/(a*d-b*c)^3*e^3/(-d/c)^(1/2)*(1+d*x^2/c)^(1
/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c
)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a*b-23/15*d/(c*f-d*e)^2*c/(a*d-b*c)^3*e/(
-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*
e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*b^2*f^2+58/15*d
^2/(c*f-d*e)^2/(a*d-b*c)^3*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(
1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+
d*e)/e/d)^(1/2))*b^2*f-8/15*d^5/(c*f-d*e)^2/c^3/(a*d-b*c)^3*e^3/(-d/c)^(1/2
)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*E
llipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a^2-11/5*d^3/(c*f-d*e)^2/
c/(a*d-b*c)^3*e^3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4
+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/
2))*b^2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((f*x**2+e)**(1/2)/(b*x**2+a)/(d*x**2+c)**(7/2),x)
[Out] Integral(sqrt(e + f*x**2)/((a + b*x**2)*(c + d*x**2)**(7/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="maxima")
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{fx^2+e}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)^(1/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="giac")
[Out] integrate(sqrt(f*x^2 + e)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\,(c+dx^2)^{7/2}}\,dx = \int \frac{\sqrt{f\,x^2+e}}{(b\,x^2+a)\,\left(d\,x^2+c\right)^{7/2}}\,dx$$

```
[In] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x)
[Out] int((e + f*x^2)^(1/2)/((a + b*x^2)*(c + d*x^2)^(7/2)), x)
```

3.71 $\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 659

$$
\begin{aligned}
\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx &= \frac{(bc-ad)^2f^2x\sqrt{c+dx^2}}{b^3d\sqrt{e+fx^2}} \\
&+ \frac{2(bc-ad)f(2de-cf)x\sqrt{c+dx^2}}{3b^2d\sqrt{e+fx^2}} \\
&+ \frac{(3d^2e^2+7cdef-2c^2f^2)\,x\sqrt{c+dx^2}}{15bd\sqrt{e+fx^2}} + \frac{(bc-ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} \\
&+ \frac{2(3de-cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15b} + \frac{fx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5b} \\
&- \frac{\sqrt{e}(15a^2d^2f^2-20abdf(de+cf)+3b^2(d^2e^2+9cdef+c^2f^2))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15b^3d\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{e^{3/2}(15a^2d^2f+3b^2c(8de+3cf)-5abd(3de+5cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15b^3c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{(bc-ad)^2e^{3/2}(be-af)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ab^3c\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

```
[Out] (-a*d+b*c)^2*f^2*x*(d*x^2+c)^(1/2)/b^3/d/(f*x^2+e)^(1/2)+2/3*(-a*d+b*c)*f*(
-c*f+2*d*e)*x*(d*x^2+c)^(1/2)/b^2/d/(f*x^2+e)^(1/2)+1/15*(-2*c^2*f^2+7*c*d*
e*f+3*d^2*e^2)*x*(d*x^2+c)^(1/2)/b/d/(f*x^2+e)^(1/2)+1/15*e^(3/2)*(15*a^2*d
^2*f+3*b^2*c*(3*c*f+8*d*e)-5*a*b*d*(5*c*f+3*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+
f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1
/2))*(d*x^2+c)^(1/2)/b^3/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e
```

```
)^(1/2)+(-a*d+b*c)^2*e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(
1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(
1/2))*(d*x^2+c)^(1/2)/a/b^3/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^
2+e)^(1/2)-1/15*(15*a^2*d^2*f^2-20*a*b*d*f*(c*f+d*e)+3*b^2*(c^2*f^2+9*c*d*e
*f+d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^
(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b^3/d/f^
(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/5*f*x*(d*x^2+c)^(3/
2)*(f*x^2+e)^(1/2)/b+1/3*(-a*d+b*c)*f*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b^2
+2/15*(-c*f+3*d*e)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 784, normalized size of antiderivative = 1.19, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {559, 427, 542, 545, 429, 506, 422, 557, 553}

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)(be-af)^2\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{ab^3\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(bc-ad)(5be-3af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3b^3\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(bc-ad)(-3adf+bcf+4bde)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3d\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{fx\sqrt{c+dx^2}(bc-ad)(-3adf+bcf+4bde)}{3b^3d\sqrt{e+fx^2}}+\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)}{3b^2}$$
$$-\frac{\sqrt{e}\sqrt{c+dx^2}(-2c^2f^2+7cdef+3d^2e^2)\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15bd\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{e^{3/2}\sqrt{c+dx^2}(9de-cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15b\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{x\sqrt{c+dx^2}(-2c^2f^2+7cdef+3d^2e^2)}{15bd\sqrt{e+fx^2}}$$
$$+\frac{fx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5b}+\frac{2x\sqrt{c+dx^2}\sqrt{e+fx^2}(3de-cf)}{15b}$$

```
[In] Int[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x]

[Out] ((b*c - a*d)*f*(4*b*d*e + b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^3*d*Sqrt
[e + f*x^2]) + ((3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2)*x*Sqrt[c + d*x^2])/(15*
```

```
b*d*Sqrt[e + f*x^2]) + ((b*c - a*d)*f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3
*b^2) + (2*(3*d*e - c*f)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(15*b) + (f*x*(
c + d*x^2)^(3/2)*Sqrt[e + f*x^2])/(5*b) - ((b*c - a*d)*Sqrt[e]*Sqrt[f]*(4*b
*d*e + b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*b^3*d*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
 + f*x^2]) - (Sqrt[e]*(3*d^2*e^2 + 7*c*d*e*f - 2*c^2*f^2)*Sqrt[c + d*x^2]*E
llipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b*d*Sqrt[f]*Sqr
t[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + ((b*c - a*d)*Sqrt[e]*
Sqrt[f]*(5*b*e - 3*a*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*b^3*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2]) + (e^(3/2)*(9*d*e - c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]
*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(15*b*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*(b*e - a*f)^2*Sqrt[e + f*x
^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e
)])/(a*b^3*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 557

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[(2*b*c - a*d + b*d*x^2)*(Sqrt
[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rule 559

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rubi steps

$$\text{integral} = \frac{d\int \sqrt{c+dx^2}(e+fx^2)^{3/2}\,dx}{b} + \frac{(bc-ad)\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx}{b}$$

$$= \frac{fx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5b} + \frac{\int \frac{\sqrt{c+dx^2}(e(5de-cf)+2f(3de-cf)x^2)}{\sqrt{e+fx^2}}\,dx}{5b} + \frac{((bc-ad)f)\int \frac{\sqrt{c+dx^2}(2be-af+bfx^2)}{\sqrt{e+fx^2}}\,dx}{b^3} + \frac{((bc-ad)(be-af)^2)\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{b^3}$$

$$= \frac{(bc-ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} + \frac{2(3de-cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15b} + \frac{fx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5b} + \frac{c^{3/2}(bc-ad)(be-af)^2\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^3\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{(bc-ad)\int \frac{cf(5be-3af)+f(4bde+bcf-3adf)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3b^3} + \frac{\int \frac{cef(9de-cf)+f(3d^2e^2+7cdef-2c^2f^2)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15bf}$$

$$= \frac{(bc-ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} + \frac{2(3de-cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15b} + \frac{fx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5b} + \frac{c^{3/2}(bc-ad)(be-af)^2\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^3\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{(c(bc-ad)f(5be-3af))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3b^3} + \frac{(ce(9de-cf))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15b} + \frac{((bc-ad)f(4bde+bcf-3adf))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3b^3} + \frac{(3d^2e^2+7cdef-2c^2f^2)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{15b}$$

$$= \frac{(bc-ad)f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^3d\sqrt{e+fx^2}} + \frac{(3d^2e^2+7cdef-2c^2f^2)\,x\sqrt{c+dx^2}}{15bd\sqrt{e+fx^2}} + \frac{(bc-ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} + \frac{2(3de-cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15b} + \frac{fx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5b} + \frac{(bc-ad)\sqrt{e}\sqrt{f}(5be-3af)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{e^{3/2}(9de-cf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15b\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{c^{3/2}(bc-ad)(be-af)^2\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^3\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{((bc-ad)ef(4bde+bcf-3adf))\int\frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{3b^3d} - \frac{(e(3d^2e^2+7cdef-2c^2f^2))\int\frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{15bd}$$

$$= \frac{(bc-ad)f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^3d\sqrt{e+fx^2}} + \frac{(3d^2e^2+7cdef-2c^2f^2)\,x\sqrt{c+dx^2}}{15bd\sqrt{e+fx^2}} + \frac{(bc-ad)fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b^2} + \frac{2(3de-cf)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{15b} + \frac{fx(c+dx^2)^{3/2}\sqrt{e+fx^2}}{5b} - \frac{(bc-ad)\sqrt{e}\sqrt{f}(4bde+bcf-3adf)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3d\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{\sqrt{e}(3d^2e^2+7cdef-2c^2f^2)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15bd\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(bc-ad)\sqrt{e}\sqrt{f}(5be-3af)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{e^{3/2}(9de-cf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15b\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{c^{3/2}(bc-ad)(be-af)^2\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^3\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.65 (sec) , antiderivative size = 445, normalized size of antiderivative = 0.68

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \frac{-iabe(15a^2d^2f^2-20abdf(de+cf)+3b^2(d^2e^2+9cdef+c^2f^2))\sqrt{1+\frac{dx^2}{c}}}{}$$

```
[In] Integrate[((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] ((-I)*a*b*e*(15*a^2*d^2*f^2 - 20*a*b*d*f*(d*e + c*f) + 3*b^2*(d^2*e^2 + 9*c
*d*e*f + c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcS
inh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-15*a^3*d^2*f^3 + 15*a^2*b*d*f^2*(d*e
 + 2*c*f) - 3*b^3*e*(d^2*e^2 + c*d*e*f - 7*c^2*f^2) + 5*a*b^2*f*(d^2*e^2 -
7*c*d*e*f - 3*c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I
*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + f*(a*b^2*Sqrt[d/c]*x*(c + d*x^2)*(e +
 f*x^2)*(-5*a*d*f + 3*b*(2*d*e + 2*c*f + d*f*x^2)) - (15*I)*(b*c - a*d)^2*(
b*e - a*f)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d)
, I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(15*a*b^4*Sqrt[d/c]*f*Sqrt[c + d*x
^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 7.95 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{x\left(-3bdf\,x^{2}+5adf-6bcf-6bde\right)\sqrt{d\,x^{2}+c}\,\sqrt{f\,x^{2}+e}}{15b^{2}}+\frac{\left(-\frac{\left(15a^{2}d^{2}f^{2}-20abcd\,f^{2}-20ab\,d^{2}ef+3b^{2}c^{2}f^{2}+27b^{2}cdef+3b^{2}d^{2}e^{2}\right)e\sqrt{1+\frac{d\,x^{2}}{c}}}{b\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+cf\,x^{2}}}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

[In] int((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] -1/15*x*(-3*b*d*f*x^2+5*a*d*f-6*b*c*f-6*b*d*e)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b^2+1/15/b^2*(-1/b*(15*a^2*d^2*f^2-20*a*b*c*d*f^2-20*a*b*d^2*e*f+3*b^2*c^2*f^2+27*b^2*c*d*e*f+3*b^2*d^2*e^2)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)))-(15*a^3*d^2*f^2-30*a^2*b*c*d*f^2-30*a^2*b*d^2*e*f+15*a*b^2*c^2*f^2+55*a*b^2*c*d*e*f+15*a*b^2*d^2*e^2-24*b^3*c^2*e*f-24*b^3*c*d*e^2)/b^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+(15*a^4*d^2*f^2-30*a^3*b*c*d*f^2-30*a^3*b*d^2*e*f+15*a^2*b^2*c^2*f^2+60*a^2*b^2*c*d*e*f+15*a^2*b^2*d^2*e^2-30*a*b^3*c^2*e*f-30*a*b^3*c*d*e^2+15*b^4*c^2*e^2)/b^2/a/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2)))*((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}\,(e+fx^2)^{3/2}}{a+bx^2}\,dx=\text{Timed out}$$

[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}(e+fx^2)^{\frac{3}{2}}}{a+bx^2}\,dx$$

```
[In] integrate((d*x**2+c)**(3/2)*(f*x**2+e)**(3/2)/(b*x**2+a),x)
[Out] Integral((c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{bx^2+a}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}{bx^2+a}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{3/2}}{b\,x^2+a}\,dx$$

```
[In] int(((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x)
[Out] int(((c + d*x^2)^(3/2)*(e + f*x^2)^(3/2))/(a + b*x^2), x)
```

3.72 $\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2} dx$

Optimal result

Integrand size = 32, antiderivative size = 403

$$
\begin{aligned}
\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2} dx &= \frac{f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^2d\sqrt{e+fx^2}} \\
&+ \frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} \\
&- \frac{\sqrt{e}\sqrt{f}(4bde+bcf-3adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2d\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{\sqrt{e}\sqrt{f}(5be-3af)\sqrt{c+dx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3b^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{c^{3/2}(be-af)^2\sqrt{e+fx^2}\operatorname{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}
\end{aligned}
$$

```
[Out] 1/3*f*(-3*a*d*f+b*c*f+4*b*d*e)*x*(d*x^2+c)^(1/2)/b^2/d/(f*x^2+e)^(1/2)-1/3*
(-3*a*d*f+b*c*f+4*b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(
x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x
^2+c)^(1/2)/b^2/d/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*(-3*a
*f+5*b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/
2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/b^2
/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*f*x*(d*x^2+c)^(1/2)*(f
*x^2+e)^(1/2)/b+c^(3/2)*(-a*f+b*e)^2*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2
)*EllipticPi(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2
))*(f*x^2+e)^(1/2)/a/b^2/e/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c)
)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {557, 553, 542, 545, 429, 506, 422}

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \frac{c^{3/2}\sqrt{e+fx^2}(be-af)^2\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(5be-3af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3b^2\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(-3adf+bcf+4bde)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2d\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{fx\sqrt{c+dx^2}(-3adf+bcf+4bde)}{3b^2d\sqrt{e+fx^2}}+\frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b}$$

```
[In] Int[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (f*(4*b*d*e + b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b^2*d*Sqrt[e + f*x^2])
 + (f*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*b) - (Sqrt[e]*Sqrt[f]*(4*b*d*e
+ b*c*f - 3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1
 - (d*e)/(c*f)])/(3*b^2*d*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*
x^2]) + (Sqrt[e]*Sqrt[f]*(5*b*e - 3*a*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b^2*Sqrt[(e*(c + d*x^2))/(c*(e +
f*x^2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*e - a*f)^2*Sqrt[e + f*x^2]*Elliptic
Pi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*b^2*S
qrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 557

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[(2*b*c - a*d + b*d*x^2)*(Sqrt
[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rubi steps

$$\text{integral} = \frac{f\int \frac{\sqrt{c+dx^2}\,(2be-af+bfx^2)}{\sqrt{e+fx^2}}\,dx}{b^2} + \frac{(be-af)^2\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{b^2}$$

$$= \frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} + \frac{c^{3/2}(be-af)^2\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+ \frac{\int \frac{cf(5be-3af)+f(4bde+bcf-3adf)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3b^2}$$

$$= \frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b} + \frac{c^{3/2}(be-af)^2\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+ \frac{(cf(5be-3af))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3b^2} + \frac{(f(4bde+bcf-3adf))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3b^2}$$

$$= \frac{f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^2d\sqrt{e+fx^2}} + \frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b}$$
$$+ \frac{\sqrt{e}\sqrt{f}(5be-3af)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{c^{3/2}(be-af)^2\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$- \frac{(ef(4bde+bcf-3adf))\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{3b^2d}$$

$$= \frac{f(4bde+bcf-3adf)x\sqrt{c+dx^2}}{3b^2d\sqrt{e+fx^2}} + \frac{fx\sqrt{c+dx^2}\sqrt{e+fx^2}}{3b}$$
$$- \frac{\sqrt{e}\sqrt{f}(4bde+bcf-3adf)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2d\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{\sqrt{e}\sqrt{f}(5be-3af)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{c^{3/2}(be-af)^2\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.64 (sec) , antiderivative size = 739, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \frac{ab^2c\sqrt{\frac{d}{c}}efx + ab^2d\sqrt{\frac{d}{c}}efx^3 + ab^2c\sqrt{\frac{d}{c}}f^2x^3 + ab^2d\sqrt{\frac{d}{c}}f^2x^5 - iabe(4bde+bc}{}$$

```
[In] Integrate[(Sqrt[c + d*x^2]*(e + f*x^2)^(3/2))/(a + b*x^2),x]
```

```
[Out] (a*b^2*c*Sqrt[d/c]*e*f*x + a*b^2*d*Sqrt[d/c]*e*f*x^3 + a*b^2*c*Sqrt[d/c]*f^
2*x^3 + a*b^2*d*Sqrt[d/c]*f^2*x^5 - I*a*b*e*(4*b*d*e + b*c*f - 3*a*d*f)*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f
)/(d*e)] - I*a*(3*a^2*d*f^2 - 3*a*b*f*(d*e + c*f) + b^2*e*(-(d*e) + 4*c*f))
*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x],
(c*f)/(d*e)] - (3*I)*b^3*c*e^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elli
pticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (3*I)*a*b^2*d*e^
2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh
[Sqrt[d/c]*x], (c*f)/(d*e)] + (6*I)*a*b^2*c*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] -
 (6*I)*a^2*b*d*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)
/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*a^2*b*c*f^2*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)] + (3*I)*a^3*d*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*a*b^3*Sqrt[
d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 8.26 (sec) , antiderivative size = 845, normalized size of antiderivative = 2.10

method	result
risch	$\frac{fx\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{3b} - \frac{\left(\frac{\left(3a^3 d\,f^2-3a^2bc\,f^2-6a^2bdef+6a\,b^2cef+3a\,b^2d\,e^2-3c\,e^2b^3\right)\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)}{b^2a\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{-\frac{3a^2d}{}}{}\right.}{}$
default	Expression too large to display
elliptic	Expression too large to display

```
[In] int((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*f*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b-1/3/b*((3*a^3*d*f^2-3*a^2*b*c*f^2
-6*a^2*b*d*e*f+6*a*b^2*c*e*f+3*a*b^2*d*e^2-3*b^3*c*e^2)/b^2/a/(-d/c)^(1/2)*
(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Ell
ipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))+1/b^2*(-3*a^2*d*f
^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^
2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-3*b^2*d*e^2
/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+
c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+3*a*b*c*f^2/(
-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*
e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-5*b^2*c*e*f/(-d
/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+6*a*b*d*e*f/(-d/c
)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-(3*a*b*d*f^2-b^2*c*
f^2-4*b^2*d*e*f)*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^
4+c*f*x^2+d*e*x^2+c*e)^(1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)
^(1/2))-EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)))))*((d*x^2+c)*(f
*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{\sqrt{c+dx^2}(e+fx^2)^{\frac{3}{2}}}{a+bx^2}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*(e + f*x**2)**(3/2)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{bx^2+a}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}{bx^2+a}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(3/2)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}(e+fx^2)^{3/2}}{a+bx^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}{b\,x^2+a}\,dx$$

```
[In] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/(a + b*x^2),x)
[Out] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(3/2))/(a + b*x^2), x)
```

3.73 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 328

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{e+fx^2}} - \frac{\sqrt{e}f^{3/2}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bd\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{bc\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{e^{3/2}(be-af)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] f^2*x*(d*x^2+c)^(1/2)/b/d/(f*x^2+e)^(1/2)-f^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+
f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1
/2))*e^(1/2)*(d*x^2+c)^(1/2)/b/d/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^
(1/2)+e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi
(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)
^(1/2)/a/b/c/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+e^(3/2
)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*
x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/b/c/(e*(d*x^2+c)/c/
(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {559, 433, 429, 506, 422, 553}

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}dx = \frac{e^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{bc\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{\sqrt{e}f^{3/2}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bd\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{e+fx^2}}$$

```
[In] Int[(e + f*x^2)^(3/2)/((a + b*x^2)*Sqrt[c + d*x^2]),x]
```

```
[Out] (f^2*x*Sqrt[c + d*x^2])/(b*d*Sqrt[e + f*x^2]) - (Sqrt[e]*f^(3/2)*Sqrt[c + d
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*d*Sqrt[(e
*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*Sqrt[f]*Sqrt[c +
 d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*c*Sqrt[
(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (e^(3/2)*(b*e - a*f)*Sq
rt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 -
(d*e)/(c*f)])/(a*b*c*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
```

```
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 559

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{f\int \frac{\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b}+\frac{(be-af)\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b}\\
&= \frac{e^{3/2}(be-af)\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\\
&\quad+\frac{(ef)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{b}+\frac{f^2\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{b}\\
&= \frac{f^2x\sqrt{c+dx^2}}{bd\sqrt{e+fx^2}}+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\\
&\quad+\frac{e^{3/2}(be-af)\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}-\frac{(ef^2)\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{bd}
\end{aligned}$$

$$= \frac{f^2 x\sqrt{c+dx^2}}{bd\sqrt{e+fx^2}} - \frac{\sqrt{e} f^{3/2}\sqrt{c+dx^2}\, E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bd\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\, F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{bc\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{e^{3/2}(be-af)\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.23 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.56

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx =$$
$$-\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(abefE\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)+(be-af)\left(af\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right)+(b\right.\right.}{ab^2\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[In] Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*Sqrt[c + d*x^2]),x]
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*e*f*EllipticE[I*ArcSinh[
Sqrt[d/c]*x], (c*f)/(d*e)] + (b*e - a*f)*(a*f*EllipticF[I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)] + (b*e - a*f)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)])))/(a*b^2*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 3.46 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.91

method	result
default	$\frac{\left(-F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2f^2+F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abef+E\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abef+\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)a^2f^2-2\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)\right.}{a\sqrt{-\frac{d}{c}}\,b^2(df\,x^4+cf\,x^2+de\,x^2+ce)}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(-\frac{f^2\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)a}{b^2\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{f\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)e}{b\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{fe\sqrt{1+\frac{d\,x^2}{c}}}{b\sqrt{-}}\right.}{}$

```
[In] int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*f^2+EllipticF(x*(-d/c)^(1/2
),(c*f/d/e)^(1/2))*a*b*e*f+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*e*
f+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*f^2-2*El
lipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*e*f+Elliptic
Pi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*e^2)*((f*x^2+e)/e)
^(1/2)*((d*x^2+c)/c)^(1/2)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(-d/c)^(1/2)/b
^2/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{(e+fx^2)^{\frac{3}{2}}}{(a+bx^2)\sqrt{c+dx^2}}\,dx$$

```
[In] integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(1/2),x)
```

```
[Out] Integral((e + f*x**2)**(3/2)/((a + b*x**2)*sqrt(c + d*x**2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)\sqrt{dx^2+c}}\,dx$$

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(1/2),x, algorithm="giac")
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*sqrt(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)\,\sqrt{d\,x^2+c}}\,dx$$

```
[In] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(1/2)),x)
[Out] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(1/2)), x)
```

3.74 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 224

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = -\frac{(de-cf)\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{e^{3/2}(be-af)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac(bc-ad)\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(
1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)
/a/c/(-a*d+b*c)/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(-c
*f+d*e)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)
/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/(-a*d+b*c)/c^(1/2)/d^
(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {556, 553, 422}

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{e^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{\sqrt{e+fx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}\sqrt{c+dx^2}(bc-ad)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[In] Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] -(((d*e - c*f)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (
c*f)/(d*e)])/(Sqrt[c]*Sqrt[d]*(b*c - a*d)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^
2))/(e*(c + d*x^2))])) + (e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1
- (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a
*d)*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 556

```
Int[((e_) + (f_.)*(x_)^2)^(3/2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2
)^(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[Sqrt[e + f*x^2]/((
a + b*x^2)*Sqrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqr
t[e + f*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{(be-af)\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{bc-ad} - \frac{(de-cf)\int \frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx}{bc-ad} \\
&= -\frac{(de-cf)\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{d}(bc-ad)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&\quad + \frac{e^{3/2}(be-af)\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.05 (sec) , antiderivative size = 492, normalized size of antiderivative = 2.20

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(abd\sqrt{\frac{d}{c}}e^2x - abc\sqrt{\frac{d}{c}}efx + abd\sqrt{\frac{d}{c}}efx^3 - abc\sqrt{\frac{d}{c}}f^2x^3 - iabe(-de+cf)\right.}{}$$

```
[In] Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[d/c]*(a*b*d*Sqrt[d/c]*e^2*x - a*b*c*Sqrt[d/c]*e*f*x + a*b*d*Sqrt[d/c]
*e*f*x^3 - a*b*c*Sqrt[d/c]*f^2*x^3 - I*a*b*e*(-(d*e) + c*f)*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I
*a*(-(a*c*f^2) + b*e*(-(d*e) + 2*c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b^2*c*e^2*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)] - (2*I)*a*b*c*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*a^2*c*f^2*S
qrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sq
rt[d/c]*x], (c*f)/(d*e)]))/(a*b*d*(-(b*c) + a*d)*Sqrt[c + d*x^2]*Sqrt[e + f
*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(274) = 548.

Time = 4.00 (sec) , antiderivative size = 630, normalized size of antiderivative = 2.81

method	result
default	$\frac{\left(-\sqrt{-\frac{d}{c}}\,abc\,f^2x^3+\sqrt{-\frac{d}{c}}\,abdef\,x^3+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2c\,f^2-2\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abcef+\sqrt{\frac{d}{}}\right.}{}$
elliptic	Expression too large to display

```
[In] int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-(-d/c)^(1/2)*a*b*c*f^2*x^3+(-d/c)^(1/2)*a*b*d*e*f*x^3+((d*x^2+c)/c)^(1/2)
*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*f^2-2*
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*a*b*c*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)
^(1/2),(c*f/d/e)^(1/2))*a*b*d*e^2+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*E
llipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*e*f-((d*x^2+c)/c)^(1/2)*((f*
x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d*e^2-((d*x^2
+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(
```

```
1/2)/(-d/c)^(1/2))*a^2*c*f^2+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elli
pticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*e*f-((d*x^2+
c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1
/2)/(-d/c)^(1/2))*b^2*c*e^2-(-d/c)^(1/2)*a*b*c*e*f*x+(-d/c)^(1/2)*a*b*d*e^2
*x)*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/a/(-d/c)^(1/2)/c/(a*d-b*c)/(d*f*x^4+c
*f*x^2+d*e*x^2+c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{(e+fx^2)^{\frac{3}{2}}}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(3/2),x)
[Out] Integral((e + f*x**2)**(3/2)/((a + b*x**2)*(c + d*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="maxima")
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(3/2),x, algorithm="giac")
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\,(c+dx^2)^{3/2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)\,(d\,x^2+c)^{3/2}}\,dx$$

```
[In] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)),x)
[Out] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(3/2)), x)
```

3.75 $\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 391

$$\begin{aligned}\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx &= -\frac{(de-cf)x\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} \\ &\quad -\frac{(bc(5de-cf)-2ad(de+cf))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}(bc-ad)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\ &\quad +\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &\quad +\frac{be^{3/2}(be-af)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

```
[Out] b*e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f
^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/
2)/a/c/(-a*d+b*c)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
+1/3*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^
(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-a*
d+b*c)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(-c*f+d*e)*x*(f*
x^2+e)^(1/2)/c/(-a*d+b*c)/(d*x^2+c)^(3/2)-1/3*(b*c*(-c*f+5*d*e)-2*a*d*(c*f+
d*e))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(3/2)/(-a*d+b*c)^2/d^
(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {558, 553, 540, 539, 429, 422}

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = -\frac{\sqrt{e+fx^2}(bc(5de-cf)-2ad(cf+de))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{be^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{x\sqrt{e+fx^2}(de-cf)}{3c(c+dx^2)^{3/2}(bc-ad)}$$

```
[In] Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] -1/3*((d*e - c*f)*x*Sqrt[e + f*x^2])/(c*(b*c - a*d)*(c + d*x^2)^(3/2)) - ((
b*c*(5*d*e - c*f) - 2*a*d*(d*e + c*f))*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*Sqrt[d]*(b*c - a*d)^2*Sqrt[
c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + (e^(3/2)*Sqrt[f]*Sqrt[c
 + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(
b*c - a*d)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b*e^(3
/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]
*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*Sqrt[(e*(c + d*x
^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 558

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b*((b*e - a*f)/(b*c - a*d)^2), Int[(c + d*x^2)^(q
 + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Dist[1/(b*c - a*d)^2, Int
[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(b*
e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && GtQ[
r, 1]
```

Rubi steps

$$\text{integral} = -\frac{\int \frac{\sqrt{e+fx^2}\left(2bcde-ad^2e-bc^2f+d^2(be-af)x^2\right)}{(c+dx^2)^{5/2}}\,dx}{(bc-ad)^2} + \frac{(b(be-af))\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{be^{3/2}(be-af)\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{\int \frac{-de(bc(5de-2cf)-ad(2de+cf))-df(bc(4de-cf)-ad(de+2cf))x^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx}{3cd(bc-ad)^2}$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} + \frac{be^{3/2}(be-af)\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(ef)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3c(bc-ad)} - \frac{(bc(5de-cf)-2ad(de+cf))\int \frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx}{3c(bc-ad)^2}$$

$$= -\frac{(de-cf)x\sqrt{e+fx^2}}{3c(bc-ad)\left(c+dx^2\right)^{3/2}} - \frac{(bc(5de-cf)-2ad(de+cf))\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{d}(bc-ad)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(bc-ad)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{be^{3/2}(be-af)\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.03 (sec) , antiderivative size = 999, normalized size of antiderivative = 2.55

$$\int \frac{\left(e+fx^2\right)^{3/2}}{\left(a+bx^2\right)\left(c+dx^2\right)^{5/2}}\,dx = \frac{3a^2cd^2\sqrt{\frac{d}{c}}e^2x - 6abc^3\left(\frac{d}{c}\right)^{3/2}e^2x + 2abc^3\sqrt{\frac{d}{c}}efx + a^2c^3\left(\frac{d}{c}\right)^{3/2}efx - 5abcd^2\sqrt{\cdots}}{\cdots}$$

```
[In] Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x]
```

```
[Out] (3*a^2*c*d^2*Sqrt[d/c]*e^2*x - 6*a*b*c^3*(d/c)^(3/2)*e^2*x + 2*a*b*c^3*Sqrt
[d/c]*e*f*x + a^2*c^3*(d/c)^(3/2)*e*f*x - 5*a*b*c*d^2*Sqrt[d/c]*e^2*x^3 + 2
*a^2*d^3*Sqrt[d/c]*e^2*x^3 + 5*a^2*c*d^2*Sqrt[d/c]*e*f*x^3 - 5*a*b*c^3*(d/c
)^(3/2)*e*f*x^3 + 2*a*b*c^3*Sqrt[d/c]*f^2*x^3 + a^2*c^3*(d/c)^(3/2)*f^2*x^3
```

```
 - 5*a*b*c*d^2*Sqrt[d/c]*e*f*x^5 + 2*a^2*d^3*Sqrt[d/c]*e*f*x^5 + 2*a^2*c*d^
2*Sqrt[d/c]*f^2*x^5 + a*b*c^3*(d/c)^(3/2)*f^2*x^5 + I*a*e*(b*c*(-5*d*e + c*
f) + 2*a*d*(d*e + c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]
*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*(-(d*e) + c*f)*(5*b*c
*e - 2*a*d*e - 3*a*c*f)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]
*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*b^2*c^3*e^2*Sqrt[1
+ (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c
]*x], (c*f)/(d*e)] + (6*I)*a*b*c^3*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*a^
2*c^3*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I
*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*b^2*c^2*d*e^2*x^2*Sqrt[1 + (d*x
^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x],
(c*f)/(d*e)] + (6*I)*a*b*c^2*d*e*f*x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)
/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*a^
2*c^2*d*f^2*x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a
*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2
*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1644 vs. 2(456) = 912.

Time = 5.19 (sec) , antiderivative size = 1645, normalized size of antiderivative = 4.21

method	result	size
elliptic	Expression too large to display	1645
default	Expression too large to display	1879

```
[In] int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/3*(c*f-d*e)
/d^2/(a*d-b*c)/c*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2+1/3*(d*f
*x^2+d*e)*(2*a*c*d*f+2*a*d^2*e+b*c^2*f-5*b*c*d*e)/d/(a*d-b*c)^2/c^2*x/((x^2
+c/d)*(d*f*x^2+d*e))^(1/2)-1/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(
1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+
d*e)/e/d)^(1/2))/d/(a*d-b*c)^2*c*b*f^2+2/3/(a*d-b*c)^2/c^2*e^2/(-d/c)^(1/2)
*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*El
lipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a*d^2-5/3/(a*d-b*c)^2/c*e^
2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2
+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*b*d-2/3/(a*d
-b*c)^2/c^2*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c
*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)
)*a*d^2-1/3/(a*d-b*c)^2*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/
(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/
e/d)^(1/2))*b*f+5/3/(a*d-b*c)^2/c*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x
```

```
^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-
1+(c*f+d*e)/e/d)^(1/2))*b*d-2/3/(a*d-b*c)^2/c*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1
/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c
)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a*d*f-1/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*
(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1
/2),(-1+(c*f+d*e)/e/d)^(1/2))/d*f^2/(a*d-b*c)-2/3/(-d/c)^(1/2)*(1+d*x^2/c)^
(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d
/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/(a*d-b*c)^2*a*f^2+1/(a*d-b*c)^2*a/(-d/c
)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(
1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*f^2+1/3/(
-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*
e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*f/(a*d-b*c)/c*e
+2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^
2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/(a*d-b*c)^2
*b*e*f-2/(a*d-b*c)^2*b/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*
f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(
1/2)/(-d/c)^(1/2))*f*e+1/(a*d-b*c)^2/a*b^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(
1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1
/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*e^2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{5/2}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(5/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\,(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="maxima")
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\,(c+dx^2)^{5/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(5/2),x, algorithm="giac")
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)\,(c+dx^2)^{5/2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)\,\left(d\,x^2+c\right)^{5/2}}\,dx$$

```
[In] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)),x)
[Out] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(5/2)), x)
```

3.76 $\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 639

$$
\begin{aligned}
\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx &= -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)(c+dx^2)^{5/2}} \\
&- \frac{(3bc(3de-cf)-2ad(2de+cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2(c+dx^2)^{3/2}} \\
&- \frac{b\sqrt{d}(be-af)\sqrt{e+fx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)^3\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&+ \frac{(ad(8d^2e^2-3cdef-2c^2f^2)-3bc(6d^2e^2-6cdef+c^2f^2))\sqrt{e+fx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{d}(bc-ad)^2(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&+ \frac{e^{3/2}\sqrt{f}(3bc(3de-2cf)-ad(4de-cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3(bc-ad)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{b^2e^{3/2}(be-af)\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac(bc-ad)^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

```
[Out] b^2*e^(3/2)*(-a*f+b*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x
*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(
1/2)/a/c/(-a*d+b*c)^3/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/
2)+1/15*e^(3/2)*(3*b*c*(-2*c*f+3*d*e)-a*d*(-c*f+4*d*e))*(1/(1+f*x^2/e))^(1/
2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c
/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c^3/(-a*d+b*c)^2/(-c*f+d*e)/(e*(d*x^2+c)
```

```
/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/5*(-c*f+d*e)*x*(f*x^2+e)^(1/2)/c/(-a*
d+b*c)/(d*x^2+c)^(5/2)-1/15*(3*b*c*(-c*f+3*d*e)-2*a*d*(c*f+2*d*e))*x*(f*x^2
+e)^(1/2)/c^2/(-a*d+b*c)^2/(d*x^2+c)^(3/2)+1/15*(a*d*(-2*c^2*f^2-3*c*d*e*f+
8*d^2*e^2)-3*b*c*(c^2*f^2-6*c*d*e*f+6*d^2*e^2))*(1/(1+d*x^2/c))^(1/2)*(1+d*
x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2
))*(f*x^2+e)^(1/2)/c^(5/2)/(-a*d+b*c)^2/(-c*f+d*e)/d^(1/2)/(d*x^2+c)^(1/2)/
(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)-b*(-a*f+b*e)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2
/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*
d^(1/2)*(f*x^2+e)^(1/2)/(-a*d+b*c)^3/c^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e
/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 639, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {558, 555, 553, 422, 540, 541, 539, 429}

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \frac{b^2e^{3/2}\sqrt{c+dx^2}(be-af)\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(bc-ad)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{e^{3/2}\sqrt{f}\sqrt{c+dx^2}(3bc(3de-2cf)-ad(4de-cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{15c^3\sqrt{e+fx^2}(bc-ad)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{\sqrt{e+fx^2}(ad(-2c^2f^2-3cdef+8d^2e^2)-3bc(c^2f^2-6cdef+6d^2e^2))\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{d}\sqrt{c+dx^2}(bc-ad)^2(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{b\sqrt{d}\sqrt{e+fx^2}(be-af)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)^3\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{x\sqrt{e+fx^2}(3bc(3de-cf)-2ad(cf+2de))}{15c^2(c+dx^2)^{3/2}(bc-ad)^2}-\frac{x\sqrt{e+fx^2}(de-cf)}{5c(c+dx^2)^{5/2}(bc-ad)}$$

```
[In] Int[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

```
[Out] -1/5*((d*e - c*f)*x*Sqrt[e + f*x^2])/(c*(b*c - a*d)*(c + d*x^2)^(5/2)) - ((
3*b*c*(3*d*e - c*f) - 2*a*d*(2*d*e + c*f))*x*Sqrt[e + f*x^2])/(15*c^2*(b*c
- a*d)^2*(c + d*x^2)^(3/2)) - (b*Sqrt[d]*(b*e - a*f)*Sqrt[e + f*x^2]*Ellipt
icE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(Sqrt[c]*(b*c - a*d)^3*S
qrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) + ((a*d*(8*d^2*e^2 -
3*c*d*e*f - 2*c^2*f^2) - 3*b*c*(6*d^2*e^2 - 6*c*d*e*f + c^2*f^2))*Sqrt[e +
f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(15*c^(5/2)
*Sqrt[d]*(b*c - a*d)^2*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*
(c + d*x^2))]) + (e^(3/2)*Sqrt[f]*(3*b*c*(3*d*e - 2*c*f) - a*d*(4*d*e - c*f
```

```
))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])
/(15*c^3*(b*c - a*d)^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sq
rt[e + f*x^2]) + (b^2*e^(3/2)*(b*e - a*f)*Sqrt[c + d*x^2]*EllipticPi[1 - (b
*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*(b*c - a*d)^
3*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 555

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rule 558

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b*((b*e - a*f)/(b*c - a*d)^2), Int[(c + d*x^2)^(q
 + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Dist[1/(b*c - a*d)^2, Int
[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(b*
e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && GtQ[
r, 1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\int \frac{\sqrt{e+fx^2}\,(2bcde-ad^2e-bc^2f+d^2(be-af)x^2)}{(c+dx^2)^{7/2}}\,dx}{(bc-ad)^2}+\frac{(b(be-af))\int \frac{\sqrt{e+fx^2}}{(a+bx^2)(c+dx^2)^{3/2}}\,dx}{(bc-ad)^2}\\
&= -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}}\\
&\quad+\frac{\int \frac{-de(bc(9de-4cf)-ad(4de+cf))-df(bc(8de-3cf)-ad(3de+2cf))x^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx}{5cd(bc-ad)^2}\\
&\quad+\frac{(b^2(be-af))\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{(bc-ad)^3}-\frac{(bd(be-af))\int \frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx}{(bc-ad)^3}
\end{aligned}
$$

$$
= -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} - \frac{(3bc(3de-cf)-2ad(2de+cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}}
- \frac{b\sqrt{d}(be-af)\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)^3\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}
+ \frac{b^2e^{3/2}(be-af)\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
- \frac{\int \frac{de(de-cf)(9bc(2de-cf)-ad(8de+cf))+df(de-cf)(3bc(3de-cf)-2ad(2de+cf))x^2}{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}dx}{15c^2d(bc-ad)^2(de-cf)}
$$

$$
= -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} - \frac{(3bc(3de-cf)-2ad(2de+cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}}
- \frac{b\sqrt{d}(be-af)\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)^3\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}
+ \frac{b^2e^{3/2}(be-af)\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
- \frac{(ef(ad(4de-cf)-b(9cde-6c^2f)))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{15c^2(bc-ad)^2(de-cf)}
+ \frac{(ad(8d^2e^2-3cdef-2c^2f^2)-3bc(6d^2e^2-6cdef+c^2f^2))\int \frac{\sqrt{e+fx^2}}{\left(c+dx^2\right)^{3/2}}dx}{15c^2(bc-ad)^2(de-cf)}
$$

$$
= -\frac{(de-cf)x\sqrt{e+fx^2}}{5c(bc-ad)\left(c+dx^2\right)^{5/2}} - \frac{(3bc(3de-cf)-2ad(2de+cf))x\sqrt{e+fx^2}}{15c^2(bc-ad)^2\left(c+dx^2\right)^{3/2}}
- \frac{b\sqrt{d}(be-af)\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)^3\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}
+ \frac{(ad(8d^2e^2-3cdef-2c^2f^2)-3bc(6d^2e^2-6cdef+c^2f^2))\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{15c^{5/2}\sqrt{d}(bc-ad)^2(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}
- \frac{e^{3/2}\sqrt{f}(ad(4de-cf)-b(9cde-6c^2f))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{15c^3(bc-ad)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
+ \frac{b^2e^{3/2}(be-af)\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^3\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.61 (sec) , antiderivative size = 570, normalized size of antiderivative = 0.89

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \frac{-a\sqrt{\frac{d}{c}}x(e+fx^2)\left(3c^2(bc-ad)^2(de-cf)^2+c(bc-ad)(-de+cf)(3bc(-3d\right.}{}$$

```
[In] Integrate[(e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x]
```

```
[Out] (-(a*Sqrt[d/c]*x*(e + f*x^2)*(3*c^2*(b*c - a*d)^2*(d*e - c*f)^2 + c*(b*c -
a*d)*(-(d*e) + c*f)*(3*b*c*(-3*d*e + c*f) + 2*a*d*(2*d*e + c*f))*(c + d*x^2
) + (a^2*d^2*(8*d^2*e^2 - 3*c*d*e*f - 2*c^2*f^2) + 3*b^2*c^2*(11*d^2*e^2 -
11*c*d*e*f + c^2*f^2) + 2*a*b*c*d*(-13*d^2*e^2 + 3*c*d*e*f + 7*c^2*f^2))*(c
 + d*x^2)^2)) + I*(c + d*x^2)^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*
e*(-3*b^2*c^2*(11*d^2*e^2 - 11*c*d*e*f + c^2*f^2) + a^2*d^2*(-8*d^2*e^2 + 3
*c*d*e*f + 2*c^2*f^2) - 2*a*b*c*d*(-13*d^2*e^2 + 3*c*d*e*f + 7*c^2*f^2))*El
lipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (d*e - c*f)*(a*(3*b^2*c^2*e*
(11*d*e - 8*c*f) + a^2*d^2*e*(8*d*e + c*f) + a*b*c*(-26*d^2*e^2 - 7*c*d*e*f
 + 15*c^2*f^2))*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - 15*b*c^3*(
b*e - a*f)^2*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))
)/(15*a*c^3*Sqrt[d/c]*(b*c - a*d)^3*(d*e - c*f)*(c + d*x^2)^(5/2)*Sqrt[e +
f*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3111 vs. 2(723) = 1446.

Time = 6.88 (sec) , antiderivative size = 3112, normalized size of antiderivative = 4.87

method	result	size
elliptic	Expression too large to display	3112
default	Expression too large to display	6211

```
[In] int((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/5*(c*f-d*e)
/d^3/(a*d-b*c)/c*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^3-2/15/(c*
f-d*e)/c/(a*d-b*c)^3*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*
f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d
)^(1/2))*a^2*d^2*f^2-1/5/(c*f-d*e)/c^2/(a*d-b*c)^3*e^2/(-d/c)^(1/2)*(1+d*x^
2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(
x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a^2*d^3*f-26/15/(c*f-d*e)/c^2/(a*d
-b*c)^3*e^3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x
```

^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a*
b*d^3-11/5/(c*f-d*e)/(a*d-b*c)^3*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^
2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1
+(c*f+d*e)/e/d)^(1/2))*b^2*d*f-1/(a*d-b*c)^3*a*b/(-d/c)^(1/2)*(1+d*x^2/c)^(
1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d
/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*f^2+2/(a*d-b*c)^3*b^2/(-d/c)^(
1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2
)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*f*e-1/(a*d-b
*c)^3/a*b^3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x
^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)
^(1/2))*e^2+1/15*(2*a*c*d*f+4*a*d^2*e+3*b*c^2*f-9*b*c*d*e)/(a*d-b*c)^2/d^2/
c^2*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2+1/15*(d*f*x^2+d*e)/d/
c^3/(c*f-d*e)*x*(2*a^2*c^2*d^2*f^2+3*a^2*c*d^3*e*f-8*a^2*d^4*e^2-14*a*b*c^3
*d*f^2-6*a*b*c^2*d^2*e*f+26*a*b*c*d^3*e^2-3*b^2*c^4*f^2+33*b^2*c^3*d*e*f-33
*b^2*c^2*d^2*e^2)/(a*d-b*c)^3/((x^2+c/d)*(d*f*x^2+d*e))^(1/2)+11/5/(-d/c)^(
1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2
)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*d/c/(a*d-b*c)^3*b^2*e^
2+2/15/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*
e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*f^2/(a*
d-b*c)^2/c*a+1/5/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+
c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2
))*f^2/d/(a*d-b*c)^2*b+14/15/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/
2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*
e)/e/d)^(1/2))/(a*d-b*c)^3*a*b*f^2-11/5/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f
*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),
(-1+(c*f+d*e)/e/d)^(1/2))/(a*d-b*c)^3*b^2*e*f+14/15/(c*f-d*e)/(a*d-b*c)^3*e
/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+
c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a*b*d*f^2+2/5
/(c*f-d*e)/c/(a*d-b*c)^3*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/
2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*
e)/e/d)^(1/2))*a*b*d^2*f-3/5/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/
2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*
e)/e/d)^(1/2))*f/(a*d-b*c)^2/c*b*e-2/15/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f
*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),
(-1+(c*f+d*e)/e/d)^(1/2))*d/c/(a*d-b*c)^3*a^2*f^2+8/15/(-d/c)^(1/2)*(1+d*x^
2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(
x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*d^3/c^3/(a*d-b*c)^3*a^2*e^2+1/5/(-
d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e
)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/d*c/(a*d-b*c)^3*
b^2*f^2+2/5/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x
^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*d/
c/(a*d-b*c)^3*a*b*e*f+4/15/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)
/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)
/e/d)^(1/2))*f*d/(a*d-b*c)^2/c^2*a*e-1/5/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+
f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2)

```
,(-1+(c*f+d*e)/e/d)^(1/2))*d^2/c^2/(a*d-b*c)^3*a^2*e*f-26/15/(-d/c)^(1/2)*(
1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Elli
pticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*d^2/c^2/(a*d-b*c)^3*a*b*e^2+
8/15/(c*f-d*e)/c^3/(a*d-b*c)^3*e^3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/
e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(
c*f+d*e)/e/d)^(1/2))*a^2*d^4+1/5/(c*f-d*e)*c/(a*d-b*c)^3*e/(-d/c)^(1/2)*(1+
d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Ellipt
icE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*b^2*f^2+11/5/(c*f-d*e)/c/(a*d-
b*c)^3*e^3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^
2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*b^2
*d^2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \text{Timed out}$$

```
[In] integrate((f*x**2+e)**(3/2)/(b*x**2+a)/(d*x**2+c)**(7/2),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="maxima")
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Giac [F]

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \int \frac{(fx^2+e)^{\frac{3}{2}}}{(bx^2+a)(dx^2+c)^{\frac{7}{2}}}\,dx$$

```
[In] integrate((f*x^2+e)^(3/2)/(b*x^2+a)/(d*x^2+c)^(7/2),x, algorithm="giac")
[Out] integrate((f*x^2 + e)^(3/2)/((b*x^2 + a)*(d*x^2 + c)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(e+fx^2)^{3/2}}{(a+bx^2)(c+dx^2)^{7/2}}\,dx = \int \frac{(f\,x^2+e)^{3/2}}{(b\,x^2+a)\,(d\,x^2+c)^{7/2}}\,dx$$

```
[In] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)),x)
[Out] int((e + f*x^2)^(3/2)/((a + b*x^2)*(c + d*x^2)^(7/2)), x)
```

3.77 $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 621

$$\begin{aligned}\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx &= \frac{d(bc-ad)x\sqrt{c+dx^2}}{b^2\sqrt{e+fx^2}} - \frac{2d(de-2cf)x\sqrt{c+dx^2}}{3bf\sqrt{e+fx^2}} \\ &+ \frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3bf} - \frac{d(bc-ad)\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{2d\sqrt{e}(de-2cf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{d(bc-ad)\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{b^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &- \frac{d\sqrt{e}(de-3cf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\ &+ \frac{c^{3/2}(bc-ad)^2\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}\end{aligned}$$

```
[Out] d*(-a*d+b*c)*x*(d*x^2+c)^(1/2)/b^2/(f*x^2+e)^(1/2)-2/3*d*(-2*c*f+d*e)*x*(d*
x^2+c)^(1/2)/b/f/(f*x^2+e)^(1/2)+2/3*d*(-2*c*f+d*e)*(1/(1+f*x^2/e))^(1/2)*(
1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^
(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f
*x^2+e)^(1/2)-1/3*d*(-3*c*f+d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*El
lipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x
```

```
^2+c)^(1/2)/b/f^(3/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-d*(-a
*d+b*c)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)
/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b^2/f^(1/2)/(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+d*(-a*d+b*c)*(1/(1+f*x^2/e))
^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d
*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e)
)^(1/2)/(f*x^2+e)^(1/2)+1/3*d^2*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/f+c^(3/
2)*(-a*d+b*c)^2*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2
)/c^(1/2)/(1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/
b^2/e/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 621, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {559, 427, 545, 429, 506, 422, 433, 553}

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)^2\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{d\sqrt{e}\sqrt{c+dx^2}(bc-ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{b^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{d\sqrt{e}\sqrt{c+dx^2}(bc-ad)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b^2\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{dx\sqrt{c+dx^2}(bc-ad)}{b^2\sqrt{e+fx^2}} - \frac{d\sqrt{e}\sqrt{c+dx^2}(de-3cf)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{2d\sqrt{e}\sqrt{c+dx^2}(de-2cf)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3bf} - \frac{2dx\sqrt{c+dx^2}(de-2cf)}{3bf\sqrt{e+fx^2}}$$

```
[In] Int[(c + d*x^2)^(5/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (d*(b*c - a*d)*x*Sqrt[c + d*x^2])/(b^2*Sqrt[e + f*x^2]) - (2*d*(d*e - 2*c*f
)*x*Sqrt[c + d*x^2])/(3*b*f*Sqrt[e + f*x^2]) + (d^2*x*Sqrt[c + d*x^2]*Sqrt[
e + f*x^2])/(3*b*f) - (d*(b*c - a*d)*Sqrt[e]*Sqrt[c + d*x^2]*EllipticE[ArcT
an[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2)
)/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (2*d*Sqrt[e]*(d*e - 2*c*f)*Sqrt[c + d
```

```
*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*b*f^(3/2)
*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(b*c - a*d)*Sq
rt[e]*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f
)])/(b^2*Sqrt[f]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (
d*Sqrt[e]*(d*e - 3*c*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e
]], 1 - (d*e)/(c*f)])/(3*b*f^(3/2)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sq
rt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)^2*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c
)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*b^2*Sqrt[d]*e*Sq
rt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

```
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 559

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rubi steps

$$\text{integral} = \frac{d\int \frac{(c+dx^2)^{3/2}}{\sqrt{e+fx^2}}\,dx}{b} + \frac{(bc-ad)\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{b}$$

$$= \frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3bf} + \frac{(d(bc-ad))\int \frac{\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx}{b^2} + \frac{(bc-ad)^2\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{b^2} + \frac{d\int \frac{-c(de-3cf)-2d(de-2cf)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3bf}$$

$$= \frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3bf} + \frac{c^{3/2}(bc-ad)^2\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{(cd(bc-ad))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{b^2} + \frac{(d^2(bc-ad))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{b^2} - \frac{(cd(de-3cf))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3bf} - \frac{(2d^2(de-2cf))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3bf}$$

$$= \frac{d(bc-ad)x\sqrt{c+dx^2}}{b^2\sqrt{e+fx^2}} - \frac{2d(de-2cf)x\sqrt{c+dx^2}}{3bf\sqrt{e+fx^2}} + \frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3bf}$$
$$+ \frac{d(bc-ad)\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{d\sqrt{e}(de-3cf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{c^{3/2}(bc-ad)^2\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$- \frac{(d(bc-ad)e)\int\frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{b^2} + \frac{(2de(de-2cf))\int\frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{3bf}$$

$$= \frac{d(bc-ad)x\sqrt{c+dx^2}}{b^2\sqrt{e+fx^2}} - \frac{2d(de-2cf)x\sqrt{c+dx^2}}{3bf\sqrt{e+fx^2}} + \frac{d^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3bf}$$
$$- \frac{d(bc-ad)\sqrt{e}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{2d\sqrt{e}(de-2cf)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{d(bc-ad)\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b^2\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{d\sqrt{e}(de-3cf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3bf^{3/2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{c^{3/2}(bc-ad)^2\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab^2\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.80 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.56

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \frac{-iabd^2e(-2bde+7bcf-3adf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ia\ldots}{}$$

```
[In] Integrate[(c + d*x^2)^(5/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*a*b*d^2*e*(-2*b*d*e + 7*b*c*f - 3*a*d*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
 (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*a*d*(3*a^2*d
^2*f^2 + 3*a*b*d*f*(d*e - 3*c*f) + b^2*(2*d^2*e^2 - 8*c*d*e*f + 9*c^2*f^2))
*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x],
(c*f)/(d*e)] + f*(a*b^2*c*d*(d/c)^(3/2)*x*(c + d*x^2)*(e + f*x^2) - (3*I)*(
b*c - a*d)^3*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*
d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(3*a*b^3*Sqrt[d/c]*f^2*Sqrt[c +
d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 8.40 (sec) , antiderivative size = 741, normalized size of antiderivative = 1.19

method	result
risch	$\frac{d^2x\sqrt{dx^2+c}\sqrt{fx^2+e}}{3bf}-\frac{\left(\frac{3\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)f\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)}{b^2a\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+ce}}+\frac{d\left(-\frac{3a^2d^2f\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\ldots}{\sqrt{-\frac{d}{c}}\sqrt{dfx^4+\ldots}}\right.}{}\right.}{}$
default	$\frac{\left(\sqrt{-\frac{d}{c}}ab^2d^3f^2x^5+\sqrt{-\frac{d}{c}}ab^2cd^2f^2x^3+\sqrt{-\frac{d}{c}}ab^2d^3efx^3+3\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^3d^3f^2-9\sqrt{\frac{dx^2+c}{c}}\sqrt{\frac{fx^2+e}{e}}F\left(\ldots\right.\right.}{}$
elliptic	Expression too large to display

```
[In] int((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*d^2*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/b/f-1/3/f/b*(3*(a^3*d^3-3*a^2*b*c
*d^2+3*a*b^2*c^2*d-b^3*c^3)*f/b^2/a/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2
/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c
/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))+d/b^2*(-3*a^2*d^2*f/(-d/c)^(1/2)*(1+d*x^2/c
)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(
-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-9*b^2*c^2*f/(-d/c)^(1/2)*(1+d*x^2/c)^
```

```
(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d
/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+b^2*d*c*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2
)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^
(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+9*a*b*c*d*f/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*
(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1
/2),(-1+(c*f+d*e)/e/d)^(1/2))-(3*a*b*d^2*f-7*b^2*c*d*f+2*b^2*d^2*e)*e/(-d/c
)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(
1/2)/f*(EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-EllipticE(x*(-d/
c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2)))))*((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)
^(1/2)/(f*x^2+e)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((d*x**2+c)**(5/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
[Out] Integral((c + d*x**2)**(5/2)/((a + b*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(d\,x^2+c)^{5/2}}{(b\,x^2+a)\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int((c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(1/2)),x)
[Out] int((c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(1/2)), x)
```

3.78 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 319

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} - \frac{d\sqrt{e}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{d\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{c^{3/2}(bc-ad)\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{ab\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] d*x*(d*x^2+c)^(1/2)/b/(f*x^2+e)^(1/2)-d*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(
1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/
2)*(d*x^2+c)^(1/2)/b/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2
)+d*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+
f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/b/f^(1/2)/(e*(d*x
^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+c^(3/2)*(-a*d+b*c)*(1/(1+d*x^2/c))
^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),1-b
*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/b/e/d^(1/2)/(d*x^2+c)^(1/2)/(c*
(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {559, 433, 429, 506, 422, 553}

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{ab\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$+\frac{d\sqrt{e}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{d\sqrt{e}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}+\frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}}$$

```
[In] Int[(c + d*x^2)^(3/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (d*x*Sqrt[c + d*x^2])/(b*Sqrt[e + f*x^2]) - (d*Sqrt[e]*Sqrt[c + d*x^2]*Elli
pticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*Sqrt[f]*Sqrt[(e*(c
+ d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*Sqrt[e]*Sqrt[c + d*x^2]*El
lipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(b*Sqrt[f]*Sqrt[(e*(
c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*Sqrt[e
 + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f
)/(d*e)])/(a*b*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2
))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
```

```
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 559

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[d/b, Int[(c + d*x^2)^(q - 1)*(e + f*x^2)^r, x], x
] + Dist[(b*c - a*d)/b, Int[(c + d*x^2)^(q - 1)*((e + f*x^2)^r/(a + b*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f, r}, x] && GtQ[q, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{d\int \frac{\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx}{b} + \frac{(bc-ad)\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{b} \\
&= \frac{c^{3/2}(bc-ad)\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&\quad + \frac{(cd)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{b} + \frac{d^2\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{b} \\
&= \frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} + \frac{d\sqrt{e}\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{c^{3/2}(bc-ad)\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{(de)\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{b}
\end{aligned}$$

$$= \frac{dx\sqrt{c+dx^2}}{b\sqrt{e+fx^2}} - \frac{d\sqrt{e}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{d\sqrt{e}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{b\sqrt{f}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{c^{3/2}(bc-ad)\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{ab\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.62

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx =$$
$$-\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(abd^2eE\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ad(bde-2bcf+adf)\operatorname{EllipticF}\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right.\right.}{ab^2\sqrt{\frac{d}{c}}f\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[In] Integrate[(c + d*x^2)^(3/2)/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*b*d^2*e*EllipticE[I*ArcSin
h[Sqrt[d/c]*x], (c*f)/(d*e)] - a*d*(b*d*e - 2*b*c*f + a*d*f)*EllipticF[I*Ar
cSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (b*c - a*d)^2*f*EllipticPi[(b*c)/(a*d),
I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*b^2*Sqrt[d/c]*f*Sqrt[c + d*x^2]*S
qrt[e + f*x^2])
```

Maple [A] (verified)

Time = 3.42 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.07

method	result
default	$\frac{\left(-F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2d^2f+2F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)abcdf-F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ab\,d^2e+E\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ab\,d^2e+\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)a^2\right.}{a\,b^2f\sqrt{-\frac{d}{c}}\,(df\,x^4+cf\,x^2+de\,x^2}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(-\frac{d^2\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)a}{b^2\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{2d\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)c}{b\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{d^2e\sqrt{1+\frac{d\,x^2}{c}}}{b\sqrt{-}}\right.}{}$

[In] int((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^2*f+2*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d*f-EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^2*e+EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^2*e+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^2*d^2*f-2*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*d*f+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^2*f)*((f*x^2+e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/a/b^2/f/(-d/c)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$$

[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)

[Out] Integral((c + d*x**2)**(3/2)/((a + b*x**2)*sqrt(e + f*x**2)), x)

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="maxima")

[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{(b\,x^2+a)\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int((c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(1/2)),x)
[Out] int((c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(1/2)), x)
```

3.79 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 102

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \frac{c^{3/2}\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/2
)/(1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/e/d^(1/2
)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {553}

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \frac{c^{3/2}\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[In] Int[Sqrt[c + d*x^2]/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/
(e*(c + d*x^2))])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
```

```
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\text{integral} = \frac{c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.67 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx =$$
$$-\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\left(ad\,\text{EllipticF}\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right)+(bc-ad)\,\text{EllipticPi}\left(\frac{bc}{ad},i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right.\right.}{ab\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[In] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*(a*d*EllipticF[I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)] + (b*c - a*d)*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)]))/(a*b*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.87

method	result
default	$\frac{\left(F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ad-\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)ad+\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)bc\right)\sqrt{\frac{f\,x^2+e}{e}}\sqrt{\frac{d\,x^2+c}{c}}\sqrt{f\,x^2+e}\sqrt{d\,x^2+c}}{ba\sqrt{-\frac{d}{c}}\,(df\,x^4+cf\,x^2+de\,x^2+ce)}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{d\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{b\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}-\frac{\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)d}{b\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{\sqrt{1+\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\Pi\left(\right.}{a\sqrt{-\frac{d}{c}}\sqrt{df\,x^4+c\right.}\right.}{\sqrt{d\,x^2+c}\sqrt{f\,x^2+e}}$

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d-EllipticPi(x*(-d/c)^(1/2),b*
c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*d+EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/
e)^(1/2)/(-d/c)^(1/2))*b*c)/b*((f*x^2+e)/e)^(1/2)*((d*x^2+c)/c)^(1/2)*(f*x^
2+e)^(1/2)*(d*x^2+c)^(1/2)/a/(-d/c)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx=\text{Timed out}$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx=\int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/((a + b*x^2)*(e + f*x^2)^(1/2)),x)
[Out] int((c + d*x^2)^(1/2)/((a + b*x^2)*(e + f*x^2)^(1/2)), x)
```

3.80 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 100

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[Out] EllipticPi(x*d^(1/2)/(-c)^(1/2),b*c/a/d,(c*f/d/e)^(1/2))*(-c)^(1/2)*(1+d*x^
2/c)^(1/2)*(1+f*x^2/e)^(1/2)/a/d^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {552, 551}

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[In] Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), A
rcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[
e + f*x^2])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\text{integral} = \frac{\sqrt{1+\frac{dx^2}{c}}\int \frac{1}{(a+bx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{e+fx^2}}\,dx}{\sqrt{c+dx^2}}$$

$$= \frac{\left(\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\right)\int \frac{1}{(a+bx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}}\,dx}{\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$= \frac{\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = -\frac{i\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(\frac{bc}{ad},i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right),\frac{cf}{de}\right)}{a\sqrt{\frac{d}{c}}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[In] Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] ((-I)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*Arc
Sinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*Sqrt[c + d*x^2]*Sqrt[e + f*x^
2])
```

Maple [A] (verified)

Time = 3.89 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{\Pi\left(x\sqrt{-\frac{d}{c}}, \frac{bc}{ad}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)\sqrt{\frac{f x^{2}+e}{e}}\, \sqrt{\frac{d x^{2}+c}{c}}\, \sqrt{f x^{2}+e}\, \sqrt{d x^{2}+c}}{a\sqrt{-\frac{d}{c}}\,(df\, x^{4}+cf\, x^{2}+de\, x^{2}+ce)}$	118
elliptic	$\frac{\sqrt{(d x^{2}+c)(f x^{2}+e)}\, \sqrt{1+\frac{d x^{2}}{c}}\, \sqrt{1+\frac{f x^{2}}{e}}\, \Pi\left(x\sqrt{-\frac{d}{c}}, \frac{bc}{ad}, \frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)}{\sqrt{d x^{2}+c}\, \sqrt{f x^{2}+e}\, a\sqrt{-\frac{d}{c}}\, \sqrt{df\, x^{4}+cf\, x^{2}+de\, x^{2}+ce}}$	133

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*((f*x^2+e)/e)^
(1/2)*((d*x^2+c)/c)^(1/2)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/a/(-d/c)^(1/2)/(d
*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.81 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 344

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = -\frac{d^{3/2}\sqrt{e+fx^2}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{d\sqrt{e}(bde-2bcf+adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c(bc-ad)^2\sqrt{f}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{b^2c^{3/2}\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}(bc-ad)^2e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] -d*(a*d*f-2*b*c*f+b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(
x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1
/2)/c/(-a*d+b*c)^2/(-c*f+d*e)/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^
2+e)^(1/2)-d^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1
/2)/c^(1/2)/(1+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/(-a*d+b*c)
/(-c*f+d*e)/c^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)+b^2*c^(
3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/2)/(
1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/(-a*d+b*c)^
2/e/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {560, 553, 539, 429, 422}

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{b^2c^{3/2}\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(bc-ad)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{d^{3/2}\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}\sqrt{c+dx^2}(bc-ad)(de-cf)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{d\sqrt{e}\sqrt{c+dx^2}(adf-2bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -((d^(3/2)*Sqrt[e + f*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)
/(d*e)])/(Sqrt[c]*(b*c - a*d)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^
2))/(e*(c + d*x^2))])) - (d*Sqrt[e]*(b*d*e - 2*b*c*f + a*d*f)*Sqrt[c + d*x^
2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^
2*Sqrt[f]*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]
) + (b^2*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*e*Sqrt[c + d*x^2]
*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
```

```
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{b^2 \int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{(bc-ad)^2} - \frac{d \int \frac{2bc-ad+bdx^2}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx}{(bc-ad)^2} \\
&= \frac{b^2 c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}(bc-ad)^2 e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&\quad - \frac{d^2 \int \frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}\,dx}{(bc-ad)(de-cf)} - \frac{(d(bde-2bcf+adf)) \int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{(bc-ad)^2(de-cf)} \\
&= -\frac{d^{3/2}\sqrt{e+fx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{\sqrt{c}(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&\quad - \frac{d\sqrt{e}(bde-2bcf+adf)\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c(bc-ad)^2\sqrt{f}(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{b^2 c^{3/2}\sqrt{e+fx^2}\,\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}(bc-ad)^2 e\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.22 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{\frac{d}{c}}\left(acd\left(\frac{d}{c}\right)^{3/2}ex + acd\left(\frac{d}{c}\right)^{3/2}fx^3 + iad^2e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(iar\right.\right.}{}$$

[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2]),x]

[Out] (Sqrt[d/c]*(a*c*d*(d/c)^(3/2)*e*x + a*c*d*(d/c)^(3/2)*f*x^3 + I*a*d^2*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*a*d*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b*c*d*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*b*c^2*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]))/(a*d*(-(b*c) + a*d)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])

Maple [A] (verified)

Time = 5.06 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.20

method	result
default	$\frac{\left(-\sqrt{-\frac{d}{c}}\,a\,d^2 f\,x^3+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)acdf-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a\,d^2e+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,E\left(x\right.\right.}{ca(ad-bc)\sqrt{-\frac{d}{c}}}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(-\frac{\left(df\,x^2+de\right)dx}{c(cf-de)(ad-bc)\sqrt{\left(x^2+\frac{c}{d}\right)\left(df\,x^2+de\right)}}+\frac{\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)d}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}\,c(ad-bc)}+\frac{d^2e\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,E\left(\right.}{c(cf-de)(ad-bc)\sqrt{-\frac{d}{c}}\,\sqrt{a}}\right.}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}$

[In] int(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)

[Out] (-(-d/c)^(1/2)*a*d^2*f*x^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*c*d*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d^2*e-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c^2*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c*d*e-(-d/c)^(1/2)*a*d^2*e*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/c/a/(a*d-b*c)/(-d/c)^(1/2)/(c*f-d*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(1/2),x)
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{3/2}\,\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)\,\left(d\,x^2+c\right)^{3/2}\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

3.82 $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 435

$$\begin{aligned}
\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx &= -\frac{d^2x\sqrt{e+fx^2}}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}} \\
&- \frac{d^{3/2}(bc(5de-7cf)-2ad(de-2cf))\sqrt{e+fx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} \\
&- \frac{d\sqrt{e}\sqrt{f}(ad(de-3cf)-2bc(2de-3cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{b^2\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}(bc-ad)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}
\end{aligned}$$

```
[Out] -1/3*d*(a*d*(-3*c*f+d*e)-2*b*c*(-3*c*f+2*d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x
^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2)
)*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c^2/(-a*d+b*c)^2/(-c*f+d*e)^2/(e*(d*x^2+c
)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*d^2*x*(f*x^2+e)^(1/2)/c/(-a*d+b*c)
/(-c*f+d*e)/(d*x^2+c)^(3/2)-1/3*d^(3/2)*(b*c*(-7*c*f+5*d*e)-2*a*d*(-2*c*f+d
*e))*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticE(x*d^(1/2)/c^(1/2)/(1
+d*x^2/c)^(1/2),(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/c^(3/2)/(-a*d+b*c)^2/(-c
*f+d*e)^2/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)+b^2*EllipticPi(x*
d^(1/2)/(-c)^(1/2),b*c/a/d,(c*f/d/e)^(1/2))*(-c)^(1/2)*(1+d*x^2/c)^(1/2)*(1
+f*x^2/e)^(1/2)/a/(-a*d+b*c)^2/d^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {560, 552, 551, 541, 539, 429, 422}

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \frac{b^2\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}\,\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)^2}$$
$$-\frac{d^{3/2}\sqrt{e+fx^2}(bc(5de-7cf)-2ad(de-2cf))E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}\sqrt{c+dx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(ad(de-3cf)-2bc(2de-3cf))\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{d^2x\sqrt{e+fx^2}}{3c(c+dx^2)^{3/2}(bc-ad)(de-cf)}$$

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] -1/3*(d^2*x*Sqrt[e + f*x^2])/(c*(b*c - a*d)*(d*e - c*f)*(c + d*x^2)^(3/2))
- (d^(3/2)*(b*c*(5*d*e - 7*c*f) - 2*a*d*(d*e - 2*c*f))*Sqrt[e + f*x^2]*Elli
pticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(3*c^(3/2)*(b*c - a*d)
^2*(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]) - (
d*Sqrt[e]*Sqrt[f]*(a*d*(d*e - 3*c*f) - 2*b*c*(2*d*e - 3*c*f))*Sqrt[c + d*x^
2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a
*d)^2*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
+ (b^2*Sqrt[-c]*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a
*d), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*S
qrt[c + d*x^2]*Sqrt[e + f*x^2])
```

<u>Rule 422</u>

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

<u>Rule 429</u>

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{b^2 \int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{(bc-ad)^2} - \frac{d \int \frac{2bc-ad+bdx^2}{(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{d^2x\sqrt{e+fx^2}}{3c(bc-ad)(de-cf)\left(c+dx^2\right)^{3/2}} + \frac{d\int \frac{-bc(5de-6cf)+ad(2de-3cf)-d(bc-ad)fx^2}{\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}}dx}{3c(bc-ad)^2(de-cf)}$$
$$+ \frac{\left(b^2\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{(a+bx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{e+fx^2}}dx}{(bc-ad)^2\sqrt{c+dx^2}}$$

$$= -\frac{d^2x\sqrt{e+fx^2}}{3c(bc-ad)(de-cf)\left(c+dx^2\right)^{3/2}}$$
$$- \frac{(df(ad(de-3cf)-2bc(2de-3cf)))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{3c(bc-ad)^2(de-cf)^2}$$
$$- \frac{(d^2(bc(5de-7cf)-2ad(de-2cf)))\int \frac{\sqrt{e+fx^2}}{(c+dx^2)^{3/2}}dx}{3c(bc-ad)^2(de-cf)^2}$$
$$+ \frac{\left(b^2\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\right)\int \frac{1}{(a+bx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}}dx}{(bc-ad)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$= -\frac{d^2x\sqrt{e+fx^2}}{3c(bc-ad)(de-cf)\left(c+dx^2\right)^{3/2}}$$
$$- \frac{d^{3/2}(bc(5de-7cf)-2ad(de-2cf))\sqrt{e+fx^2}E\left(\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{3c^{3/2}(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$- \frac{d\sqrt{e}\sqrt{f}(ad(de-3cf)-2bc(2de-3cf))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{b^2\sqrt{-c}\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{a\sqrt{d}(bc-ad)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.33 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx^2)\left(c+dx^2\right)^{5/2}\sqrt{e+fx^2}}dx = \frac{acd\left(\frac{d}{c}\right)^{3/2}x(e+fx^2)\left(bc(-6cde+8c^2f-5d^2ex^2+7cdfx^2)+ad(-5\right.}{}$$

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (a*c*d*(d/c)^(3/2)*x*(e + f*x^2)*(b*c*(-6*c*d*e + 8*c^2*f - 5*d^2*e*x^2 + 7
*c*d*f*x^2) + a*d*(-5*c^2*f + 2*d^2*e*x^2 + c*d*(3*e - 4*f*x^2))) + I*a*d^2
```

```
*e*(2*a*d*(d*e - 2*c*f) + b*c*(-5*d*e + 7*c*f))*(c + d*x^2)*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I
*a*d*(-(d*e) + c*f)*(a*d*(2*d*e - 3*c*f) + b*c*(-5*d*e + 6*c*f))*(c + d*x^2
)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x],
 (c*f)/(d*e)] - (3*I)*b^2*c^2*(d*e - c*f)^2*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]
*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)])/(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2*(d*e - c*f)^2*(c + d*x^2)^(3/2)*Sq
rt[e + f*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. $2(460) = 920$.

Time = 6.55 (sec) , antiderivative size = 1325, normalized size of antiderivative = 3.05

method	result	size
elliptic	Expression too large to display	1325
default	Expression too large to display	2062

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/3/c/(c*f-d*
e)*x/(a*d-b*c)*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(x^2+c/d)^2-1/3*(d*f*x^2
+d*e)*d/c^2/(c*f-d*e)^2*x*(4*a*c*d*f-2*a*d^2*e-7*b*c^2*f+5*b*c*d*e)/(a*d-b*
c)^2/((x^2+c/d)*(d*f*x^2+d*e))^(1/2)-1/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+
f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2)
,(-1+(c*f+d*e)/e/d)^(1/2))*d*f/c/(c*f-d*e)/(a*d-b*c)+4/3/(-d/c)^(1/2)*(1+d*
x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Elliptic
F(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*e)*d^2/c/(a*d-b*c)^2*a*f-
2/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x
^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*e)*
d^3/c^2/(a*d-b*c)^2*a*e-7/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2
)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e
)/e/d)^(1/2))/(c*f-d*e)*d/(a*d-b*c)^2*b*f+5/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2
)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^
(1/2),(-1+(c*f+d*e)/e/d)^(1/2))/(c*f-d*e)*d^2/c/(a*d-b*c)^2*b*e+4/3*d^3/(c*
f-d*e)^2/c/(a*d-b*c)^2*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(
d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e
/d)^(1/2))*a*f-2/3*d^4/(c*f-d*e)^2/c^2/(a*d-b*c)^2*e^2/(-d/c)^(1/2)*(1+d*x^
2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(
x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*a-7/3*d^2/(c*f-d*e)^2/(a*d-b*c)^2*
e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2
+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*b*f+5/3*d^3/
(c*f-d*e)^2/c/(a*d-b*c)^2*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1
/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d
```

```
*e)/e/d)^(1/2))*b+b^2/(a*d-b*c)^2/a/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2
/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c
/a/d,(-f/e)^(1/2)/(-d/c)^(1/2)))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx=\text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx=\int \frac{1}{(a+bx^2)\,(c+dx^2)^{\frac{5}{2}}\,\sqrt{e+fx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(5/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)\,(c+dx^2)^{5/2}\,\sqrt{e+fx^2}}\,dx=\int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)\,(d\,x^2+c)^{5/2}\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(1/2)), x)
```

3.83 $\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 980

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b(be-af)^2\sqrt{e+fx^2}}$$
$$+\frac{(be(6d^2e^2-7cdef-c^2f^2)-af(8d^2e^2-13cdef+3c^2f^2))\,x\sqrt{c+dx^2}}{3ef(be-af)^2\sqrt{e+fx^2}}$$
$$+\frac{(de-cf)x(c+dx^2)^{3/2}}{e(be-af)\sqrt{e+fx^2}}+\frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3(be-af)^2}$$
$$+\frac{d(af(4de-3cf)-be(3de-2cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef(be-af)^2}$$
$$-\frac{(bc-ad)\sqrt{e}(bde+4bcf-3adf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{(be(6d^2e^2-7cdef-c^2f^2)-af(8d^2e^2-13cdef+3c^2f^2))\,\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3\sqrt{e}f^{3/2}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{d(5bc-3ad)(bc-ad)e^{3/2}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3bc\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}(2adf(2de-3cf)-b(3d^2e^2-2cdef-3c^2f^2))\,\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3f^{3/2}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{(bc-ad)^3e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{abc\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] (-c*f+d*e)*x*(d*x^2+c)^(3/2)/e/(-a*f+b*e)/(f*x^2+e)^(1/2)+1/3*(-a*d+b*c)*(-
3*a*d*f+4*b*c*f+b*d*e)*x*(d*x^2+c)^(1/2)/b/(-a*f+b*e)^2/(f*x^2+e)^(1/2)+1/3
*(b*e*(-c^2*f^2-7*c*d*e*f+6*d^2*e^2)-a*f*(3*c^2*f^2-13*c*d*e*f+8*d^2*e^2))*
x*(d*x^2+c)^(1/2)/e/f/(-a*f+b*e)^2/(f*x^2+e)^(1/2)-1/3*(b*e*(-c^2*f^2-7*c*d
*e*f+6*d^2*e^2)-a*f*(3*c^2*f^2-13*c*d*e*f+8*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)
*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f
)^(1/2))*(d*x^2+c)^(1/2)/f^(3/2)/(-a*f+b*e)^2/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2
+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*(2*a*d*f*(-3*c*f+2*d*e)-b*(-3*c^2*f^2-2*c*d*
e*f+3*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)
/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/f^(3/
2)/(-a*f+b*e)^2/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d*(-3*a
*d+5*b*c)*(-a*d+b*c)*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellipt
icF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/
b/c/(-a*f+b*e)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+(-
a*d+b*c)^3*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(
1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)
/a/b/c/(-a*f+b*e)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
-1/3*(-a*d+b*c)*(-3*a*d*f+4*b*c*f+b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^
(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1
/2)*(d*x^2+c)^(1/2)/b/(-a*f+b*e)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/
(f*x^2+e)^(1/2)+1/3*d*(-a*d+b*c)*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(-a*f+b*
e)^2+1/3*d*(a*f*(-3*c*f+4*d*e)-b*e*(-2*c*f+3*d*e))*x*(d*x^2+c)^(1/2)*(f*x^2
+e)^(1/2)/e/f/(-a*f+b*e)^2
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 980, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used

$= \{558, 557, 553, 542, 545, 429, 506, 422, 540\}$

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \frac{e^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)(bc-ad)^3}{abc\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$-\frac{\sqrt{e}(bde+4bcf-3adf)\sqrt{dx^2+c}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)(bc-ad)}{3b\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$+\frac{d(5bc-3ad)e^{3/2}\sqrt{dx^2+c}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)(bc-ad)}{3bc\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$+\frac{dx\sqrt{dx^2+c}\sqrt{fx^2+e}(bc-ad)}{3(be-af)^2}+\frac{(bde+4bcf-3adf)x\sqrt{dx^2+c}(bc-ad)}{3b(be-af)^2\sqrt{fx^2+e}}$$
$$-\frac{(be(6d^2e^2-7cdfe-c^2f^2)-af(8d^2e^2-13cdfe+3c^2f^2))\sqrt{dx^2+c}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3\sqrt{e}f^{3/2}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$-\frac{\sqrt{e}(2adf(2de-3cf)-b(3d^2e^2-2cdfe-3c^2f^2))\sqrt{dx^2+c}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3f^{3/2}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$+\frac{d(af(4de-3cf)-be(3de-2cf))x\sqrt{dx^2+c}\sqrt{fx^2+e}}{3ef(be-af)^2}+\frac{(de-cf)x(dx^2+c)^{3/2}}{e(be-af)\sqrt{fx^2+e}}$$
$$+\frac{(be(6d^2e^2-7cdfe-c^2f^2)-af(8d^2e^2-13cdfe+3c^2f^2))x\sqrt{dx^2+c}}{3ef(be-af)^2\sqrt{fx^2+e}}$$

```
[In] Int[(c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((b*c - a*d)*(b*d*e + 4*b*c*f - 3*a*d*f)*x*Sqrt[c + d*x^2])/(3*b*(b*e - a*f
)^2*Sqrt[e + f*x^2]) + ((b*e*(6*d^2*e^2 - 7*c*d*e*f - c^2*f^2) - a*f*(8*d^2
*e^2 - 13*c*d*e*f + 3*c^2*f^2))*x*Sqrt[c + d*x^2])/(3*e*f*(b*e - a*f)^2*Sqr
t[e + f*x^2]) + ((d*e - c*f)*x*(c + d*x^2)^(3/2))/(e*(b*e - a*f)*Sqrt[e + f
*x^2]) + (d*(b*c - a*d)*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(3*(b*e - a*f)^2
) + (d*(a*f*(4*d*e - 3*c*f) - b*e*(3*d*e - 2*c*f))*x*Sqrt[c + d*x^2]*Sqrt[e
 + f*x^2])/(3*e*f*(b*e - a*f)^2) - ((b*c - a*d)*Sqrt[e]*(b*d*e + 4*b*c*f -
3*a*d*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(3*b*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqr
t[e + f*x^2]) - ((b*e*(6*d^2*e^2 - 7*c*d*e*f - c^2*f^2) - a*f*(8*d^2*e^2 -
13*c*d*e*f + 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[
e]], 1 - (d*e)/(c*f)])/(3*Sqrt[e]*f^(3/2)*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2)
)/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*(5*b*c - 3*a*d)*(b*c - a*d)*e^(3/2
)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/
(3*b*c*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2]) - (Sqrt[e]*(2*a*d*f*(2*d*e - 3*c*f) - b*(3*d^2*e^2 - 2*c*d*e*f - 3
```

```
*c^2*f^2))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(3*f^(3/2)*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqr
t[e + f*x^2]) + ((b*c - a*d)^3*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)
/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(a*b*c*Sqrt[f]*(b*e
- a*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 557

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[(2*b*c - a*d + b*d*x^2)*(Sqrt
[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rule 558

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b*((b*e - a*f)/(b*c - a*d)^2), Int[(c + d*x^2)^(q
 + 2)*((e + f*x^2)^(r - 1)/(a + b*x^2)), x], x] - Dist[1/(b*c - a*d)^2, Int
[(c + d*x^2)^q*(e + f*x^2)^(r - 1)*(2*b*c*d*e - a*d^2*e - b*c^2*f + d^2*(b*
e - a*f)*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[q, -1] && GtQ[
r, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{(c+dx^2)^{3/2}(-bde^2+2bcef-acf^2+(bc-ad)f^2x^2)}{(e+fx^2)^{3/2}}\,dx}{(be-af)^2} + \frac{(b(bc-ad))\int \frac{(c+dx^2)^{3/2}\sqrt{e+fx^2}}{a+bx^2}\,dx}{(be-af)^2} \\
&= \frac{(de-cf)x(c+dx^2)^{3/2}}{e(be-af)\sqrt{e+fx^2}} + \frac{(d(bc-ad))\int \frac{(2bc-ad+bdx^2)\sqrt{e+fx^2}}{\sqrt{c+dx^2}}\,dx}{b(be-af)^2} \\
&\quad + \frac{(bc-ad)^3\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{b(be-af)^2} \\
&\quad + \frac{\int \frac{\sqrt{c+dx^2}(-c(bc-ad)ef^2+df(af(4de-3cf)-be(3de-2cf))x^2)}{\sqrt{e+fx^2}}\,dx}{ef(be-af)^2}
\end{aligned}$$

$$= \frac{(de-cf)x(c+dx^2)^{3/2}}{e(be-af)\sqrt{e+fx^2}} + \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3(be-af)^2} + \frac{d(af(4de-3cf)-be(3de-2cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef(be-af)^2} + \frac{(bc-ad)^3e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(bc-ad)\int \frac{d(5bc-3ad)e+d(bde+4bcf-3adf)x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{3b(be-af)^2} + \frac{\int \frac{-cef(2adf(2de-3cf)-b(3d^2e^2-2cdef-3c^2f^2))+df(be(6d^2e^2-7cdef-c^2f^2)-af(8d^2e^2-13cdef+3c^2f^2))x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{3ef^2(be-af)^2}$$

$$= \frac{(de-cf)x(c+dx^2)^{3/2}}{e(be-af)\sqrt{e+fx^2}} + \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3(be-af)^2} + \frac{d(af(4de-3cf)-be(3de-2cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef(be-af)^2} + \frac{(bc-ad)^3e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(d(5bc-3ad)(bc-ad)e)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{3b(be-af)^2} + \frac{(d(bc-ad)(bde+4bcf-3adf))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{3b(be-af)^2} - \frac{(c(2adf(2de-3cf)-b(3d^2e^2-2cdef-3c^2f^2)))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{3f(be-af)^2} + \frac{(d(be(6d^2e^2-7cdef-c^2f^2)-af(8d^2e^2-13cdef+3c^2f^2)))\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}dx}{3ef(be-af)^2}$$

$$
\begin{aligned}
= &\frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b(be-af)^2\sqrt{e+fx^2}} \\
&+ \frac{\left(be(6d^2e^2-7cdef-c^2f^2)-af(8d^2e^2-13cdef+3c^2f^2)\right)x\sqrt{c+dx^2}}{3ef(be-af)^2\sqrt{e+fx^2}} \\
&+ \frac{(de-cf)x\left(c+dx^2\right)^{3/2}}{e(be-af)\sqrt{e+fx^2}} + \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3(be-af)^2} \\
&+ \frac{d(af(4de-3cf)-be(3de-2cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef(be-af)^2} \\
&+ \frac{d(5bc-3ad)(bc-ad)e^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3bc\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&- \frac{\sqrt{e}(2adf(2de-3cf)-b(3d^2e^2-2cdef-3c^2f^2))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{3/2}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{(bc-ad)^3e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&- \frac{((bc-ad)e(bde+4bcf-3adf))\int\frac{\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx}{3b(be-af)^2} \\
&- \frac{(be(6d^2e^2-7cdef-c^2f^2)-af(8d^2e^2-13cdef+3c^2f^2))\int\frac{\sqrt{c+dx^2}}{\left(e+fx^2\right)^{3/2}}\,dx}{3f(be-af)^2}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{(bc-ad)(bde+4bcf-3adf)x\sqrt{c+dx^2}}{3b(be-af)^2\sqrt{e+fx^2}} \\
&+ \frac{(be(6d^2e^2-7cdef-c^2f^2)-af(8d^2e^2-13cdef+3c^2f^2))\,x\sqrt{c+dx^2}}{3ef(be-af)^2\sqrt{e+fx^2}} \\
&+ \frac{(de-cf)x(c+dx^2)^{3/2}}{e(be-af)\sqrt{e+fx^2}} + \frac{d(bc-ad)x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3(be-af)^2} \\
&+ \frac{d(af(4de-3cf)-be(3de-2cf))x\sqrt{c+dx^2}\sqrt{e+fx^2}}{3ef(be-af)^2} \\
&- \frac{(bc-ad)\sqrt{e}(bde+4bcf-3adf)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3b\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&- \frac{(be(6d^2e^2-7cdef-c^2f^2)-af(8d^2e^2-13cdef+3c^2f^2))\,\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3\sqrt{e}f^{3/2}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{d(5bc-3ad)(bc-ad)e^{3/2}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3bc\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&- \frac{\sqrt{e}(2adf(2de-3cf)-b(3d^2e^2-2cdef-3c^2f^2))\,\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3f^{3/2}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{(bc-ad)^3e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{abc\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.83 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.36

$$
\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \frac{-iabde(-ad^2ef+b(2d^2e^2-2cdef+c^2f^2))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\Big(i\text{arcsinh}\Big(
$$

```
[In] Integrate[(c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((-I)*a*b*d*e*(-(a*d^2*e*f) + b*(2*d^2*e^2 - 2*c*d*e*f + c^2*f^2))*Sqrt[1 +
 (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*
e)] - I*a*d^2*e*(b*e - a*f)*(-2*b*d*e + 3*b*c*f - a*d*f)*Sqrt[1 + (d*x^2)/c
]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - f*(a
*b^2*Sqrt[d/c]*(d*e - c*f)^2*x*(c + d*x^2) + I*(b*c - a*d)^3*e*f*Sqrt[1 + (
```

```
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)]))/(a*b^2*Sqrt[d/c]*e*f^2*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e
 + f*x^2])
```

Maple [A] (verified)

Time = 4.21 (sec) , antiderivative size = 1063, normalized size of antiderivative = 1.08

method	result	size
default	Expression too large to display	1063
elliptic	Expression too large to display	1255

```
[In] int((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-d/c)^(1/2)*a*b^2*c^2*d*f^3*x^3-2*(-d/c)^(1/2)*a*b^2*c*d^2*e*f^2*x^3+(-d/
c)^(1/2)*a*b^2*d^3*e^2*f*x^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellipt
icF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^3*d^3*e*f^2+3*((d*x^2+c)/c)^(1/2)*((f
*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*b*c*d^2*e*f^
2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a^2*b*d^3*e^2*f-3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Elliptic
F(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d^2*e^2*f+2*((d*x^2+c)/c)^(1/2)*(
(f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*d^3*e^3+
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^
(1/2))*a^2*b*d^3*e^2*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*
(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c^2*d*e*f^2+2*((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b^2*c*d^2*e^2*f-
2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e
)^(1/2))*a*b^2*d^3*e^3+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x
*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a^3*d^3*e*f^2-3*((d*x^2+c)
/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2
)/(-d/c)^(1/2))*a^2*b*c*d^2*e*f^2+3*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)
*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b^2*c^2*d*e
*f^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/
a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^3*c^3*e*f^2+(-d/c)^(1/2)*a*b^2*c^3*f^3*x-2
*(-d/c)^(1/2)*a*b^2*c^2*d*e*f^2*x+(-d/c)^(1/2)*a*b^2*c*d^2*e^2*f*x)*(f*x^2+
e)^(1/2)*(d*x^2+c)^(1/2)/e/f^2/a/(-d/c)^(1/2)/b^2/(a*f-b*e)/(d*f*x^4+c*f*x^
2+d*e*x^2+c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^{\frac{5}{2}}}{(a+bx^2)(e+fx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**(5/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
[Out] Integral((c + d*x**2)**(5/2)/((a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{5}{2}}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(5/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(5/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{5/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^{5/2}}{(b\,x^2+a)\,(f\,x^2+e)^{3/2}}\,dx$$

```
[In] int((c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x)
[Out] int((c + d*x^2)^(5/2)/((a + b*x^2)*(e + f*x^2)^(3/2)), x)
```

3.84 $\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 223

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \frac{(de-cf)\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{c^{3/2}(bc-ad)\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e(be-af)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] (-c*f+d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1
/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/(-a*f+b*e)/e^(1/2)
/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+c^(3/2)*(-a*d+b*c)
*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/2)/(1+d*
x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/e/(-a*f+b*e)/d^
(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {556, 553, 422}

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \frac{c^{3/2}\sqrt{e+fx^2}(bc-ad)\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} + \frac{\sqrt{c+dx^2}(de-cf)E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[In] Int[(c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((d*e - c*f)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*
e)/(c*f)])/(Sqrt[e]*Sqrt[f]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2]) + (c^(3/2)*(b*c - a*d)*Sqrt[e + f*x^2]*EllipticPi[1 - (
b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*(b*
e - a*f)*Sqrt[c + d*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 556

```
Int[((e_) + (f_.)*(x_)^2)^(3/2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2
)^(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[Sqrt[e + f*x^2]/((
a + b*x^2)*Sqrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqr
t[e + f*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && P
osQ[d/c] && PosQ[f/e]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{(bc-ad)\int\frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{be-af}+\frac{(de-cf)\int\frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{be-af}\\&=\frac{(de-cf)\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{f}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}\\&\quad+\frac{c^{3/2}(bc-ad)\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e(be-af)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.84 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.36

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \frac{ab\sqrt{\frac{d}{c}}f(de-cf)x(c+dx^2)-iabde(-de+cf)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\Big(i\text{arcsin}}{}$$

```
[In] Integrate[(c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (a*b*Sqrt[d/c]*f*(d*e - c*f)*x*(c + d*x^2) - I*a*b*d*e*(-(d*e) + c*f)*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/
(d*e)] - I*a*d^2*e*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elli
pticF[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*(b*c - a*d)^2*e*f*Sqrt[1 + (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x
], (c*f)/(d*e)])/(a*b*Sqrt[d/c]*e*f*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*
x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 593 vs. 2(273) = 546.

Time = 4.12 (sec) , antiderivative size = 594, normalized size of antiderivative = 2.66

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\,abcd\,f^2x^3-\sqrt{-\frac{d}{c}}\,ab\,d^2ef\,x^3+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^2d^2ef-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ab\,d^2e^2-\right.}{}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{\left(df\,x^2+cf\right)(cf-de)x}{f(af-be)e\sqrt{\left(x^2+\frac{e}{f}\right)\left(df\,x^2+cf\right)}}+\frac{\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)d^2}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}\,bf}-\frac{d\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,E\left(x\sqrt{-\frac{d}{c}}\right.}{(af-be)\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x}}\right.}{}$

```
[In] int((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-d/c)^(1/2)*a*b*c*d*f^2*x^3-(-d/c)^(1/2)*a*b*d^2*e*f*x^3+((d*x^2+c)/c)^(1
/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*d^2*e
*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/
e)^(1/2))*a*b*d^2*e^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(
-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d*e*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^2*e^2-((d*x^2+c)/c)^(
1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d
/c)^(1/2))*a^2*d^2*e*f+2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi
(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*b*c*d*e*f-((d*x^2+c)/c
```

```
)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/
(-d/c)^(1/2))*b^2*c^2*e*f+(-d/c)^(1/2)*a*b*c^2*f^2*x-(-d/c)^(1/2)*a*b*c*d*e
*f*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/b/a/(-d/c)^(1/2)/e/f/(a*f-b*e)/(d*f*x
^4+c*f*x^2+d*e*x^2+c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{(c+dx^2)^{\frac{3}{2}}}{(a+bx^2)(e+fx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**(3/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
[Out] Integral((c + d*x**2)**(3/2)/((a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="maxima")
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\,(e+fx^2)^{3/2}}\,dx = \int \frac{(dx^2+c)^{\frac{3}{2}}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(3/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate((d*x^2 + c)^(3/2)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx^2)^{3/2}}{(a+bx^2)\,(e+fx^2)^{3/2}}\,dx = \int \frac{(d\,x^2+c)^{3/2}}{(b\,x^2+a)\,(f\,x^2+e)^{3/2}}\,dx$$

```
[In] int((c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x)
[Out] int((c + d*x^2)^(3/2)/((a + b*x^2)*(e + f*x^2)^(3/2)), x)
```

3.85 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 209

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = -\frac{\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{bc^{3/2}\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e(be-af)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] -(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x
^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/(-a*f+b*e)/e^(1/2)/(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+b*c^(3/2)*(1/(1+d*x^2/c))^(1
/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),1-b*c/
a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2)/a/e/(-a*f+b*e)/d^(1/2)/(d*x^2+c)^(1/
2)/(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {555, 553, 422}

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \frac{bc^{3/2}\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[In] Int[Sqrt[c + d*x^2]/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -((Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/(Sqrt[e]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e
+ f*x^2])) + (b*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[
(Sqrt[d]*x)/Sqrt[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*e*(b*e - a*f)*Sqrt[c + d
*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 555

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rubi steps

$$\text{integral} = \frac{b\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{be-af} - \frac{f\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{be-af}$$

$$= -\frac{\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{bc^{3/2}\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}e(be-af)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.38 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \frac{-a\sqrt{\frac{d}{c}}fx(c+dx^2)-iade\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)\middle|\frac{cf}{de}\right)-i(bc\cdots}{a\sqrt{\frac{d}{c}}e(be-af)\sqrt{c+dx^2}\cdots}$$

```
[In] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] (-(a*Sqrt[d/c]*f*x*(c + d*x^2)) - I*a*d*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x
^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*(b*c - a*d)*e*Sqr
t[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*e*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2])
```

Maple [A] (verified)

Time = 4.25 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.36

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\,adf\,x^3-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,E\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)ade+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)ade-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\Pi\left(x\sqrt{\cdots}\right.\right.}{ea\sqrt{-\frac{d}{c}}\,(af-be)(df\,x^4+cf\,x^2+de\,x^2+ce)}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{\left(df\,x^2+cf\right)x}{e(af-be)\sqrt{\left(x^2+\frac{e}{f}\right)\left(df\,x^2+cf\right)}}-\frac{d\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,E\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{(af-be)\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}}+\frac{\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\cdots\right.}{(af-be)\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^{\cdots}}}\right.}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}$

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-d/c)^(1/2)*a*d*f*x^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x
*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2
)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*a*d*e-((d*x^
2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^
(1/2)/(-d/c)^(1/2))*b*c*e+(-d/c)^(1/2)*a*c*f*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(
1/2)/e/a/(-d/c)^(1/2)/(a*f-b*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)/(f*x**2+e)**(3/2),x)
[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)\,\left(f\,x^2+e\right)^{3/2}}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/((a + b*x^2)*(e + f*x^2)^(3/2)),x)
[Out] int((c + d*x^2)^(1/2)/((a + b*x^2)*(e + f*x^2)^(3/2)), x)
```

3.86 $\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 344

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{f^{3/2}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{e}\sqrt{f}(2bde-bcf-adf)\sqrt{c+dx^2}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c(be-af)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{b^2e^{3/2}\sqrt{c+dx^2}\,\mathrm{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

```
[Out] f^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)
/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/(-a*f+b*e)/(-c*f+d*e)
/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+b^2*e^(3/2)*(1/(1+
f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^
(1/2),1-b*e/a/f,(1-d*e/c/f)^(1/2))*(d*x^2+c)^(1/2)/a/c/(-a*f+b*e)^2/f^(1/2)
/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-(-a*d*f-b*c*f+2*b*d*e)*(1/
(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e
)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c/(-a*f+b*e)^2/(
-c*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.156, Rules used = {560, 553, 539, 429, 422}

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{b^2e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac\sqrt{f}\sqrt{e+fx^2}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{f^{3/2}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} - \frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}(-adf-bcf+2bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{e+fx^2}(be-af)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[In] Int[1/((a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] (f^(3/2)*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(
c*f)])/(Sqrt[e]*(b*e - a*f)*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2)
)]*Sqrt[e + f*x^2]) - (Sqrt[e]*Sqrt[f]*(2*b*d*e - b*c*f - a*d*f)*Sqrt[c + d
*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*e - a*
f)^2*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (
b^2*e^(3/2)*Sqrt[c + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/
Sqrt[e]], 1 - (d*e)/(c*f)])/(a*c*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))
/(c*(e + f*x^2))]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
```

```
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{b^2\int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{(be-af)^2} - \frac{f\int \frac{2be-af+bfx^2}{\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx}{(be-af)^2} \\
&= \frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{f^2\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{(be-af)(de-cf)} - \frac{(f(2bde-bcf-adf))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{(be-af)^2(de-cf)} \\
&= \frac{f^{3/2}\sqrt{c+dx^2}\,E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{\sqrt{e}\sqrt{f}(2bde-bcf-adf)\sqrt{c+dx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c(be-af)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&\quad + \frac{b^2e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.64

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{-a\sqrt{\frac{d}{c}}f^2x(c+dx^2)-iadef\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)\right.}{a\sqrt{\frac{d}{c}}e(-be+af)(}$$

```
[In] Integrate[1/((a + b*x^2)*Sqrt[c + d*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] (-(a*Sqrt[d/c]*f^2*x*(c + d*x^2)) - I*a*d*e*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 +
(f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*b*e*(-(d*e) +
 c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*Arc
Sinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]*e*(-(b*e) + a*f)*(d*e - c*f)*
Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 4.98 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.88

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\,a d\,f^2x^3-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,E\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)adef-\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\Pi\left(x\sqrt{-\frac{d}{c}},\frac{bc}{ad},\frac{\sqrt{-\frac{f}{e}}}{\sqrt{-\frac{d}{c}}}\right)bcef+\sqrt{\frac{d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,\Pi\left(\right.\right.}{ae(af-be)\sqrt{-\frac{d}{c}}\,(cf-de)(df\,x^4+cf\,x^2+de\,x^2+ce)}$
elliptic	$\frac{\sqrt{(d\,x^2+c)(f\,x^2+e)}\left(\frac{\left(df\,x^2+cf\right)fx}{e(cf-de)(af-be)\sqrt{\left(x^2+\frac{e}{f}\right)\left(df\,x^2+cf\right)}}+\frac{\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)f}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2+ce}\,e(af-be)}-\frac{\sqrt{1+\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{-\right.}{\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^4+cf\,x^2+de\,x^2}}\right.}{}$

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-d/c)^(1/2)*a*d*f^2*x^3-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE
(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*d*e*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^
(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b*c*e*f+
((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(
-f/e)^(1/2)/(-d/c)^(1/2))*b*d*e^2+(-d/c)^(1/2)*a*c*f^2*x)*(f*x^2+e)^(1/2)*(
d*x^2+c)^(1/2)/a/e/(a*f-b*e)/(-d/c)^(1/2)/(c*f-d*e)/(d*f*x^4+c*f*x^2+d*e*x^
2+c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*sqrt(c + d*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+c}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + c)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{c+dx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{d\,x^2+c}\,(f\,x^2+e)^{3/2}}\,dx$$

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

3.87 $\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 539

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = -\frac{d^2x}{c(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}}$$
$$-\frac{b^2\sqrt{f}\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{(bc-ad)^2\sqrt{e}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{d\sqrt{f}(2bc^2f-ad(de+cf))\sqrt{c+dx^2}\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c(bc-ad)^2\sqrt{e}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$-\frac{d^2\sqrt{e}(bde-3bcf+2adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c(bc-ad)^2\sqrt{f}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+\frac{b^3c^{3/2}\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}(bc-ad)^2e(be-af)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

```
[Out] -d^2*x/c/(-a*d+b*c)/(-c*f+d*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)-d^2*(2*a*d*f
-3*b*c*f+b*d*e)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)
/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)/c/(-a
*d+b*c)^2/(-c*f+d*e)^2/f^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1
/2)-b^2*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)
/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/(-a*d+b*c)^2/
(-a*f+b*e)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-d*(2*b*c
^2*f-a*d*(c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(
1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/2)*(d*x^2+c)^(1/2)/c
/(-a*d+b*c)^2/(-c*f+d*e)^2/e^(1/2)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e
```

```
)^(1/2)+b^3*c^(3/2)*(1/(1+d*x^2/c))^(1/2)*(1+d*x^2/c)^(1/2)*EllipticPi(x*d^
(1/2)/c^(1/2)/(1+d*x^2/c)^(1/2),1-b*c/a/d,(1-c*f/d/e)^(1/2))*(f*x^2+e)^(1/2
)/a/(-a*d+b*c)^2/e/(-a*f+b*e)/d^(1/2)/(d*x^2+c)^(1/2)/(c*(f*x^2+e)/e/(d*x^2
+c))^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {560, 555, 553, 422, 541, 539, 429}

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \frac{b^3c^{3/2}\sqrt{e+fx^2}\,\text{EllipticPi}\left(1-\frac{bc}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{cf}{de}\right)}{a\sqrt{d}e\sqrt{c+dx^2}(bc-ad)^2(be-af)\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$
$$-\frac{b^2\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{\sqrt{e}\sqrt{e+fx^2}(bc-ad)^2(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{d\sqrt{f}\sqrt{c+dx^2}(2bc^2f-ad(cf+de))\,E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c\sqrt{e}\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{d^2\sqrt{e}\sqrt{c+dx^2}(2adf-3bcf+bde)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c\sqrt{f}\sqrt{e+fx^2}(bc-ad)^2(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$-\frac{d^2x}{c\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)(de-cf)}$$

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -((d^2*x)/(c*(b*c - a*d)*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])) - (b
^2*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)
/(c*f)])/((b*c - a*d)^2*Sqrt[e]*(b*e - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*
x^2))]*Sqrt[e + f*x^2]) - (d*Sqrt[f]*(2*b*c^2*f - a*d*(d*e + c*f))*Sqrt[c +
 d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c -
a*d)^2*Sqrt[e]*(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e +
 f*x^2]) - (d^2*Sqrt[e]*(b*d*e - 3*b*c*f + 2*a*d*f)*Sqrt[c + d*x^2]*Ellipti
cF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^2*Sqrt[f]*
(d*e - c*f)^2*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b^3
*c^(3/2)*Sqrt[e + f*x^2]*EllipticPi[1 - (b*c)/(a*d), ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (c*f)/(d*e)])/(a*Sqrt[d]*(b*c - a*d)^2*e*(b*e - a*f)*Sqrt[c + d*
x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
```

```
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 555

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
```

```
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{b^2 \int \frac{\sqrt{c+dx^2}}{(a+bx^2)(e+fx^2)^{3/2}}\,dx}{(bc-ad)^2} - \frac{d \int \frac{2bc-ad+bdx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{d^2x}{c(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{b^3 \int \frac{\sqrt{c+dx^2}}{(a+bx^2)\sqrt{e+fx^2}}\,dx}{(bc-ad)^2(be-af)} - \frac{(b^2f)\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{(bc-ad)^2(be-af)} + \frac{d \int \frac{-c(bde-2bcf+adf)-d(bc-ad)fx^2}{\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx}{c(bc-ad)^2(de-cf)}$$

$$= -\frac{d^2x}{c(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{b^2\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{(bc-ad)^2\sqrt{e}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{b^3c^{3/2}\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}(bc-ad)^2e(be-af)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}} - \frac{(d^2(bde-3bcf+2adf))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{(bc-ad)^2(de-cf)^2} - \frac{(df(2bc^2f-ad(de+cf)))\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{c(bc-ad)^2(de-cf)^2}$$

$$= -\frac{d^2x}{c(bc-ad)(de-cf)\sqrt{c+dx^2}\sqrt{e+fx^2}}$$
$$- \frac{b^2\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{(bc-ad)^2\sqrt{e}(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{d\sqrt{f}(2bc^2f-ad(de+cf))\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c(bc-ad)^2\sqrt{e}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{d^2\sqrt{e}(bde-3bcf+2adf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c(bc-ad)^2\sqrt{f}(de-cf)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{b^3c^{3/2}\sqrt{e+fx^2}\Pi\left(1-\frac{bc}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{cf}{de}\right)}{a\sqrt{d}(bc-ad)^2e(be-af)\sqrt{c+dx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.13 (sec) , antiderivative size = 418, normalized size of antiderivative = 0.78

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \frac{\frac{adx(-adf(c^2f^2+cdf^2x^2+d^2e(e+fx^2))+b(c^3f^3+c^2df^3x^2+d^3e^2(e+fx^2)))}{c}+iad\sqrt{}}{}$$

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((a*d*x*(-(a*d*f*(c^2*f^2 + c*d*f^2*x^2 + d^2*e*(e + f*x^2))) + b*(c^3*f^3
+ c^2*d*f^3*x^2 + d^3*e^2*(e + f*x^2))))/c + I*a*d*Sqrt[d/c]*e*(-(a*d*f*(d*
e + c*f)) + b*(d^2*e^2 + c^2*f^2))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*
EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*a*c*d*(d/c)^(3/2)*e*(b*e
 - a*f)*(-(d*e) + c*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*
ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + I*b^2*c*Sqrt[d/c]*e*(d*e - c*f)^2*Sqrt
[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[
d/c]*x], (c*f)/(d*e)])/(a*d*(-(b*c) + a*d)*e*(b*e - a*f)*(d*e - c*f)^2*Sqrt
[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 6.60 (sec) , antiderivative size = 956, normalized size of antiderivative = 1.77

method	result
default	$\frac{\left(\sqrt{-\frac{d}{c}}\, a^{2} c\, d^{2} f^{3} x^{3}+\sqrt{-\frac{d}{c}}\, a^{2} d^{3} e\, f^{2} x^{3}-\sqrt{-\frac{d}{c}}\, a b\, c^{2} d\, f^{3} x^{3}-\sqrt{-\frac{d}{c}}\, a b\, d^{3} e^{2} f\, x^{3}-\sqrt{\frac{d\, x^{2}+c}{c}}\, \sqrt{\frac{f\, x^{2}+e}{e}}\, F\left(x \sqrt{-\frac{d}{c}}, \sqrt{\frac{c f}{d e}}\right) a^{2} c\, d^{2} e\, f^{2}+\sqrt{\frac{d}{}}\right.}{}$
elliptic	Expression too large to display

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-d/c)^(1/2)*a^2*c*d^2*f^3*x^3+(-d/c)^(1/2)*a^2*d^3*e*f^2*x^3-(-d/c)^(1/2)
*a*b*c^2*d*f^3*x^3-(-d/c)^(1/2)*a*b*d^3*e^2*f*x^3-((d*x^2+c)/c)^(1/2)*((f*x
^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d^2*e*f^2+((
d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1
/2))*a^2*d^3*e^2*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(c*f/d/e)^(1/2))*a*b*c*d^2*e^2*f-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)
^(1/2)*EllipticF(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^3*e^3-((d*x^2+c)/c)^
(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a^2*c*d
^2*e*f^2-((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(
c*f/d/e)^(1/2))*a^2*d^3*e^2*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*Ellip
ticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*c^2*d*e*f^2+((d*x^2+c)/c)^(1/2)*((
f*x^2+e)/e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(c*f/d/e)^(1/2))*a*b*d^3*e^3+((d
*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/
e)^(1/2)/(-d/c)^(1/2))*b^2*c^3*e*f^2-2*((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1
/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c^2*d*
e^2*f+((d*x^2+c)/c)^(1/2)*((f*x^2+e)/e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c
/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*b^2*c*d^2*e^3+(-d/c)^(1/2)*a^2*c^2*d*f^3*x+
(-d/c)^(1/2)*a^2*d^3*e^2*f*x-(-d/c)^(1/2)*a*b*c^3*f^3*x-(-d/c)^(1/2)*a*b*d^
3*e^3*x)*(f*x^2+e)^(1/2)*(d*x^2+c)^(1/2)/c/e/(c*f-d*e)^2/a/(-d/c)^(1/2)/(a*
d-b*c)/(a*f-b*e)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\left(c+dx^2\right)^{3/2}\left(e+fx^2\right)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{3}{2}}(e+fx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(3/2)/(f*x**2+e)**(3/2),x)
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(3/2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="maxima"
)
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)\,(d\,x^2+c)^{3/2}\,(f\,x^2+e)^{3/2}}\,dx$$

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

3.88 $\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 814

$$
\begin{aligned}
\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx &= -\frac{d^2x}{3c(bc-ad)(de-cf)(c+dx^2)^{3/2}\sqrt{e+fx^2}} \\
&- \frac{d^2(bc(5de-9cf)-2ad(de-3cf))x}{3c^2(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}} \\
&+ \frac{b^2f^{3/2}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{(bc-ad)^2\sqrt{e}(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&- \frac{d\sqrt{f}(bc(5d^2e^2-7cdef-6c^2f^2)-ad(2d^2e^2-7cdef-3c^2f^2))\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2\sqrt{e}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&- \frac{b^2\sqrt{e}\sqrt{f}(2bde-bcf-adf)\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{c(bc-ad)^2(be-af)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{d^2\sqrt{e}\sqrt{f}(bc(7de-15cf)-ad(de-9cf))\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{b^4e^{3/2}\sqrt{c+dx^2}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}
\end{aligned}
$$

```
[Out] -1/3*d^2*x/c/(-a*d+b*c)/(-c*f+d*e)/(d*x^2+c)^(3/2)/(f*x^2+e)^(1/2)-1/3*d^2*
(b*c*(-9*c*f+5*d*e)-2*a*d*(-3*c*f+d*e))*x/c^2/(-a*d+b*c)^2/(-c*f+d*e)^2/(d*
x^2+c)^(1/2)/(f*x^2+e)^(1/2)+b^2*f^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^
(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*(d*x
```

```
^2+c)^(1/2)/(-a*d+b*c)^2/(-a*f+b*e)/(-c*f+d*e)/e^(1/2)/(e*(d*x^2+c)/c/(f*x^
2+e))^(1/2)/(f*x^2+e)^(1/2)+b^4*e^(3/2)*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(
1/2)*EllipticPi(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),1-b*e/a/f,(1-d*e/c/f)^(
1/2))*(d*x^2+c)^(1/2)/a/c/(-a*d+b*c)^2/(-a*f+b*e)^2/f^(1/2)/(e*(d*x^2+c)/c/
(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/3*d*(b*c*(-6*c^2*f^2-7*c*d*e*f+5*d^2*e^2
)-a*d*(-3*c^2*f^2-7*c*d*e*f+2*d^2*e^2))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(
1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*f^(1/
2)*(d*x^2+c)^(1/2)/c^2/(-a*d+b*c)^2/(-c*f+d*e)^3/e^(1/2)/(e*(d*x^2+c)/c/(f*
x^2+e))^(1/2)/(f*x^2+e)^(1/2)-b^2*(-a*d*f-b*c*f+2*b*d*e)*(1/(1+f*x^2/e))^(1
/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/
c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c/(-a*d+b*c)^2/(-a*f+b*e)^2/(-c
*f+d*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/3*d^2*(b*c*(-15*c
*f+7*d*e)-a*d*(-9*c*f+d*e))*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*Ellipti
cF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(
d*x^2+c)^(1/2)/c^2/(-a*d+b*c)^2/(-c*f+d*e)^3/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2
)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 814, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {560, 553, 539, 429, 422, 541}

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \frac{e^{3/2}\sqrt{dx^2+c}\,\text{EllipticPi}\left(1-\frac{be}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)b^4}{ac(bc-ad)^2\sqrt{f}(be-af)^2\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$+\frac{f^{3/2}\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)b^2}{(bc-ad)^2\sqrt{e}(be-af)(de-cf)\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$-\frac{\sqrt{e}\sqrt{f}(2bde-bcf-adf)\sqrt{dx^2+c}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)b^2}{c(bc-ad)^2(be-af)^2(de-cf)\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$-\frac{d\sqrt{f}\left(bc(5d^2e^2-7cdfe-6c^2f^2)-ad(2d^2e^2-7cdfe-3c^2f^2)\right)\sqrt{dx^2+c}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2\sqrt{e}(de-cf)^3\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$+\frac{d^2\sqrt{e}\sqrt{f}(bc(7de-15cf)-ad(de-9cf))\sqrt{dx^2+c}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2(de-cf)^3\sqrt{\frac{e(dx^2+c)}{c(fx^2+e)}}\sqrt{fx^2+e}}$$
$$-\frac{d^2(bc(5de-9cf)-2ad(de-3cf))x}{3c^2(bc-ad)^2(de-cf)^2\sqrt{dx^2+c}\sqrt{fx^2+e}}-\frac{d^2x}{3c(bc-ad)(de-cf)\left(dx^2+c\right)^{3/2}\sqrt{fx^2+e}}$$

```
[In] Int[1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] -1/3*(d^2*x)/(c*(b*c - a*d)*(d*e - c*f)*(c + d*x^2)^(3/2)*Sqrt[e + f*x^2])
- (d^2*(b*c*(5*d*e - 9*c*f) - 2*a*d*(d*e - 3*c*f))*x)/(3*c^2*(b*c - a*d)^2*
(d*e - c*f)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]) + (b^2*f^(3/2)*Sqrt[c + d*x^
2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/((b*c - a*d)^2*
Sqrt[e]*(b*e - a*f)*(d*e - c*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[
e + f*x^2]) - (d*Sqrt[f]*(b*c*(5*d^2*e^2 - 7*c*d*e*f - 6*c^2*f^2) - a*d*(2*
d^2*e^2 - 7*c*d*e*f - 3*c^2*f^2))*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]
*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)^2*Sqrt[e]*(d*e - c*f)^3*
Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (b^2*Sqrt[e]*Sqrt[
f]*(2*b*d*e - b*c*f - a*d*f)*Sqrt[c + d*x^2]*EllipticF[ArcTan[(Sqrt[f]*x)/S
qrt[e]], 1 - (d*e)/(c*f)])/(c*(b*c - a*d)^2*(b*e - a*f)^2*(d*e - c*f)*Sqrt[
(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d^2*Sqrt[e]*Sqrt[f]*(b
*c*(7*d*e - 15*c*f) - a*d*(d*e - 9*c*f))*Sqrt[c + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(3*c^2*(b*c - a*d)^2*(d*e - c*f)^3*S
qrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (b^4*e^(3/2)*Sqrt[c
 + d*x^2]*EllipticPi[1 - (b*e)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e
)/(c*f)])/(a*c*(b*c - a*d)^2*Sqrt[f]*(b*e - a*f)^2*Sqrt[(e*(c + d*x^2))/(c*
(e + f*x^2))]*Sqrt[e + f*x^2])
```

<u>Rule 422</u>

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

<u>Rule 429</u>

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

<u>Rule 539</u>

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

<u>Rule 541</u>

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
```

```
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\text{integral} = \frac{b^2 \int \frac{1}{(a+bx^2)\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx}{(bc-ad)^2} - \frac{d \int \frac{2bc-ad+bdx^2}{(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx}{(bc-ad)^2}$$

$$= -\frac{d^2x}{3c(bc-ad)(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} + \frac{b^4 \int \frac{\sqrt{e+fx^2}}{(a+bx^2)\sqrt{c+dx^2}}\,dx}{(bc-ad)^2(be-af)^2} - \frac{(b^2f)\int \frac{2be-af+bfx^2}{\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx}{(bc-ad)^2(be-af)^2} + \frac{d\int \frac{-bc(5de-6cf)+ad(2de-3cf)-3d(bc-ad)fx^2}{(c+dx^2)^{3/2}(e+fx^2)^{3/2}}\,dx}{3c(bc-ad)^2(de-cf)}$$

$$= -\frac{d^2x}{3c(bc-ad)(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} - \frac{d^2(bc(5de-9cf)-2ad(de-3cf))x}{3c^2(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{b^4e^{3/2}\sqrt{c+dx^2}\,\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{d\int \frac{-cf(2bc(de-3cf)+ad(de+3cf))+df(bc(5de-9cf)-2ad(de-3cf))x^2}{\sqrt{c+dx^2}(e+fx^2)^{3/2}}\,dx}{3c^2(bc-ad)^2(de-cf)^2} + \frac{(b^2f^2)\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{(bc-ad)^2(be-af)(de-cf)} - \frac{(b^2f(2bde-bcf-adf))\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{(bc-ad)^2(be-af)^2(de-cf)}$$

$$= -\frac{d^2x}{3c(bc-ad)(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} - \frac{d^2(bc(5de-9cf)-2ad(de-3cf))x}{3c^2(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{b^2f^{3/2}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{(bc-ad)^2\sqrt{e}(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{b^2\sqrt{e}\sqrt{f}(2bde-bcf-adf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c(bc-ad)^2(be-af)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{b^4e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{\left(d^2f(bc(7de-15cf)-ad(de-9cf))\right)\int\frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{3c(bc-ad)^2(de-cf)^3} - \frac{\left(df(bc(5d^2e^2-7cdef-6c^2f^2)-ad(2d^2e^2-7cdef-3c^2f^2))\right)\int\frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{3c^2(bc-ad)^2(de-cf)^3}$$

$$= -\frac{d^2x}{3c(bc-ad)(de-cf)\left(c+dx^2\right)^{3/2}\sqrt{e+fx^2}} - \frac{d^2(bc(5de-9cf)-2ad(de-3cf))x}{3c^2(bc-ad)^2(de-cf)^2\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{b^2f^{3/2}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{(bc-ad)^2\sqrt{e}(be-af)(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{d\sqrt{f}(bc(5d^2e^2-7cdef-6c^2f^2)-ad(2d^2e^2-7cdef-3c^2f^2))\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2\sqrt{e}(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{b^2\sqrt{e}\sqrt{f}(2bde-bcf-adf)\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{c(bc-ad)^2(be-af)^2(de-cf)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{d^2\sqrt{e}\sqrt{f}(bc(7de-15cf)-ad(de-9cf))\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{3c^2(bc-ad)^2(de-cf)^3\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{b^4e^{3/2}\sqrt{c+dx^2}\Pi\left(1-\frac{be}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{ac(bc-ad)^2\sqrt{f}(be-af)^2\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.59 (sec) , antiderivative size = 1645, normalized size of antiderivative = 2.02

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \frac{-iade(2abd(de-3cf)(de+cf)^2+a^2d^2f(-2d^2e^2+7cdef+3c^2f^2)}{}$$

```
[In] Integrate[1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((-I)*a*d*e*(2*a*b*d*(d*e - 3*c*f)*(d*e + c*f)^2 + a^2*d^2*f*(-2*d^2*e^2 +
7*c*d*e*f + 3*c^2*f^2) + b^2*c*(-5*d^3*e^3 + 10*c*d^2*e^2*f + 3*c^3*f^3))*(
c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt
[d/c]*x], (c*f)/(d*e)] + (Sqrt[d/c]*(6*a*b^2*c^2*d^5*e^4*x - 3*a^2*b*c*d^6*
e^4*x - 11*a*b^2*c^3*d^4*e^3*f*x + 2*a^2*b*c^2*d^5*e^3*f*x + 3*a^3*c*d^6*e^
3*f*x + 11*a^2*b*c^3*d^4*e^2*f^2*x - 8*a^3*c^2*d^5*e^2*f^2*x - 3*a*b^2*c^6*
d*f^4*x + 6*a^2*b*c^5*d^2*f^4*x - 3*a^3*c^4*d^3*f^4*x + 5*a*b^2*c*d^6*e^4*x
^3 - 2*a^2*b*d^7*e^4*x^3 - 4*a*b^2*c^2*d^5*e^3*f*x^3 - a^2*b*c*d^6*e^3*f*x^
3 + 2*a^3*d^7*e^3*f*x^3 - 11*a*b^2*c^3*d^4*e^2*f^2*x^3 + 12*a^2*b*c^2*d^5*e
^2*f^2*x^3 - 4*a^3*c*d^6*e^2*f^2*x^3 + 11*a^2*b*c^3*d^4*e*f^3*x^3 - 8*a^3*c
^2*d^5*e*f^3*x^3 - 6*a*b^2*c^5*d^2*f^4*x^3 + 12*a^2*b*c^4*d^3*f^4*x^3 - 6*a
^3*c^3*d^4*f^4*x^3 + 5*a*b^2*c*d^6*e^3*f*x^5 - 2*a^2*b*d^7*e^3*f*x^5 - 10*a
*b^2*c^2*d^5*e^2*f^2*x^5 + 2*a^2*b*c*d^6*e^2*f^2*x^5 + 2*a^3*d^7*e^2*f^2*x^
5 + 10*a^2*b*c^2*d^5*e*f^3*x^5 - 7*a^3*c*d^6*e*f^3*x^5 - 3*a*b^2*c^4*d^3*f^
4*x^5 + 6*a^2*b*c^3*d^4*f^4*x^5 - 3*a^3*c^2*d^5*f^4*x^5 - I*a*c*d^2*Sqrt[d/
c]*e*(b*e - a*f)*(-(d*e) + c*f)*(2*a*d*(d*e - 3*c*f) + b*c*(-5*d*e + 9*c*f)
)*(c + d*x^2)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[S
qrt[d/c]*x], (c*f)/(d*e)] + (3*I)*b^3*c^4*d^3*Sqrt[d/c]*e^4*Sqrt[1 + (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c
*f)/(d*e)] - (9*I)*b^3*c^7*(d/c)^(5/2)*e^3*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (
f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (9
*I)*b^3*c^7*(d/c)^(3/2)*e^2*f^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ell
ipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - (3*I)*b^3*c^7*S
qrt[d/c]*e*f^3*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*
d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + (3*I)*b^3*c^3*d^4*Sqrt[d/c]*e^4*
x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSi
nh[Sqrt[d/c]*x], (c*f)/(d*e)] - (9*I)*b^3*c^4*d^3*Sqrt[d/c]*e^3*f*x^2*Sqrt[
1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d
/c]*x], (c*f)/(d*e)] + (9*I)*b^3*c^7*(d/c)^(5/2)*e^2*f^2*x^2*Sqrt[1 + (d*x^
2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)] - (3*I)*b^3*c^7*(d/c)^(3/2)*e*f^3*x^2*Sqrt[1 + (d*x^2)/c]*Sqrt[
1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)]
))/d)/(3*a*c^2*Sqrt[d/c]*(b*c - a*d)^2*e*(b*e - a*f)*(-(d*e) + c*f)^3*(c +
d*x^2)^(3/2)*Sqrt[e + f*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2126 vs. 2(923) = 1846.

Time = 7.73 (sec) , antiderivative size = 2127, normalized size of antiderivative = 2.61

method	result	size
elliptic	Expression too large to display	2127
default	Expression too large to display	4115

```
[In] int(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/3*d/c/(c*f-d
*e)*x/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)
/(x^2+c/d)^2+1/3*(d*f*x^2+d*e)*d^2/c^2/(c*f-d*e)^2*x*(7*a*c*d*f-2*a*d^2*e-1
0*b*c^2*f+5*b*c*d*e)/(a*d-b*c)/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)/((x^2+c/d)
*(d*f*x^2+d*e))^(1/2)+(d*f*x^2+c*f)*f^3/e/(c*f-d*e)^3*x/(a*f-b*e)/((x^2+e/f
)*(d*f*x^2+c*f))^(1/2)-7/3*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/
2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*f*d^4/(c*f-d*e)^2/c/(a*d-b*c)/(a*c*d
*f-a*d^2*e-b*c^2*f+b*c*d*e)*a*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(
1/2))+2/3*e^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f
*x^2+d*e*x^2+c*e)^(1/2)*d^5/(c*f-d*e)^2/c^2/(a*d-b*c)/(a*c*d*f-a*d^2*e-b*c^
2*f+b*c*d*e)*a*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-7/3/(-d/c
)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(
1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*d^3/(c*f-d*e)/c/(a*
d-b*c)/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)*a*f+2/3/(-d/c)^(1/2)*(1+d*x^2/c)^(
1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*d^4/(c*f-d*e)/c^2/(a*d-b*c)/(a*c*d*f-a*d
^2*e-b*c^2*f+b*c*d*e)*a*e-5/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1
/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d
*e)/e/d)^(1/2))*d^3/(c*f-d*e)/c/(a*d-b*c)/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)
*b*e+10/3*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x
^2+d*e*x^2+c*e)^(1/2)*f*d^3/(c*f-d*e)^2/(a*d-b*c)/(a*c*d*f-a*d^2*e-b*c^2*f+
b*c*d*e)*b*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-5/3*e^2/(-d/c
)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(
1/2)*d^4/(c*f-d*e)^2/c/(a*d-b*c)/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)*b*Ellipt
icE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1/3/(-d/c)^(1/2)*(1+d*x^2/c)^(
1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/
c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*d^2/c/(c*f-d*e)*f/(a*c*d*f-a*d^2*e-b*c^2
*f+b*c*d*e)-1/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f
*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*
c*f^4/e/(c*f-d*e)^3/(a*f-b*e)+1/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^
(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*f^3*d/(c*f-d*e)^3/(a*f-b*e)*Ellip
ticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-1/(-d/c)^(1/2)*(1+d*x^2/c)^(1
/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*f^3*d/(c*f-d*e)^3
/(a*f-b*e)*EllipticE(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1/(-d/c)^(1/2
```

```
)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*E
llipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))*f^3/(c*f-d*e)^2/e/(a*f-b*
e)-b^3/(a*d-b*c)^2/(a*f-b*e)/a/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(
1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,
(-f/e)^(1/2)/(-d/c)^(1/2))+10/3/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^
(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f
+d*e)/e/d)^(1/2))*d^2/(c*f-d*e)/(a*d-b*c)/(a*c*d*f-a*d^2*e-b*c^2*f+b*c*d*e)
*b*f)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(a+bx^2)(c+dx^2)^{\frac{5}{2}}(e+fx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x**2+a)/(d*x**2+c)**(5/2)/(f*x**2+e)**(3/2),x)
```

```
[Out] Integral(1/((a + b*x**2)*(c + d*x**2)**(5/2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)(c+dx^2)^{5/2}(e+fx^2)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx = \int \frac{1}{(bx^2+a)(dx^2+c)^{\frac{5}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+c)^(5/2)/(f*x^2+e)^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)*(d*x^2 + c)^(5/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\left(c+dx^2\right)^{5/2}\left(e+fx^2\right)^{3/2}}\,dx = \int \frac{1}{(b\,x^2+a)\,\left(d\,x^2+c\right)^{5/2}\,\left(f\,x^2+e\right)^{3/2}}\,dx$$

```
[In] int(1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)),x)
[Out] int(1/((a + b*x^2)*(c + d*x^2)^(5/2)*(e + f*x^2)^(3/2)), x)
```

3.89 $\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 242

$$\begin{aligned}\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}dx &= -\frac{(a-2b)x\sqrt{2+x^2}}{b^2\sqrt{1+x^2}}+\frac{x\sqrt{1+x^2}\sqrt{2+x^2}}{3b}\\ &+\frac{\sqrt{2}(a-2b)\sqrt{2+x^2}E\left(\arctan(x)\left|\frac{1}{2}\right.\right)}{b^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}-\frac{(3a-7b)\sqrt{2+x^2}\,\mathrm{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{2}b^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}\\ &+\frac{(a-2b)(a-b)\sqrt{2+x^2}\,\mathrm{EllipticPi}\left(1-\frac{b}{a},\arctan(x),\frac{1}{2}\right)}{\sqrt{2}ab^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}\end{aligned}$$

```
[Out] -(a-2*b)*x*(x^2+2)^(1/2)/b^2/(x^2+1)^(1/2)+1/3*x*(x^2+1)^(1/2)*(x^2+2)^(1/2
)/b-1/6*(3*a-7*b)*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*
(x^2+2)^(1/2)/b^2*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)+1/2*(a-2*b)*(a-b)*(1/(x^2
+1))^(1/2)*EllipticPi(x/(x^2+1)^(1/2),1-b/a,1/2*2^(1/2))*(x^2+2)^(1/2)/a/b^
2*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)+(a-2*b)*(1/(x^2+1))^(1/2)*EllipticE(x/(x^
2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2+2)^(1/2)/b^2/((x^2+2)/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

$= \{557, 553, 542, 545, 429, 506, 422\}$

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = -\frac{\sqrt{2}\sqrt{x^2+2}(3a-5b)\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3b^2\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}} + \frac{\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{b^2\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}} + \frac{2\sqrt{x^2+1}(a-b)^2\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{ab^2\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}} - \frac{x\sqrt{x^2+2}(a-2b)}{b^2\sqrt{x^2+1}} + \frac{x\sqrt{x^2+1}\sqrt{x^2+2}}{3b}$$

```
[In] Int[((1 + x^2)^(3/2)*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] -(((a - 2*b)*x*Sqrt[2 + x^2])/(b^2*Sqrt[1 + x^2])) + (x*Sqrt[1 + x^2]*Sqrt[
2 + x^2])/(3*b) + (Sqrt[2]*(a - 2*b)*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2
])/(b^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) - (Sqrt[2]*(3*a - 5*b)*Sqr
t[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(3*b^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(
1 + x^2)]) + (2*(a - b)^2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sq
rt[2]], -1])/(a*b^2*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 542

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[f*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^q/(
b*(n*(p + q + 1) + 1))), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 557

```
Int[(((c_) + (d_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[(b*c - a*d)^2/b^2, Int[Sqrt[e + f*x^2]/((a + b*x^
2)*Sqrt[c + d*x^2]), x], x] + Dist[d/b^2, Int[(2*b*c - a*d + b*d*x^2)*(Sqrt
[e + f*x^2]/Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && Pos
Q[d/c] && PosQ[f/e]
```

Rubi steps

$$\text{integral} = \frac{\int \frac{\sqrt{2+x^2}\left(-a+2b+bx^2\right)}{\sqrt{1+x^2}}\,dx}{b^2} + \frac{(a-b)^2\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\left(a+bx^2\right)}\,dx}{b^2}$$

$$= \frac{x\sqrt{1+x^2}\sqrt{2+x^2}}{3b} + \frac{2(a-b)^2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{ab^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}} + \frac{\int \frac{-2(3a-5b)-3(a-2b)x^2}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx}{3b^2}$$

$$= \frac{x\sqrt{1+x^2}\sqrt{2+x^2}}{3b} + \frac{2(a-b)^2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{ab^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}} - \frac{(2(3a-5b))\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx}{3b^2} - \frac{(a-2b)\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx}{b^2}$$

$$= -\frac{(a-2b)x\sqrt{2+x^2}}{b^2\sqrt{1+x^2}} + \frac{x\sqrt{1+x^2}\sqrt{2+x^2}}{3b} - \frac{\sqrt{2}(3a-5b)\sqrt{2+x^2}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3b^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} + \frac{2(a-b)^2\sqrt{1+x^2}\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{ab^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}} + \frac{(a-2b)\int\frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}}\,dx}{b^2}$$

$$= -\frac{(a-2b)x\sqrt{2+x^2}}{b^2\sqrt{1+x^2}} + \frac{x\sqrt{1+x^2}\sqrt{2+x^2}}{3b} + \frac{\sqrt{2}(a-2b)\sqrt{2+x^2}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{b^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{\sqrt{2}(3a-5b)\sqrt{2+x^2}F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3b^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} + \frac{2(a-b)^2\sqrt{1+x^2}\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{ab^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.33 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.84

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = \frac{ab^2x\sqrt{1+x^2}\sqrt{2+x^2} + 3ia(a-2b)bE\left(i\operatorname{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - ia(3a^2-9ab+7b^2)}{}$$

```
[In] Integrate[((1 + x^2)^(3/2)*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] (a*b^2*x*Sqrt[1 + x^2]*Sqrt[2 + x^2] + (3*I)*a*(a - 2*b)*b*EllipticE[I*ArcS
inh[x/Sqrt[2]], 2] - I*a*(3*a^2 - 9*a*b + 7*b^2)*EllipticF[I*ArcSinh[x/Sqrt
[2]], 2] + (3*I)*a^3*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (12*I)*
a^2*b*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] + (15*I)*a*b^2*EllipticP
i[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (6*I)*b^3*EllipticPi[(2*b)/a, I*ArcSi
nh[x/Sqrt[2]], 2])/(3*a*b^3)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 6.70 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.10

method	result
risch	$\frac{x\sqrt{x^2+1}\,\sqrt{x^2+2}}{3b}-\frac{\left(\frac{3i(a-2b)\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,\left(F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-E\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)\right)}{2b\sqrt{x^4+3x^2+2}}+\frac{i\left(3a^2-12ab+13b^2\right)\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2b^2\sqrt{x^4+3x^2+2}}\right.}{3b\sqrt{x^2+1}\,\sqrt{x^2+2}}$
default	$-\frac{\sqrt{x^2+1}\,\sqrt{x^2+2}\,\left(-a\,b^2x^5+3i\sqrt{x^2+1}\,\sqrt{x^2+2}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^3-9i\sqrt{x^2+1}\,\sqrt{x^2+2}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^2b+7i\sqrt{x^2+1}\,\sqrt{x^2+2}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right.\right.}{}$
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\,\left(\frac{x\sqrt{x^4+3x^2+2}}{3b}-\frac{7i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{6b\sqrt{x^4+3x^2+2}}+\frac{ia^2\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{b^3\sqrt{x^4+3x^2+2}}-\frac{2i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\right.}{a\sqrt{x^4+3x^2}}\right.}{}$

```
[In] int((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x*(x^2+1)^(1/2)*(x^2+2)^(1/2)/b-1/3/b*(3/2*I*(a-2*b)/b*2^(1/2)*(2*x^2+4
)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*(EllipticF(1/2*I*x*2^(1/2),2^(1/2
))-EllipticE(1/2*I*x*2^(1/2),2^(1/2)))+1/2*I*(3*a^2-12*a*b+13*b^2)/b^2*2^(1
/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*x*2^(
1/2),2^(1/2))-I*(3*a^3-12*a^2*b+15*a*b^2-6*b^3)/b^2/a*2^(1/2)*(1+1/2*x^2)^(
1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(
1/2)))*((x^2+1)*(x^2+2))^(1/2)/(x^2+1)^(1/2)/(x^2+2)^(1/2)
```

Fricas [F]

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx=\int \frac{\sqrt{x^2+2}(x^2+1)^{\frac{3}{2}}}{bx^2+a}\,dx$$

```
[In] integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx=\int \frac{(x^2+1)^{\frac{3}{2}}\sqrt{x^2+2}}{a+bx^2}\,dx$$

```
[In] integrate((x**2+1)**(3/2)*(x**2+2)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral((x**2 + 1)**(3/2)*sqrt(x**2 + 2)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+2}(x^2+1)^{\frac{3}{2}}}{bx^2+a}\,dx$$

```
[In] integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+2}(x^2+1)^{\frac{3}{2}}}{bx^2+a}\,dx$$

```
[In] integrate((x^2+1)^(3/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 2)*(x^2 + 1)^(3/2)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(1+x^2)^{3/2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{(x^2+1)^{3/2}\sqrt{x^2+2}}{b\,x^2+a}\,dx$$

```
[In] int(((x^2 + 1)^(3/2)*(x^2 + 2)^(1/2))/(a + b*x^2),x)
[Out] int(((x^2 + 1)^(3/2)*(x^2 + 2)^(1/2))/(a + b*x^2), x)
```

3.90 $\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 192

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \frac{x\sqrt{2+x^2}}{b\sqrt{1+x^2}} - \frac{\sqrt{2}\sqrt{2+x^2}\,E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{b\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} + \frac{\sqrt{2+x^2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}b\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{(a-2b)\sqrt{2+x^2}\,\text{EllipticPi}\left(1-\frac{b}{a},\arctan(x),\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}$$

```
[Out] x*(x^2+2)^(1/2)/b/(x^2+1)^(1/2)+1/2*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(
1/2),1/2*2^(1/2))*(x^2+2)^(1/2)/b*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)-1/2*(a-2*
b)*(1/(x^2+1))^(1/2)*EllipticPi(x/(x^2+1)^(1/2),1-b/a,1/2*2^(1/2))*(x^2+2)^
(1/2)/a/b*2^(1/2)/((x^2+2)/(x^2+1))^(1/2)-(1/(x^2+1))^(1/2)*EllipticE(x/(x^
2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2+2)^(1/2)/b/((x^2+2)/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used

= {548, 433, 429, 506, 422, 553}

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = -\frac{\sqrt{x^2+2}(a-2b)\,\text{EllipticPi}\left(1-\frac{b}{a},\arctan(x),\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}} + \frac{\sqrt{x^2+2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{\sqrt{2}b\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}} - \frac{\sqrt{2}\sqrt{x^2+2}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{b\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}} + \frac{\sqrt{x^2+2}x}{b\sqrt{x^2+1}}$$

```
[In] Int[(Sqrt[1 + x^2]*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] (x*Sqrt[2 + x^2])/(b*Sqrt[1 + x^2]) - (Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcT
an[x], 1/2])/(b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) + (Sqrt[2 + x^2]*E
llipticF[ArcTan[x], 1/2])/(Sqrt[2]*b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)
]) - ((a - 2*b)*Sqrt[2 + x^2]*EllipticPi[1 - b/a, ArcTan[x], 1/2])/(Sqrt[2]
*a*b*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 548

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Dist[d/b, Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
Dist[(b*c - a*d)/b, Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[d/c, 0] && GtQ[f/e, 0] &&  !Simpl
erSqrtQ[d/c, f/e]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\int \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}}\,dx}{b} + \frac{(-a+2b)\int \frac{\sqrt{1+x^2}}{\sqrt{2+x^2}(a+bx^2)}\,dx}{b} \\
&= -\frac{(a-2b)\sqrt{2+x^2}\,\Pi\left(1-\frac{b}{a};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} + \frac{\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx}{b} + \frac{\int \frac{x^2}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx}{b} \\
&= \frac{x\sqrt{2+x^2}}{b\sqrt{1+x^2}} + \frac{\sqrt{2+x^2}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}b\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{(a-2b)\sqrt{2+x^2}\,\Pi\left(1-\frac{b}{a};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}}\,dx}{b} \\
&= \frac{x\sqrt{2+x^2}}{b\sqrt{1+x^2}} - \frac{\sqrt{2}\sqrt{2+x^2}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{b\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} + \frac{\sqrt{2+x^2}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}b\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} \\
&\quad - \frac{(a-2b)\sqrt{2+x^2}\,\Pi\left(1-\frac{b}{a};\tan^{-1}(x)\middle|\frac{1}{2}\right)}{\sqrt{2}ab\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.02 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.41

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx$$

$$= \frac{i\left(-abE\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right)+(a-2b)\left(a\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)+(-a+b)\,\text{EllipticPi}\left(\frac{2b}{a},i\text{arcsinh}\right.\right.\right.}{ab^2}$$

```
[In] Integrate[(Sqrt[1 + x^2]*Sqrt[2 + x^2])/(a + b*x^2),x]
```

```
[Out] (I*(-(a*b*EllipticE[I*ArcSinh[x/Sqrt[2]], 2]) + (a - 2*b)*(a*EllipticF[I*Ar
cSinh[x/Sqrt[2]], 2] + (-a + b)*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2
])))/(a*b^2)
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 3.33 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.63

method	result
default	$\frac{i\left(F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^{2}-2F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)ba-a^{2}\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)+3\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)ba-2\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)b^{2}-E\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)ba\right)}{a\,b^{2}}$
elliptic	$\frac{\sqrt{(x^{2}+1)(x^{2}+2)}\left(-\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{b\sqrt{x^{4}+3x^{2}+2}}+\frac{i\sqrt{2}\,\sqrt{2x^{2}+4}\,\sqrt{x^{2}+1}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a}{2\sqrt{x^{4}+3x^{2}+2}\,b^{2}}-\frac{ia\sqrt{2}\,\sqrt{1+\frac{x^{2}}{2}}\,\sqrt{x^{2}+1}\,\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{b^{2}\sqrt{x^{4}+3x^{2}+2}}-2i\sqrt{2}\cdots\right.}{\sqrt{x^{2}+1}\,\sqrt{x^{2}+2}}$

```
[In] int((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] I*(EllipticF(1/2*I*x*2^(1/2),2^(1/2))*a^2-2*EllipticF(1/2*I*x*2^(1/2),2^(1/
2))*b*a-a^2*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))+3*EllipticPi(1/2*I*x*
2^(1/2),2*b/a,2^(1/2))*b*a-2*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b^2-
EllipticE(1/2*I*x*2^(1/2),2^(1/2))*b*a)/a/b^2
```

Fricas [F]

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a}\,dx$$

```
[In] integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+1}\sqrt{x^2+2}}{a+bx^2}\,dx$$

```
[In] integrate((x**2+1)**(1/2)*(x**2+2)**(1/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(x**2 + 1)*sqrt(x**2 + 2)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a}\,dx$$

```
[In] integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+2}\sqrt{x^2+1}}{bx^2+a}\,dx$$

```
[In] integrate((x^2+1)^(1/2)*(x^2+2)^(1/2)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x^2}\sqrt{2+x^2}}{a+bx^2}\,dx = \int \frac{\sqrt{x^2+1}\,\sqrt{x^2+2}}{b\,x^2+a}\,dx$$

```
[In] int(((x^2 + 1)^(1/2)*(x^2 + 2)^(1/2))/(a + b*x^2),x)
[Out] int(((x^2 + 1)^(1/2)*(x^2 + 2)^(1/2))/(a + b*x^2), x)
```

3.91 $\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}(a+bx^2)}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 58

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx=\frac{2\sqrt{1+x^2}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}$$

```
[Out] 2*(1/(2*x^2+4))^(1/2)*(2*x^2+4)^(1/2)*EllipticPi(x*2^(1/2)/(2*x^2+4)^(1/2),
1-2*b/a,I)*(x^2+1)^(1/2)/a/((x^2+1)/(x^2+2))^(1/2)/(x^2+2)^(1/2)
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.036, Rules used = {553}

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx=\frac{2\sqrt{x^2+1}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}}$$

```
[In] Int[Sqrt[2 + x^2]/(Sqrt[1 + x^2]*(a + b*x^2)),x]
```

```
[Out] (2*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a*Sqrt[(1
 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
```

[d/c]

Rubi steps

$$\text{integral} = \frac{2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = -\frac{i\left(a\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)-(a-2b)\,\text{EllipticPi}\left(\frac{2b}{a},i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)\right)}{ab}$$

```
[In] Integrate[Sqrt[2 + x^2]/(Sqrt[1 + x^2]*(a + b*x^2)),x]
[Out] ((-I)*(a*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] - (a - 2*b)*EllipticPi[(2*b)/a,
 I*ArcSinh[x/Sqrt[2]], 2]))/(a*b)
```

Maple [A] (verified)

Time = 3.34 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.10

method	result
default	$-\frac{i\left(aF\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)-a\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)+2b\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)\right)}{ab}$
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\left(-\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2b\sqrt{x^4+3x^2+2}}+\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{b\sqrt{x^4+3x^2+2}}-\frac{2i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{a\sqrt{x^4+3x^2+2}}\right)}{\sqrt{x^2+1}\,\sqrt{x^2+2}}$

```
[In] int((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
[Out] -I*(a*EllipticF(1/2*I*x*2^(1/2),2^(1/2))-a*EllipticPi(1/2*I*x*2^(1/2),2*b/a
,2^(1/2))+2*b*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2)))/a/b
```

Fricas [F]

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)\sqrt{x^2+1}}\,dx$$

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="fricas")
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^4 + (a + b)*x^2 + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(a+bx^2)\,\sqrt{x^2+1}}\,dx$$

```
[In] integrate((x**2+2)**(1/2)/(x**2+1)**(1/2)/(b*x**2+a),x)
[Out] Integral(sqrt(x**2 + 2)/((a + b*x**2)*sqrt(x**2 + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)\sqrt{x^2+1}}\,dx$$

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)), x)
```

Giac [F]

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)\sqrt{x^2+1}}\,dx$$

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(1/2)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*sqrt(x^2 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{\sqrt{x^2+1}\,\left(b\,x^2+a\right)}\,dx$$

```
[In] int((x^2 + 2)^(1/2)/((x^2 + 1)^(1/2)*(a + b*x^2)),x)
[Out] int((x^2 + 2)^(1/2)/((x^2 + 1)^(1/2)*(a + b*x^2)), x)
```

3.92 $\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)} dx$

Optimal result

Integrand size = 28, antiderivative size = 121

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)} dx = \frac{\sqrt{2}\sqrt{2+x^2}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{(a-b)\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{2b\sqrt{1+x^2}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a(a-b)\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}$$

```
[Out] -2*b*(1/(2*x^2+4))^(1/2)*(2*x^2+4)^(1/2)*EllipticPi(x*2^(1/2)/(2*x^2+4)^(1/
2),1-2*b/a,I)*(x^2+1)^(1/2)/a/(a-b)/((x^2+1)/(x^2+2))^(1/2)/(x^2+2)^(1/2)+(
1/(x^2+1))^(1/2)*EllipticE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2+2)^(1/
2)/(a-b)/((x^2+2)/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {555, 553, 422}

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)} dx = \frac{\sqrt{2}\sqrt{x^2+2}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)} - \frac{2b\sqrt{x^2+1}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}(a-b)}$$

```
[In] Int[Sqrt[2 + x^2]/((1 + x^2)^(3/2)*(a + b*x^2)),x]
```

```
[Out] (Sqrt[2]*Sqrt[2 + x^2]*EllipticE[ArcTan[x], 1/2])/((a - b)*Sqrt[1 + x^2]*Sq
rt[(2 + x^2)/(1 + x^2)]) - (2*b*Sqrt[1 + x^2]*EllipticPi[1 - (2*b)/a, ArcTa
n[x/Sqrt[2]], -1])/(a*(a - b)*Sqrt[(1 + x^2)/(2 + x^2)]*Sqrt[2 + x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 555

```
Int[Sqrt[(e_) + (f_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[b/(b*c - a*d), Int[Sqrt[e + f*x^2]/((a + b*x^2)*S
qrt[c + d*x^2]), x], x] - Dist[d/(b*c - a*d), Int[Sqrt[e + f*x^2]/(c + d*x^
2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[d/c] && PosQ[f/e]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{b\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}(a+bx^2)}\,dx}{a-b} - \frac{\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}}\,dx}{-a+b} \\ &= \frac{\sqrt{2}\sqrt{2+x^2}E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{(a-b)\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{2b\sqrt{1+x^2}\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a(a-b)\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}} \end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx = \frac{\frac{x}{\sqrt{\frac{1+x^2}{2+x^2}}} + iE\left(i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right)\middle|2\right) - \frac{i(a-2b)\,\text{EllipticPi}\left(\frac{2b}{a},i\text{arcsinh}\left(\frac{x}{\sqrt{2}}\right),2\right)}{a}}{a-b}$$

```
[In] Integrate[Sqrt[2 + x^2]/((1 + x^2)^(3/2)*(a + b*x^2)),x]
[Out] (x/Sqrt[(1 + x^2)/(2 + x^2)] + I*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - (I*(a
 - 2*b)*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2])/a)/(a - b)
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.21

method	result
default	$\frac{\left(iE\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a\sqrt{x^2+2}\,\sqrt{x^2+1}-i\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)a\sqrt{x^2+2}\,\sqrt{x^2+1}+2i\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)b\sqrt{x^2+2}\,\sqrt{x^2+1}+a\,x^3+2ax\right)\sqrt{x^2+1}\,\sqrt{x^2+2}}{a(x^4+3x^2+2)(a-b)}$
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\left(\frac{\left(x^2+2\right)x}{(a-b)\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}+\frac{i\sqrt{2}\,\sqrt{2x^2+4}\,\sqrt{x^2+1}\,E\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2(a-b)\sqrt{x^4+3x^2+2}}-\frac{i\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+1}\,\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)}{(a-b)\sqrt{x^4+3x^2+2}}+\frac{2ib\sqrt{2}\,\sqrt{1+\frac{x^2}{2}}\,\sqrt{x^2+}}{(a-b)a\sqrt{x}}\right.}{\sqrt{x^2+1}\,\sqrt{x^2+2}}$

```
[In] int((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] (I*EllipticE(1/2*I*x*2^(1/2),2^(1/2))*a*(x^2+2)^(1/2)*(x^2+1)^(1/2)-I*Ellip
ticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*a*(x^2+2)^(1/2)*(x^2+1)^(1/2)+2*I*Elli
pticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))*b*(x^2+2)^(1/2)*(x^2+1)^(1/2)+a*x^3+2
*a*x)*(x^2+1)^(1/2)*(x^2+2)^(1/2)/a/(x^4+3*x^2+2)/(a-b)
```

Fricas [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}\,(a+bx^2)}\,dx=\int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^6 + (a + 2*b)*x^4 + (2*a + b)*x^2
 + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}\,(a+bx^2)}\,dx=\int \frac{\sqrt{x^2+2}}{(a+bx^2)\,(x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x**2+2)**(1/2)/(x**2+1)**(3/2)/(b*x**2+a),x)
```

```
[Out] Integral(sqrt(x**2 + 2)/((a + b*x**2)*(x**2 + 1)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(3/2)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{3/2}(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(x^2+1)^{3/2}\,(b\,x^2+a)}\,dx$$

```
[In] int((x^2 + 2)^(1/2)/((x^2 + 1)^(3/2)*(a + b*x^2)),x)
[Out] int((x^2 + 2)^(1/2)/((x^2 + 1)^(3/2)*(a + b*x^2)), x)
```

3.93 $\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 215

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)} \, dx = \frac{x\sqrt{2+x^2}}{3(a-b)(1+x^2)^{3/2}} + \frac{\sqrt{2}(a-2b)\sqrt{2+x^2}E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{(a-b)^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} - \frac{\sqrt{2}\sqrt{2+x^2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3(a-b)\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}} + \frac{2b^2\sqrt{1+x^2}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a(a-b)^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}$$

```
[Out] 2*b^2*(1/(2*x^2+4))^(1/2)*(2*x^2+4)^(1/2)*EllipticPi(x*2^(1/2)/(2*x^2+4)^(1
/2),1-2*b/a,I)*(x^2+1)^(1/2)/a/(a-b)^2/((x^2+1)/(x^2+2))^(1/2)/(x^2+2)^(1/2
)+1/3*x*(x^2+2)^(1/2)/(a-b)/(x^2+1)^(3/2)+(a-2*b)*(1/(x^2+1))^(1/2)*Ellipti
cE(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(1/2)*(x^2+2)^(1/2)/(a-b)^2/((x^2+2)/(x^2
+1))^(1/2)-1/3*(1/(x^2+1))^(1/2)*EllipticF(x/(x^2+1)^(1/2),1/2*2^(1/2))*2^(
1/2)*(x^2+2)^(1/2)/(a-b)/((x^2+2)/(x^2+1))^(1/2)
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used

= {560, 553, 540, 539, 429, 422}

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \frac{2b^2\sqrt{x^2+1}\,\text{EllipticPi}\left(1-\frac{2b}{a},\arctan\left(\frac{x}{\sqrt{2}}\right),-1\right)}{a\sqrt{\frac{x^2+1}{x^2+2}}\sqrt{x^2+2}(a-b)^2}$$
$$-\frac{\sqrt{2}\sqrt{x^2+2}\,\text{EllipticF}\left(\arctan(x),\frac{1}{2}\right)}{3\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)}$$
$$+\frac{\sqrt{2}\sqrt{x^2+2}(a-2b)E\left(\arctan(x)\,\middle|\,\frac{1}{2}\right)}{\sqrt{x^2+1}\sqrt{\frac{x^2+2}{x^2+1}}(a-b)^2}+\frac{x\sqrt{x^2+2}}{3\left(x^2+1\right)^{3/2}(a-b)}$$

```
[In] Int[Sqrt[2 + x^2]/((1 + x^2)^(5/2)*(a + b*x^2)),x]
```

```
[Out] (x*Sqrt[2 + x^2])/(3*(a - b)*(1 + x^2)^(3/2)) + (Sqrt[2]*(a - 2*b)*Sqrt[2 +
 x^2]*EllipticE[ArcTan[x], 1/2])/((a - b)^2*Sqrt[1 + x^2]*Sqrt[(2 + x^2)/(1
 + x^2)]) - (Sqrt[2]*Sqrt[2 + x^2]*EllipticF[ArcTan[x], 1/2])/(3*(a - b)*Sq
rt[1 + x^2]*Sqrt[(2 + x^2)/(1 + x^2)]) + (2*b^2*Sqrt[1 + x^2]*EllipticPi[1
- (2*b)/a, ArcTan[x/Sqrt[2]], -1])/(a*(a - b)^2*Sqrt[(1 + x^2)/(2 + x^2)]*S
qrt[2 + x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 539

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 540

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
```

```
+ d*x^n)^q/(a*b*n*(p + 1))), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(
p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p
 + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rule 560

```
Int[(((c_) + (d_.)*(x_)^2)^(q_)*((e_) + (f_.)*(x_)^2)^(r_))/((a_) + (b_.)*(
x_)^2), x_Symbol] :> Dist[b^2/(b*c - a*d)^2, Int[(c + d*x^2)^(q + 2)*((e +
f*x^2)^r/(a + b*x^2)), x], x] - Dist[d/(b*c - a*d)^2, Int[(c + d*x^2)^q*(e
+ f*x^2)^r*(2*b*c - a*d + b*d*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, r},
x] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\int \frac{\sqrt{2+x^2}\left(-a+2b+bx^2\right)}{\left(1+x^2\right)^{5/2}}\,dx}{(a-b)^2}+\frac{b^2\int \frac{\sqrt{2+x^2}}{\sqrt{1+x^2}\left(a+bx^2\right)}\,dx}{(a-b)^2}\\
&= \frac{x\sqrt{2+x^2}}{3(a-b)\left(1+x^2\right)^{3/2}}+\frac{2b^2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a(a-b)^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}+\frac{\int \frac{2(2a-5b)+(a-4b)x^2}{\left(1+x^2\right)^{3/2}\sqrt{2+x^2}}\,dx}{3(a-b)^2}\\
&= \frac{x\sqrt{2+x^2}}{3(a-b)\left(1+x^2\right)^{3/2}}+\frac{2b^2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a(a-b)^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}\\
&\quad+\frac{(a-2b)\int \frac{\sqrt{2+x^2}}{\left(1+x^2\right)^{3/2}}\,dx}{(a-b)^2}-\frac{2\int \frac{1}{\sqrt{1+x^2}\sqrt{2+x^2}}\,dx}{3(a-b)}\\
&= \frac{x\sqrt{2+x^2}}{3(a-b)\left(1+x^2\right)^{3/2}}+\frac{\sqrt{2}(a-2b)\sqrt{2+x^2}\,E\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{(a-b)^2\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}\\
&\quad-\frac{\sqrt{2}\sqrt{2+x^2}\,F\left(\tan^{-1}(x)\middle|\frac{1}{2}\right)}{3(a-b)\sqrt{1+x^2}\sqrt{\frac{2+x^2}{1+x^2}}}+\frac{2b^2\sqrt{1+x^2}\,\Pi\left(1-\frac{2b}{a};\tan^{-1}\left(\frac{x}{\sqrt{2}}\right)\middle|-1\right)}{a(a-b)^2\sqrt{\frac{1+x^2}{2+x^2}}\sqrt{2+x^2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.08 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.64

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \frac{4a^2x\sqrt{1+x^2}\sqrt{2+x^2}-7abx\sqrt{1+x^2}\sqrt{2+x^2}+3a^2x^3\sqrt{1+x^2}\sqrt{2+x^2}-6ab}{}$$

```
[In] Integrate[Sqrt[2 + x^2]/((1 + x^2)^(5/2)*(a + b*x^2)),x]
```

```
[Out] (4*a^2*x*Sqrt[1 + x^2]*Sqrt[2 + x^2] - 7*a*b*x*Sqrt[1 + x^2]*Sqrt[2 + x^2]
+ 3*a^2*x^3*Sqrt[1 + x^2]*Sqrt[2 + x^2] - 6*a*b*x^3*Sqrt[1 + x^2]*Sqrt[2 +
x^2] + (3*I)*a*(a - 2*b)*(1 + x^2)^2*EllipticE[I*ArcSinh[x/Sqrt[2]], 2] - I
*a*(a - b)*(1 + x^2)^2*EllipticF[I*ArcSinh[x/Sqrt[2]], 2] + (3*I)*a*b*Ellip
ticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (6*I)*b^2*EllipticPi[(2*b)/a, I*A
rcSinh[x/Sqrt[2]], 2] + (6*I)*a*b*x^2*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[
2]], 2] - (12*I)*b^2*x^2*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] + (3*
I)*a*b*x^4*EllipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2] - (6*I)*b^2*x^4*Ell
ipticPi[(2*b)/a, I*ArcSinh[x/Sqrt[2]], 2])/(3*a*(a - b)^2*(1 + x^2)^2)
```

Maple [A] (verified)

Time = 4.23 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.73

method	result
elliptic	$\frac{\sqrt{(x^2+1)(x^2+2)}\left(\frac{x\sqrt{x^4+3x^2+2}}{3(a-b)\left(x^2+1\right)^2}+\frac{\left(x^2+2\right)x(a-2b)}{(a-b)^2\sqrt{\left(x^2+1\right)\left(x^2+2\right)}}-\frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,F\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{6\sqrt{x^4+3x^2+2}\,(a-b)}+\frac{i\sqrt{2}\sqrt{2x^2+4}\sqrt{x^2+1}\,aE\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)}{2(a-b)^2\sqrt{x^4+3x^2+2}}-\right.}{\sqrt{x^2+1}\sqrt{x^2+2}}$
default	$-\frac{iF\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^2\sqrt{x^2+2}\sqrt{x^2+1}-3iE\left(\frac{ix\sqrt{2}}{2},\sqrt{2}\right)a^2\sqrt{x^2+2}\sqrt{x^2+1}+6i\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)b^2\sqrt{x^2+2}\sqrt{x^2+1}-3i\Pi\left(\frac{ix\sqrt{2}}{2},\frac{2b}{a},\sqrt{2}\right)ab}{}$

```
[In] int((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] ((x^2+1)*(x^2+2))^(1/2)/(x^2+1)^(1/2)/(x^2+2)^(1/2)*(1/3*x/(a-b)*(x^4+3*x^2
+2)^(1/2)/(x^2+1)^2+(x^2+2)*x*(a-2*b)/(a-b)^2/((x^2+1)*(x^2+2))^(1/2)-1/6*I
*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticF(1/2*I*
x*2^(1/2),2^(1/2))/(a-b)+1/2*I/(a-b)^2*2^(1/2)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2
)/(x^4+3*x^2+2)^(1/2)*a*EllipticE(1/2*I*x*2^(1/2),2^(1/2))-I/(a-b)^2*2^(1/2
)*(2*x^2+4)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*b*EllipticE(1/2*I*x*2^(
1/2),2^(1/2))+I/(a-b)^2*b*2^(1/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^
2+2)^(1/2)*EllipticPi(1/2*I*x*2^(1/2),2*b/a,2^(1/2))-2*I/(a-b)^2/a*b^2*2^(1
/2)*(1+1/2*x^2)^(1/2)*(x^2+1)^(1/2)/(x^4+3*x^2+2)^(1/2)*EllipticPi(1/2*I*x*
2^(1/2),2*b/a,2^(1/2)))
```

Fricas [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="fricas")
[Out] integral(sqrt(x^2 + 2)*sqrt(x^2 + 1)/(b*x^8 + (a + 3*b)*x^6 + 3*(a + b)*x^4
 + (3*a + b)*x^2 + a), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \text{Timed out}$$

```
[In] integrate((x**2+2)**(1/2)/(x**2+1)**(5/2)/(b*x**2+a),x)
[Out] Timed out
```

Maxima [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{(bx^2+a)(x^2+1)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((x^2+2)^(1/2)/(x^2+1)^(5/2)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sqrt(x^2 + 2)/((b*x^2 + a)*(x^2 + 1)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+x^2}}{(1+x^2)^{5/2}\,(a+bx^2)}\,dx = \int \frac{\sqrt{x^2+2}}{{\left(x^2+1\right)}^{5/2}\,\left(b\,x^2+a\right)}\,dx$$

```
[In] int((x^2 + 2)^(1/2)/((x^2 + 1)^(5/2)*(a + b*x^2)),x)
[Out] int((x^2 + 2)^(1/2)/((x^2 + 1)^(5/2)*(a + b*x^2)), x)
```

3.94 $\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 298

$$\begin{aligned}\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx &= \frac{fx\sqrt{2+dx^2}}{b\sqrt{3+fx^2}}-\frac{\sqrt{2}\sqrt{f}\sqrt{2+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{b\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}\\&+\frac{3d\sqrt{2+dx^2}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}b\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}\\&+\frac{3(2b-ad)\sqrt{2+dx^2}\operatorname{EllipticPi}\left(1-\frac{3b}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}ab\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}\end{aligned}$$

```
[Out] f*x*(d*x^2+2)^(1/2)/b/(f*x^2+3)^(1/2)+3/2*d*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2+
9)^(1/2)*EllipticF(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1/2*(4-6*d/f)^(1/2))
*(d*x^2+2)^(1/2)/b*2^(1/2)/f^(1/2)/((d*x^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)^(1
/2)+3/2*(-a*d+2*b)*(1/(3*f*x^2+9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticPi(x*f^(
1/2)*3^(1/2)/(3*f*x^2+9)^(1/2),1-3*b/a/f,1/2*(4-6*d/f)^(1/2))*(d*x^2+2)^(1/
2)/a/b*2^(1/2)/f^(1/2)/((d*x^2+2)/(f*x^2+3))^(1/2)/(f*x^2+3)^(1/2)-(1/(3*f*
x^2+9))^(1/2)*(3*f*x^2+9)^(1/2)*EllipticE(x*f^(1/2)*3^(1/2)/(3*f*x^2+9)^(1/
2),1/2*(4-6*d/f)^(1/2))*2^(1/2)*f^(1/2)*(d*x^2+2)^(1/2)/b/((d*x^2+2)/(f*x^2
+3))^(1/2)/(f*x^2+3)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {549, 433, 429, 506, 422, 553}

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx = \frac{3\sqrt{dx^2+2}(2b-ad)\,\text{EllipticPi}\left(1-\frac{3b}{af},\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}ab\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}} + \frac{3d\sqrt{dx^2+2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right),1-\frac{3d}{2f}\right)}{\sqrt{2}b\sqrt{f}\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}} - \frac{\sqrt{2}\sqrt{f}\sqrt{dx^2+2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{b\sqrt{fx^2+3}\sqrt{\frac{dx^2+2}{fx^2+3}}} + \frac{fx\sqrt{dx^2+2}}{b\sqrt{fx^2+3}}$$

```
[In] Int[(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(a + b*x^2),x]
```

```
[Out] (f*x*Sqrt[2 + d*x^2])/(b*Sqrt[3 + f*x^2]) - (Sqrt[2]*Sqrt[f]*Sqrt[2 + d*x^2
]*EllipticE[ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(b*Sqrt[(2 + d*x
^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (3*d*Sqrt[2 + d*x^2]*EllipticF[ArcTan[(
Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*b*Sqrt[f]*Sqrt[(2 + d*x^2)/
(3 + f*x^2)]*Sqrt[3 + f*x^2]) + (3*(2*b - a*d)*Sqrt[2 + d*x^2]*EllipticPi[1
 - (3*b)/(a*f), ArcTan[(Sqrt[f]*x)/Sqrt[3]], 1 - (3*d)/(2*f)])/(Sqrt[2]*a*b
*Sqrt[f]*Sqrt[(2 + d*x^2)/(3 + f*x^2)]*Sqrt[3 + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
```

```
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 549

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2), x_Symbol] :> Dist[d/b, Int[Sqrt[e + f*x^2]/Sqrt[c + d*x^2], x], x] +
Dist[(b*c - a*d)/b, Int[Sqrt[e + f*x^2]/((a + b*x^2)*Sqrt[c + d*x^2]), x],
x] /; FreeQ[{a, b, c, d, e, f}, x] &&  !SimplerSqrtQ[-f/e, -d/c]
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{d\int \frac{\sqrt{3+fx^2}}{\sqrt{2+dx^2}}\,dx}{b} + \frac{(2b-ad)\int \frac{\sqrt{3+fx^2}}{(a+bx^2)\sqrt{2+dx^2}}\,dx}{b} \\
&= \frac{3(2b-ad)\sqrt{2+dx^2}\,\Pi\left(1-\frac{3b}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}ab\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} \\
&\quad + \frac{(3d)\int \frac{1}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx}{b} + \frac{(df)\int \frac{x^2}{\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx}{b} \\
&= \frac{fx\sqrt{2+dx^2}}{b\sqrt{3+fx^2}} + \frac{3d\sqrt{2+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}b\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} \\
&\quad + \frac{3(2b-ad)\sqrt{2+dx^2}\,\Pi\left(1-\frac{3b}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}ab\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} - \frac{(3f)\int \frac{\sqrt{2+dx^2}}{(3+fx^2)^{3/2}}\,dx}{b}
\end{aligned}
$$

$$= \frac{fx\sqrt{2+dx^2}}{b\sqrt{3+fx^2}} - \frac{\sqrt{2}\sqrt{f}\sqrt{2+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{b\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} + \frac{3d\sqrt{2+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}b\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}} + \frac{3(2b-ad)\sqrt{2+dx^2}\Pi\left(1-\frac{3b}{af};\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{3}}\right)|1-\frac{3d}{2f}\right)}{\sqrt{2}ab\sqrt{f}\sqrt{\frac{2+dx^2}{3+fx^2}}\sqrt{3+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.45

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx = \frac{i\left(-3abdE\left(i\text{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|\frac{2f}{3d}\right)+(-2b+ad)\left(af\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)+(3b-af)\,\text{EllipticPi}\right.\right.}{\sqrt{3}ab^2\sqrt{d}}$$

```
[In] Integrate[(Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2])/(a + b*x^2),x]
```

```
[Out] (I*(-3*a*b*d*EllipticE[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (-2*b
 + a*d)*(a*f*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (3*b
- a*f)*EllipticPi[(2*b)/(a*d), I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]
)))/(Sqrt[3]*a*b^2*Sqrt[d])
```

Maple [A] (verified)

Time = 3.38 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.98

method	result
default	$-\frac{\left(F\left(\frac{x\sqrt{3}\sqrt{-f}}{3},\frac{\sqrt{2}\sqrt{3}\sqrt{\frac{d}{f}}}{2}\right)a^2df-a^2\Pi\left(\frac{x\sqrt{3}\sqrt{-f}}{3},\frac{3b}{af},\frac{\sqrt{2}\sqrt{-d}\sqrt{3}}{2\sqrt{-f}}\right)df-3F\left(\frac{x\sqrt{3}\sqrt{-f}}{3},\frac{\sqrt{2}\sqrt{3}\sqrt{\frac{d}{f}}}{2}\right)dba-2fE\left(\frac{x\sqrt{3}\sqrt{-f}}{3},\frac{\sqrt{2}\sqrt{3}\sqrt{\frac{d}{f}}}{2}\right.\right.}{2a\,b^2\sqrt{-f}}$
elliptic	$\frac{\sqrt{(f\,x^2+3)(d\,x^2+2)}\left(-\frac{\sqrt{3f\,x^2+9}\,\sqrt{2d\,x^2+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)adf}{2b^2\sqrt{-3f}\,\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}}+\frac{3\sqrt{3f\,x^2+9}\,\sqrt{2d\,x^2+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)d}{2b\sqrt{-3f}\,\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}}+\frac{f\sqrt{3f\,x}}{}\right.}{}$

```
[In] int((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*a
^2*d*f-a^2*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/
2)*3^(1/2)/(-f)^(1/2))*d*f-3*EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)
*3^(1/2)*(d/f)^(1/2))*d*b*a-2*f*EllipticE(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1
/2)*3^(1/2)*(d/f)^(1/2))*b*a+3*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,
1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*d*b*a+2*EllipticPi(1/3*x*3^(1/2)
*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*f*b*a-6*Elli
pticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2)/(-f)
^(1/2))*b^2)*2^(1/2)/a/b^2/(-f)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{bx^2+a}\,dx$$

```
[In] integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="fricas")
[Out] integral(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

Sympy [F]

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{a+bx^2}\,dx$$

```
[In] integrate((d*x**2+2)**(1/2)*(f*x**2+3)**(1/2)/(b*x**2+a),x)
[Out] Integral(sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{bx^2+a}\,dx$$

```
[In] integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="maxima")
[Out] integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{dx^2+2}\sqrt{fx^2+3}}{bx^2+a}\,dx$$

```
[In] integrate((d*x^2+2)^(1/2)*(f*x^2+3)^(1/2)/(b*x^2+a),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+dx^2}\sqrt{3+fx^2}}{a+bx^2}\,dx = \int \frac{\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}}{b\,x^2+a}\,dx$$

```
[In] int(((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2))/(a + b*x^2),x)
[Out] int(((d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2))/(a + b*x^2), x)
```

3.95 $\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 93

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = \frac{2\sqrt{3+fx^2}\,\text{EllipticPi}\left(1-\frac{2b}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),1-\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{d}\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}}$$

```
[Out] 2/3*(1/(2*d*x^2+4))^(1/2)*(2*d*x^2+4)^(1/2)*EllipticPi(x*d^(1/2)*2^(1/2)/(2
*d*x^2+4)^(1/2),1-2*b/a/d,1/3*(9-6*f/d)^(1/2))*(f*x^2+3)^(1/2)/a*3^(1/2)/d^
(1/2)/(d*x^2+2)^(1/2)/((f*x^2+3)/(d*x^2+2))^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {553}

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = \frac{2\sqrt{fx^2+3}\,\text{EllipticPi}\left(1-\frac{2b}{ad},\arctan\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),1-\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{d}\sqrt{dx^2+2}\sqrt{\frac{fx^2+3}{dx^2+2}}}$$

```
[In] Int[Sqrt[2 + d*x^2]/((a + b*x^2)*Sqrt[3 + f*x^2]),x]
```

```
[Out] (2*Sqrt[3 + f*x^2]*EllipticPi[1 - (2*b)/(a*d), ArcTan[(Sqrt[d]*x)/Sqrt[2]],
 1 - (2*f)/(3*d)])/(Sqrt[3]*a*Sqrt[d]*Sqrt[2 + d*x^2]*Sqrt[(3 + f*x^2)/(2 +
 d*x^2)])
```

Rule 553

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)*Sqrt[(e_) + (f_.)*(x_)
^2]), x_Symbol] :> Simp[c*(Sqrt[e + f*x^2]/(a*e*Rt[d/c, 2]*Sqrt[c + d*x^2]*
```

```
Sqrt[c*((e + f*x^2)/(e*(c + d*x^2)))]))*EllipticPi[1 - b*(c/(a*d)), ArcTan[
Rt[d/c, 2]*x], 1 - c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ
[d/c]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{3+fx^2}\,\Pi\left(1-\frac{2b}{ad};\tan^{-1}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right)|1-\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{d}\sqrt{2+dx^2}\sqrt{\frac{3+fx^2}{2+dx^2}}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.55 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = -\frac{i\left(ad\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)+(2b-ad)\,\text{EllipticPi}\left(\frac{2b}{ad},i\text{arcsinh}\left(\frac{\sqrt{d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)\right)}{\sqrt{3}ab\sqrt{d}}$$

```
[In] Integrate[Sqrt[2 + d*x^2]/((a + b*x^2)*Sqrt[3 + f*x^2]),x]
```

```
[Out] ((-I)*(a*d*EllipticF[I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)] + (2*b -
a*d)*EllipticPi[(2*b)/(a*d), I*ArcSinh[(Sqrt[d]*x)/Sqrt[2]], (2*f)/(3*d)]))
/(Sqrt[3]*a*b*Sqrt[d])
```

Maple [A] (verified)

Time = 3.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.43

method	result
default	$\frac{\left(F\left(\frac{x\sqrt{3}\sqrt{-f}}{3},\frac{\sqrt{2}\sqrt{3}\sqrt{\frac{d}{f}}}{2}\right)ad-\Pi\left(\frac{x\sqrt{3}\sqrt{-f}}{3},\frac{3b}{af},\frac{\sqrt{2}\sqrt{-d}\sqrt{3}}{2\sqrt{-f}}\right)ad+2\Pi\left(\frac{x\sqrt{3}\sqrt{-f}}{3},\frac{3b}{af},\frac{\sqrt{2}\sqrt{-d}\sqrt{3}}{2\sqrt{-f}}\right)b\right)\sqrt{2}}{2ab\sqrt{-f}}$
elliptic	$\frac{\sqrt{(f\,x^2+3)(d\,x^2+2)}\left(\frac{\sqrt{3f\,x^2+9}\,\sqrt{2d\,x^2+4}\,F\left(\frac{x\sqrt{-3f}}{3},\frac{\sqrt{-4+\frac{6d+4f}{f}}}{2}\right)d}{2b\sqrt{-3f}\,\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}}-\frac{\sqrt{1+\frac{f\,x^2}{3}}\,\sqrt{1+\frac{d\,x^2}{2}}\,\Pi\left(\sqrt{-\frac{f}{3}}\,x,\frac{3b}{af},\frac{\sqrt{-\frac{d}{2}}}{\sqrt{-\frac{f}{3}}}\right)d}{b\sqrt{-\frac{f}{3}}\,\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}}+\frac{2\sqrt{1+\frac{f\,x^2}{3}}\,\sqrt{1+\frac{d}{}}}{a\sqrt{-\frac{f}{3}}\,\sqrt{d}}\right.}{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}}$

```
[In] int((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(EllipticF(1/3*x*3^(1/2)*(-f)^(1/2),1/2*2^(1/2)*3^(1/2)*(d/f)^(1/2))*a*
d-EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(1/2
)/(-f)^(1/2))*a*d+2*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)
*(-d)^(1/2)*3^(1/2)/(-f)^(1/2))*b)*2^(1/2)/a/b/(-f)^(1/2)
```

Fricas [F]

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = \int \frac{\sqrt{dx^2+2}}{(bx^2+a)\sqrt{fx^2+3}}\,dx$$

```
[In] integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)/(b*f*x^4 + (a*f + 3*b)*x^2 + 3*a),
 x)
```

Sympy [F]

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = \int \frac{\sqrt{dx^2+2}}{(a+bx^2)\sqrt{fx^2+3}}\,dx$$

```
[In] integrate((d*x**2+2)**(1/2)/(b*x**2+a)/(f*x**2+3)**(1/2),x)
```

```
[Out] Integral(sqrt(d*x**2 + 2)/((a + b*x**2)*sqrt(f*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = \int \frac{\sqrt{dx^2+2}}{(bx^2+a)\sqrt{fx^2+3}}\,dx$$

```
[In] integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = \int \frac{\sqrt{dx^2+2}}{(bx^2+a)\sqrt{fx^2+3}}\,dx$$

```
[In] integrate((d*x^2+2)^(1/2)/(b*x^2+a)/(f*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + 2)/((b*x^2 + a)*sqrt(f*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2+dx^2}}{(a+bx^2)\sqrt{3+fx^2}}\,dx = \int \frac{\sqrt{d\,x^2+2}}{(b\,x^2+a)\,\sqrt{f\,x^2+3}}\,dx$$

```
[In] int((d*x^2 + 2)^(1/2)/((a + b*x^2)*(f*x^2 + 3)^(1/2)),x)
[Out] int((d*x^2 + 2)^(1/2)/((a + b*x^2)*(f*x^2 + 3)^(1/2)), x)
```

3.96 $\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 49

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx=\frac{\text{EllipticPi}\left(\frac{2b}{ad},\arcsin\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

```
[Out] 1/3*EllipticPi(1/2*x*(-d)^(1/2)*2^(1/2),2*b/a/d,1/3*6^(1/2)*(f/d)^(1/2))/a*
3^(1/2)/(-d)^(1/2)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.031, Rules used = {551}

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx=\frac{\text{EllipticPi}\left(\frac{2b}{ad},\arcsin\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right),\frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

```
[In] Int[1/((a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
```

```
[Out] EllipticPi[(2*b)/(a*d), ArcSin[(Sqrt[-d]*x)/Sqrt[2]], (2*f)/(3*d)]/(Sqrt[3]
*a*Sqrt[-d])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rubi steps

$$\text{integral} = \frac{\Pi\left(\frac{2b}{ad}; \sin^{-1}\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right) | \frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a + bx^2)\sqrt{2 + dx^2}\sqrt{3 + fx^2}} \, dx = \frac{\text{EllipticPi}\left(\frac{2b}{ad}, \arcsin\left(\frac{\sqrt{-d}x}{\sqrt{2}}\right), \frac{2f}{3d}\right)}{\sqrt{3}a\sqrt{-d}}$$

```
[In] Integrate[1/((a + b*x^2)*Sqrt[2 + d*x^2]*Sqrt[3 + f*x^2]),x]
[Out] EllipticPi[(2*b)/(a*d), ArcSin[(Sqrt[-d]*x)/Sqrt[2]], (2*f)/(3*d)]/(Sqrt[3]
*a*Sqrt[-d])
```

Maple [A] (verified)

Time = 3.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

method	result	size
default	$\frac{\Pi\left(\frac{x\sqrt{3}\,\sqrt{-f}}{3}, \frac{3b}{af}, \frac{\sqrt{2}\,\sqrt{-d}\,\sqrt{3}}{2\sqrt{-f}}\right)\sqrt{2}}{2a\sqrt{-f}}$	53
elliptic	$\frac{\sqrt{(f\,x^2+3)(d\,x^2+2)}\,\sqrt{1+\frac{f\,x^2}{3}}\,\sqrt{1+\frac{d\,x^2}{2}}\,\Pi\left(\sqrt{-\frac{f}{3}}\,x, \frac{3b}{af}, \frac{\sqrt{-\frac{d}{2}}}{\sqrt{-\frac{f}{3}}}\right)}{\sqrt{f\,x^2+3}\,\sqrt{d\,x^2+2}\,a\sqrt{-\frac{f}{3}}\,\sqrt{df\,x^4+3d\,x^2+2f\,x^2+6}}$	115

```
[In] int(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*EllipticPi(1/3*x*3^(1/2)*(-f)^(1/2),3*b/a/f,1/2*2^(1/2)*(-d)^(1/2)*3^(1
/2)/(-f)^(1/2))*2^(1/2)/a/(-f)^(1/2)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a + bx^2)\sqrt{2 + dx^2}\sqrt{3 + fx^2}} \, dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="fricas"
)
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{1}{(a+bx^2)\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

```
[In] integrate(1/(b*x**2+a)/(d*x**2+2)**(1/2)/(f*x**2+3)**(1/2),x)
[Out] Integral(1/((a + b*x**2)*sqrt(d*x**2 + 2)*sqrt(f*x**2 + 3)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="maxima"
)
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{1}{(bx^2+a)\sqrt{dx^2+2}\sqrt{fx^2+3}}\,dx$$

```
[In] integrate(1/(b*x^2+a)/(d*x^2+2)^(1/2)/(f*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*x^2 + a)*sqrt(d*x^2 + 2)*sqrt(f*x^2 + 3)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)\sqrt{2+dx^2}\sqrt{3+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)\,\sqrt{d\,x^2+2}\,\sqrt{f\,x^2+3}}\,dx$$

```
[In] int(1/((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)),x)
[Out] int(1/((a + b*x^2)*(d*x^2 + 2)^(1/2)*(f*x^2 + 3)^(1/2)), x)
```

3.97 $\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 30, antiderivative size = 36

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{1+\frac{bx^2}{a}}\,\text{EllipticF}\left(\arcsin(x),-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

```
[Out] -EllipticF(x,(-b/a)^(1/2))*(1+b*x^2/a)^(1/2)/(b*x^2+a)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {21, 432, 430}

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{\frac{bx^2}{a}+1}\,\text{EllipticF}\left(\arcsin(x),-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

```
[In] Int[Sqrt[1 - x^2]/((-1 + x^2)*Sqrt[a + b*x^2]),x]
```

```
[Out] -((Sqrt[1 + (b*x^2)/a]*EllipticF[ArcSin[x], -(b/a)])/Sqrt[a + b*x^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= -\int \frac{1}{\sqrt{1-x^2}\sqrt{a+bx^2}}\,dx \\ &= -\frac{\sqrt{1+\frac{bx^2}{a}}\int \frac{1}{\sqrt{1-x^2}\sqrt{1+\frac{bx^2}{a}}}\,dx}{\sqrt{a+bx^2}} \\ &= -\frac{\sqrt{1+\frac{bx^2}{a}}F\left(\sin^{-1}(x)|-\frac{b}{a}\right)}{\sqrt{a+bx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = -\frac{\sqrt{\frac{a+bx^2}{a}}\,\text{EllipticF}\left(\arcsin(x),-\frac{b}{a}\right)}{\sqrt{a+bx^2}}$$

```
[In] Integrate[Sqrt[1 - x^2]/((-1 + x^2)*Sqrt[a + b*x^2]),x]
[Out] -((Sqrt[(a + b*x^2)/a]*EllipticF[ArcSin[x], -(b/a)])/Sqrt[a + b*x^2])
```

Maple [A] (verified)

Time = 3.60 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{\sqrt{\frac{b\,x^2+a}{a}}\,F\left(x,\sqrt{-\frac{b}{a}}\right)}{\sqrt{b\,x^2+a}}$	35
elliptic	$-\frac{\sqrt{-(x^2-1)(b\,x^2+a)}\,\sqrt{1+\frac{b\,x^2}{a}}\,F\left(x,\sqrt{-1-\frac{-a+b}{a}}\right)}{\sqrt{b\,x^2+a}\,\sqrt{-b\,x^4-a\,x^2+b\,x^2+a}}$	77

```
[In] int((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/(b*x^2+a)^(1/2)*((b*x^2+a)/a)^(1/2)*EllipticF(x,(-b/a)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = -\frac{F\left(\arcsin\left(x\right)\mid -\frac{b}{a}\right)}{\sqrt{a}}$$

```
[In] integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] -elliptic_f(arcsin(x), -b/a)/sqrt(a)
```

Sympy [A] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.53

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = \begin{cases} -\frac{F\left(\operatorname{asin}\left(x\right)\middle| -\frac{b}{a}\right)}{\sqrt{a}} & \text{for}\: x > -1 \wedge x < 1 \end{cases}$$

```
[In] integrate((-x**2+1)**(1/2)/(x**2-1)/(b*x**2+a)**(1/2),x)
[Out] Piecewise((-elliptic_f(asin(x), -b/a)/sqrt(a), (x > -1) & (x < 1)))
```

Maxima [F]

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{bx^2+a}(x^2-1)}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{-x^2+1}}{\sqrt{bx^2+a}(x^2-1)}\,dx$$

```
[In] integrate((-x^2+1)^(1/2)/(x^2-1)/(b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(-x^2 + 1)/(sqrt(b*x^2 + a)*(x^2 - 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x^2}}{(-1+x^2)\sqrt{a+bx^2}}\,dx = -\int \frac{1}{\sqrt{1-x^2}\,\sqrt{b\,x^2+a}}\,dx$$

```
[In] int(-1/((1 - x^2)^(1/2)*(a + b*x^2)^(1/2)),x)
[Out] -int(1/((1 - x^2)^(1/2)*(a + b*x^2)^(1/2)), x)
```

3.98 $\int \frac{a+bx^2}{\sqrt{c+dx^2}(e+fx^2)^2} dx$

Optimal result

Integrand size = 28, antiderivative size = 113

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\,(e+fx^2)}-\frac{(bce-2ade+acf)\text{arctanh}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{e}\sqrt{c+dx^2}}\right)}{2e^{3/2}(de-cf)^{3/2}}$$

```
[Out] -1/2*(a*c*f-2*a*d*e+b*c*e)*arctanh(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1/
2))/e^(3/2)/(-c*f+d*e)^(3/2)+1/2*(-a*f+b*e)*x*(d*x^2+c)^(1/2)/e/(-c*f+d*e)/
(f*x^2+e)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {541, 12, 385, 214}

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\frac{x\sqrt{c+dx^2}(be-af)}{2e\,(e+fx^2)\,(de-cf)}-\frac{(acf-2ade+bce)\text{arctanh}\left(\frac{x\sqrt{de-cf}}{\sqrt{e}\sqrt{c+dx^2}}\right)}{2e^{3/2}(de-cf)^{3/2}}$$

```
[In] Int[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
```

```
[Out] ((b*e - a*f)*x*Sqrt[c + d*x^2])/(2*e*(d*e - c*f)*(e + f*x^2)) - ((b*c*e - 2
*a*d*e + a*c*f)*ArcTanh[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])])/(2*
e^(3/2)*(d*e - c*f)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\,(e+fx^2)}+\frac{\int \frac{-bce+2ade-acf}{\sqrt{c+dx^2}(e+fx^2)}\,dx}{2e(de-cf)}\\
&= \frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\,(e+fx^2)}-\frac{(bce-2ade+acf)\int \frac{1}{\sqrt{c+dx^2}(e+fx^2)}\,dx}{2e(de-cf)}\\
&= \frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\,(e+fx^2)}-\frac{(bce-2ade+acf)\text{Subst}\left(\int \frac{1}{e-(de-cf)x^2}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{2e(de-cf)}\\
&= \frac{(be-af)x\sqrt{c+dx^2}}{2e(de-cf)\,(e+fx^2)}-\frac{(bce-2ade+acf)\tanh^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)}{2e^{3/2}(de-cf)^{3/2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.16

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\frac{\frac{\sqrt{e}(be-af)x\sqrt{c+dx^2}}{(de-cf)(e+fx^2)}-\frac{(bce-2ade+acf)\arctan\left(\frac{-fx\sqrt{c+dx^2}+\sqrt{d}\left(e+fx^2\right)}{\sqrt{e}\sqrt{-de+cf}}\right)}{(-de+cf)^{3/2}}}{2e^{3/2}}$$

```
[In] Integrate[(a + b*x^2)/(Sqrt[c + d*x^2]*(e + f*x^2)^2),x]
[Out] ((Sqrt[e]*(b*e - a*f)*x*Sqrt[c + d*x^2])/((d*e - c*f)*(e + f*x^2)) - ((b*c*
e - 2*a*d*e + a*c*f)*ArcTan[(-(f*x*Sqrt[c + d*x^2]) + Sqrt[d]*(e + f*x^2))/
(Sqrt[e]*Sqrt[-(d*e) + c*f])])/(-(d*e) + c*f)^(3/2))/(2*e^(3/2))
```

Maple [A] (verified)

Time = 3.48 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{\frac{(af-be)\sqrt{dx^2+c}\,x}{fx^2+e}-\frac{(acf-2ade+bce)\arctan\left(\frac{e\sqrt{dx^2+c}}{x\sqrt{(cf-de)e}}\right)}{\sqrt{(cf-de)e}}}{2(cf-de)e}$
default	$\frac{(-af+be)\left(-\frac{f\sqrt{d\left(x+\frac{\sqrt{-ef}}{f}\right)^2-\frac{2d\sqrt{-ef}\left(x+\frac{\sqrt{-ef}}{f}\right)}{f}+\frac{cf-de}{f}}}{(cf-de)\left(x+\frac{\sqrt{-ef}}{f}\right)}-\frac{d\sqrt{-ef}\ln\left(\frac{\frac{2cf-2de}{f}-\frac{2d\sqrt{-ef}\left(x+\frac{\sqrt{-ef}}{f}\right)}{f}+2\sqrt{\frac{cf-de}{f}}\sqrt{d\left(x+\cdots\right.}}{x+\frac{\sqrt{-ef}}{f}}\cdots\right)}{(cf-de)\sqrt{\frac{cf-de}{f}}}\cdots\right)}{4ef^2}$

```
[In] int((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(c*f-d*e)/e*((a*f-b*e)*(d*x^2+c)^(1/2)*x/(f*x^2+e)-(a*c*f-2*a*d*e+b*c*e
)/((c*f-d*e)*e)^(1/2)*arctan(e*(d*x^2+c)^(1/2)/x/((c*f-d*e)*e)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. 2(97) = 194.

Time = 1.11 (sec) , antiderivative size = 513, normalized size of antiderivative = 4.54

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$
$$=\left[\frac{4\,(bde^3+acef^2-(bc+ad)e^2f)\sqrt{dx^2+c}x-(acef+(bc-2\,ad)e^2+(acf^2+(bc-2\,ad)ef)x^2)\sqrt{de^2-\cdots}}{8\,(d^2e^5-2\,cde^4f+c^2e^3f^2+(d^2e^4f-2\,c\cdots}\right.$$

```
[In] integrate((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*(b*d*e^3 + a*c*e*f^2 - (b*c + a*d)*e^2*f)*sqrt(d*x^2 + c)*x - (a*c*
e*f + (b*c - 2*a*d)*e^2 + (a*c*f^2 + (b*c - 2*a*d)*e*f)*x^2)*sqrt(d*e^2 - c
*e*f)*log(((8*d^2*e^2 - 8*c*d*e*f + c^2*f^2)*x^4 + c^2*e^2 + 2*(4*c*d*e^2 -
 3*c^2*e*f)*x^2 + 4*((2*d*e - c*f)*x^3 + c*e*x)*sqrt(d*e^2 - c*e*f)*sqrt(d*
x^2 + c))/(f^2*x^4 + 2*e*f*x^2 + e^2)))/(d^2*e^5 - 2*c*d*e^4*f + c^2*e^3*f^
2 + (d^2*e^4*f - 2*c*d*e^3*f^2 + c^2*e^2*f^3)*x^2), 1/4*(2*(b*d*e^3 + a*c*e
*f^2 - (b*c + a*d)*e^2*f)*sqrt(d*x^2 + c)*x + (a*c*e*f + (b*c - 2*a*d)*e^2
+ (a*c*f^2 + (b*c - 2*a*d)*e*f)*x^2)*sqrt(-d*e^2 + c*e*f)*arctan(1/2*sqrt(-
d*e^2 + c*e*f)*((2*d*e - c*f)*x^2 + c*e)*sqrt(d*x^2 + c)/((d^2*e^2 - c*d*e*
f)*x^3 + (c*d*e^2 - c^2*e*f)*x)))/(d^2*e^5 - 2*c*d*e^4*f + c^2*e^3*f^2 + (d
^2*e^4*f - 2*c*d*e^3*f^2 + c^2*e^2*f^3)*x^2)]
```

Sympy [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx$$

[In] integrate((b*x**2+a)/(f*x**2+e)**2/(d*x**2+c)**(1/2),x)

[Out] Integral((a + b*x**2)/(sqrt(c + d*x**2)*(e + f*x**2)**2), x)

Maxima [F]

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\int \frac{bx^2+a}{\sqrt{dx^2+c}(fx^2+e)^2}\,dx$$

[In] integrate((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate((b*x^2 + a)/(sqrt(d*x^2 + c)*(f*x^2 + e)^2), x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(97) = 194$.

Time = 0.94 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.91

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx=\frac{\left(bc\sqrt{d}e-2\,ad^{\frac{3}{2}}e+ac\sqrt{d}f\right)\arctan\left(\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2f+2\,de-cf}{2\,\sqrt{-d^2e^2+cdef}}\right)}{2\,\sqrt{-d^2e^2+cdef}(de^2-cef)}+\frac{2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bd^{\frac{3}{2}}e^2-\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}ef-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2ad^{\frac{3}{2}}ef+\left(\sqrt{d}x-\sqrt{\right.}}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^4f+4\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2de-2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2cf+\right.}$$

[In] integrate((b*x^2+a)/(f*x^2+e)^2/(d*x^2+c)^(1/2),x, algorithm="giac")

[Out] 1/2*(b*c*sqrt(d)*e - 2*a*d^(3/2)*e + a*c*sqrt(d)*f)*arctan(1/2*((sqrt(d)*x - sqrt(d*x^2 + c))^2*f + 2*d*e - c*f)/sqrt(-d^2*e^2 + c*d*e*f))/(sqrt(-d^2*e^2 + c*d*e*f)*(d*e^2 - c*e*f)) + (2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*b*d^(3/2)*e^2 - (sqrt(d)*x - sqrt(d*x^2 + c))^2*b*c*sqrt(d)*e*f - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*a*d^(3/2)*e*f + (sqrt(d)*x - sqrt(d*x^2 + c))^2*a*c*sqrt(d)*f^2 + b*c^2*sqrt(d)*e*f - a*c^2*sqrt(d)*f^2)/(((sqrt(d)*x - sqrt(d*x^2 + c))^4*f + 4*(sqrt(d)*x - sqrt(d*x^2 + c))^2*d*e - 2*(sqrt(d)*x - sqrt(d*x^2 + c))^2*c*f + c^2*f)*(d*e^2*f - c*e*f^2))

Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx^2}{\sqrt{c+dx^2}\,(e+fx^2)^2}\,dx = \int \frac{b\,x^2+a}{\sqrt{d\,x^2+c}\,(f\,x^2+e)^2}\,dx$$

```
[In] int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^2),x)
[Out] int((a + b*x^2)/((c + d*x^2)^(1/2)*(e + f*x^2)^2), x)
```

3.99 $\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 359

$$\begin{aligned}
&\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx \\
&= \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)} + \frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}\,E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2ab\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}} \\
&\quad - \frac{\sqrt{c}\sqrt{d}(be+af)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2ab^2\sqrt{c-dx^2}\sqrt{e+fx^2}} \\
&\quad + \frac{\sqrt{c}(b^2ce+a^2df)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(-\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}}
\end{aligned}$$

```
[Out] 1/2*x*(-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(b*x^2+a)+1/2*EllipticE(x*d^(1/2)/
c^(1/2),(-c*f/d/e)^(1/2))*c^(1/2)*d^(1/2)*(1-d*x^2/c)^(1/2)*(f*x^2+e)^(1/2)
/a/b/(-d*x^2+c)^(1/2)/(1+f*x^2/e)^(1/2)+1/2*(a^2*d*f+b^2*c*e)*EllipticPi(x*
d^(1/2)/c^(1/2),-b*c/a/d,(-c*f/d/e)^(1/2))*c^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x
^2/e)^(1/2)/a^2/b^2/d^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)-1/2*(a*f+b*e)*
EllipticF(x*d^(1/2)/c^(1/2),(-c*f/d/e)^(1/2))*c^(1/2)*d^(1/2)*(1-d*x^2/c)^(
1/2)*(1+f*x^2/e)^(1/2)/a/b^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {562, 552, 551, 538, 438, 437, 435, 432, 430}

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$
$$=\frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(a^2df+b^2ce)\,\text{EllipticPi}\left(-\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}}$$
$$-\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(af+be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2ab^2\sqrt{c-dx^2}\sqrt{e+fx^2}}$$
$$+\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2ab\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}}+\frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\,(a+bx^2)}$$

```
[In] Int[(Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] (x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(a + b*x^2)) + (Sqrt[c]*Sqrt[d]*Sq
rt[1 - (d*x^2)/c]*Sqrt[e + f*x^2]*EllipticE[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -(
(c*f)/(d*e))])/(2*a*b*Sqrt[c - d*x^2]*Sqrt[1 + (f*x^2)/e]) - (Sqrt[c]*Sqrt[
d]*(b*e + a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[ArcSin[(Sq
rt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a*b^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2
]) + (Sqrt[c]*(b^2*c*e + a^2*d*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*
a^2*b^2*Sqrt[d]*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] && !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 562

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(a + b*x^
2))), x] + (Dist[(b^2*c*e - a^2*d*f)/(2*a*b^2), Int[1/((a + b*x^2)*Sqrt[c +
 d*x^2]*Sqrt[e + f*x^2]), x], x] + Dist[d*(f/(2*a*b^2)), Int[(a - b*x^2)/(S
qrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\text{integral} = \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} - \frac{(df)\int \frac{a-bx^2}{\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx}{2ab^2} + \frac{1}{2}\left(\frac{ce}{a}+\frac{adf}{b^2}\right)\int \frac{1}{\left(a+bx^2\right)\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$

$$= \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{d\int \frac{\sqrt{e+fx^2}}{\sqrt{c-dx^2}}\,dx}{2ab} - \frac{(d(be+af))\int \frac{1}{\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx}{2ab^2} + \frac{\left(\left(\frac{ce}{a}+\frac{adf}{b^2}\right)\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{\left(a+bx^2\right)\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}}\,dx}{2\sqrt{c-dx^2}}$$

$$= \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{\left(d\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{e+fx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{2ab\sqrt{c-dx^2}} - \frac{\left(d(be+af)\sqrt{1+\frac{fx^2}{e}}\right)\int \frac{1}{\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}}\,dx}{2ab^2\sqrt{e+fx^2}} + \frac{\left(\left(\frac{ce}{a}+\frac{adf}{b^2}\right)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\right)\int \frac{1}{\left(a+bx^2\right)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}}\,dx}{2\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

$$= \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{\sqrt{c}\left(\frac{ce}{a}+\frac{adf}{b^2}\right)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(-\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}} + \frac{\left(d\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}\right)\int \frac{\sqrt{1+\frac{fx^2}{e}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{2ab\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}} - \frac{\left(d(be+af)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\right)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}}\,dx}{2ab^2\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

$$= \frac{x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)} + \frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2ab\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}}$$
$$- \frac{\sqrt{c}\sqrt{d}(be+af)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2ab^2\sqrt{c-dx^2}\sqrt{e+fx^2}}$$
$$+ \frac{\sqrt{c}\left(\frac{ce}{a}+\frac{adf}{b^2}\right)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(-\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.32 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$
$$= \frac{\frac{cex}{a+bx^2} - \frac{dex^3}{a+bx^2} + \frac{cfx^3}{a+bx^2} - \frac{dfx^5}{a+bx^2} + \frac{ic\sqrt{-\frac{d}{c}}e\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{cf}{de}\right)}{b} - \frac{ic\sqrt{-\frac{d}{c}}(be+af)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\mathrm{E}}{}}{}$$

```
[In] Integrate[(Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((c*e*x)/(a + b*x^2) - (d*e*x^3)/(a + b*x^2) + (c*f*x^3)/(a + b*x^2) - (d*f
*x^5)/(a + b*x^2) + (I*c*Sqrt[-(d/c)]*e*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2
)/e]*EllipticE[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))])/b - (I*c*Sqrt[-(
d/c)]*(b*e + a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSi
nh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))])/b^2 + (I*d*e*Sqrt[1 - (d*x^2)/c]*Sqrt[
1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f
)/(d*e))])/(a*(-(d/c))^(3/2)) + (I*a*c*Sqrt[-(d/c)]*f*Sqrt[1 - (d*x^2)/c]*S
qrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -(
(c*f)/(d*e))])/b^2)/(2*a*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Maple [A] (verified)

Time = 3.40 (sec) , antiderivative size = 574, normalized size of antiderivative = 1.60

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)(f\,x^2+e)}\left(\frac{x\sqrt{-df\,x^4+cf\,x^2-de\,x^2+ce}}{2a\left(b\,x^2+a\right)}-\frac{df\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{cf-de}{ed}}\right)}{2b^2\sqrt{\frac{d}{c}}\,\sqrt{-df\,x^4+cf\,x^2-de\,x^2+ce}}-\frac{de\sqrt{1-\frac{d\,x^2}{c}}\,\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1}\right.}{2ab\sqrt{\frac{d}{c}}\,\sqrt{-df\,x^4+cf\,x^2-de\,x^2}}\right.}{\sqrt{-}}$
default	$-\frac{\sqrt{-d\,x^2+c}\,\sqrt{f\,x^2+e}\left(\sqrt{\frac{d}{c}}\,a\,b^2df\,x^5+\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{cf}{de}}\right)a^2bdf\,x^2+\sqrt{\frac{-d\,x^2+c}{c}}\,\sqrt{\frac{f\,x^2+e}{e}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-\frac{cf}{de}}\right)a\,b\right.}{}$

```
[In] int((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((-d*x^2+c)*(f*x^2+e))^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/2*x/a*(-d*
f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)/(b*x^2+a)-1/2*d*f/b^2/(d/c)^(1/2)*(1-d*x^2
/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticF(
x*(d/c)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))-1/2*d/a/b*e/(d/c)^(1/2)*(1-d*x^2/c)
^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticF(x*(
d/c)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))+1/2*d/a/b*e/(d/c)^(1/2)*(1-d*x^2/c)^(1
/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticE(x*(d/c
)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))+1/2/b^2/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+
f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticPi(x*(d/c)^(1/2
),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*d*f+1/2/a^2/(d/c)^(1/2)*(1-d*x^2/c)^(1
/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticPi(x*(d/
c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{\left(a+bx^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas
")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

```
[In] integrate((-d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**2,x)
[Out] Integral(sqrt(c - d*x**2)*sqrt(e + f*x**2)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^2}\,dx$$

```
[In] integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima
")
[Out] integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{-dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^2}\,dx$$

```
[In] integrate((-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c-dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{c-d\,x^2}\,\sqrt{f\,x^2+e}}{(b\,x^2+a)^2}\,dx$$

```
[In] int(((c - d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2,x)
[Out] int(((c - d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2, x)
```

3.100 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 381

$$\begin{aligned}
&\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx \\
&= -\frac{fx\sqrt{c+dx^2}}{2ab\sqrt{e+fx^2}} + \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)} + \frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2ab\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{2b^2c\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} \\
&+ \frac{\sqrt{-c}(b^2ce-a^2df)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}
\end{aligned}$$

```
[Out] -1/2*f*x*(d*x^2+c)^(1/2)/a/b/(f*x^2+e)^(1/2)+1/2*(1/(1+f*x^2/e))^(1/2)*(1+f
*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/
2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/a/b/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*
x^2+e)^(1/2)+1/2*d*(1/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1
/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^
(1/2)/b^2/c/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/2*x*(d*x^2+c)
^(1/2)*(f*x^2+e)^(1/2)/a/(b*x^2+a)+1/2*(-a^2*d*f+b^2*c*e)*EllipticPi(x*d^(1
/2)/(-c)^(1/2),b*c/a/d,(c*f/d/e)^(1/2))*(-c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x
^2/e)^(1/2)/a^2/b^2/d^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {562, 552, 551, 545, 429, 506, 422}

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(b^2ce-a^2df)\,\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{2a^2b^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2ab\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} + \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)} - \frac{fx\sqrt{c+dx^2}}{2ab\sqrt{e+fx^2}} + \frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] -1/2*(f*x*Sqrt[c + d*x^2])/(a*b*Sqrt[e + f*x^2]) + (x*Sqrt[c + d*x^2]*Sqrt[
e + f*x^2])/(2*a*(a + b*x^2)) + (Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[
ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*a*b*Sqrt[(e*(c + d*x^2))/
(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (d*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*Elli
pticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*b^2*c*Sqrt[(e*(c +
d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[-c]*(b^2*c*e - a^2*d*f)*S
qrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), ArcSin[(Sqrt
[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/(2*a^2*b^2*Sqrt[d]*Sqrt[c + d*x^2]*Sqrt[e +
 f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 562

```
Int[(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x_)^2])/((a_) + (b_.)*(x_
)^2)^2, x_Symbol] :> Simp[x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(a + b*x^
2))), x] + (Dist[(b^2*c*e - a^2*d*f)/(2*a*b^2), Int[1/((a + b*x^2)*Sqrt[c +
 d*x^2]*Sqrt[e + f*x^2]), x], x] + Dist[d*(f/(2*a*b^2)), Int[(a - b*x^2)/(S
qrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\text{integral} = \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\left(a+bx^2\right)} + \frac{(df)\int \frac{a-bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2ab^2} + \frac{1}{2}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\int \frac{1}{\left(a+bx^2\right)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$= \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\,(a+bx^2)} + \frac{(df)\int \frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2b^2} - \frac{(df)\int \frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2ab} + \frac{\left(\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\sqrt{1+\frac{dx^2}{c}}\right)\int \frac{1}{(a+bx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{e+fx^2}}\,dx}{2\sqrt{c+dx^2}}$$

$$= -\frac{fx\sqrt{c+dx^2}}{2ab\sqrt{e+fx^2}} + \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\,(a+bx^2)} + \frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2b^2c\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(ef)\int \frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{2ab} + \frac{\left(\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\right)\int \frac{1}{(a+bx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}}\,dx}{2\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$= -\frac{fx\sqrt{c+dx^2}}{2ab\sqrt{e+fx^2}} + \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a\,(a+bx^2)} + \frac{\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2ab\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2b^2c\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{\sqrt{-c}\left(\frac{ce}{a}-\frac{adf}{b^2}\right)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{2a\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.05 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \frac{\frac{cex}{a+bx^2} + \frac{dex^3}{a+bx^2} + \frac{cfx^3}{a+bx^2} + \frac{dfx^5}{a+bx^2} + \frac{ic\sqrt{\frac{d}{c}}e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\text{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)}{b} - \frac{ic\sqrt{\frac{d}{c}}(be+af)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticF}}{b^2}}{2a\sqrt{}}$$

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(a + b*x^2)^2,x]
```

```
[Out] ((c*e*x)/(a + b*x^2) + (d*e*x^3)/(a + b*x^2) + (c*f*x^3)/(a + b*x^2) + (d*f
*x^5)/(a + b*x^2) + (I*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e
]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/b - (I*c*Sqrt[d/c]*(b*e +
 a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcSinh[Sqrt[d/c]
*x], (c*f)/(d*e)])/b^2 - (I*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Ell
ipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*Sqrt[d/c]) +
```

```
(I*a*c*Sqrt[d/c]*f*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)
/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/b^2)/(2*a*Sqrt[c + d*x^2]*Sqr
t[e + f*x^2])
```

Maple [A] (verified)

Time = 3.35 (sec) , antiderivative size = 559, normalized size of antiderivative = 1.47

method	result
elliptic	$\frac{\sqrt{(d\,x^{2}+c)(f\,x^{2}+e)}\left(\frac{x\sqrt{df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce}}{2a\left(b\,x^{2}+a\right)}+\frac{df\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{f\,x^{2}}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{2b^{2}\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+cf\,x^{2}+de\,x^{2}+ce}}+\frac{de\sqrt{1+\frac{d\,x^{2}}{c}}\,\sqrt{1+\frac{f\,x^{2}}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\cdots}\right.}{2ab\sqrt{-\frac{d}{c}}\,\sqrt{df\,x^{4}+cf\,x^{2}+de\,x^{2}+\cdots}}\right.}{\sqrt{\cdots}}$
default	$\frac{\sqrt{d\,x^{2}+c}\,\sqrt{f\,x^{2}+e}\left(\sqrt{-\frac{d}{c}}\,a\,b^{2}df\,x^{5}+\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{f\,x^{2}+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a^{2}bdf\,x^{2}+\sqrt{\frac{d\,x^{2}+c}{c}}\,\sqrt{\frac{f\,x^{2}+e}{e}}\,F\left(x\sqrt{-\frac{d}{c}},\sqrt{\frac{cf}{de}}\right)a\,b^{2}de\,x\cdots\right.}{\cdots}$

```
[In] int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/2*x/a*(d*f*x
^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(b*x^2+a)+1/2*d*f/b^2/(-d/c)^(1/2)*(1+d*x^2/c
)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(
-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1/2*d/a/b*e/(-d/c)^(1/2)*(1+d*x^2/c)^
(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d
/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-1/2*d/a/b*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1
/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c
)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-1/2/b^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1
+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/
2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*d*f+1/2/a^2/(-d/c)^(1/2)*(1+d*x^2/c)^
(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-
d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\left(a+bx^2\right)^2}\,dx=\text{Timed out}$$

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="fricas"
)
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**2,x)
[Out] Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^2}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="maxima"
)
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{(bx^2+a)^2}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{(a+bx^2)^2}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{(b\,x^2+a)^2}\,dx$$

```
[In] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2,x)
[Out] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^2, x)
```

3.101 $\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 33, antiderivative size = 426

$$\begin{aligned}
&\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx \\
&= \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)(a+bx^2)} + \frac{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a(bc+ad)(be-af)\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}} \\
&\quad - \frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a(bc+ad)\sqrt{c-dx^2}\sqrt{e+fx^2}} \\
&\quad + \frac{\sqrt{c}(b^2ce-3a^2df+ab(2de-2cf))\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\operatorname{EllipticPi}\left(-\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a^2\sqrt{d}(bc+ad)(be-af)\sqrt{c-dx^2}\sqrt{e+fx^2}}
\end{aligned}$$

```
[Out] 1/2*b^2*x*(-d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/a/(a*d+b*c)/(-a*f+b*e)/(b*x^2+a)
+1/2*b*EllipticE(x*d^(1/2)/c^(1/2),(-c*f/d/e)^(1/2))*c^(1/2)*d^(1/2)*(1-d*x
^2/c)^(1/2)*(f*x^2+e)^(1/2)/a/(a*d+b*c)/(-a*f+b*e)/(-d*x^2+c)^(1/2)/(1+f*x^
2/e)^(1/2)+1/2*(b^2*c*e-3*a^2*d*f+a*b*(-2*c*f+2*d*e))*EllipticPi(x*d^(1/2)/
c^(1/2),-b*c/a/d,(-c*f/d/e)^(1/2))*c^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1
/2)/a^2/(a*d+b*c)/(-a*f+b*e)/d^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)-1/2*E
llipticF(x*d^(1/2)/c^(1/2),(-c*f/d/e)^(1/2))*c^(1/2)*d^(1/2)*(1-d*x^2/c)^(1
/2)*(1+f*x^2/e)^(1/2)/a/(a*d+b*c)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {563, 552, 551, 538, 438, 437, 435, 432, 430}

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$
$$= \frac{\sqrt{c}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}(-3a^2df+ab(2de-2cf)+b^2ce)\,\text{EllipticPi}\left(-\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a^2\sqrt{d}\sqrt{c-dx^2}\sqrt{e+fx^2}(ad+bc)(be-af)}$$
$$-\frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{\frac{fx^2}{e}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),-\frac{cf}{de}\right)}{2a\sqrt{c-dx^2}\sqrt{e+fx^2}(ad+bc)}$$
$$+\frac{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\arcsin\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a\sqrt{c-dx^2}\sqrt{\frac{fx^2}{e}+1}(ad+bc)(be-af)}+\frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)(ad+bc)(be-af)}$$

```
[In] Int[1/((a + b*x^2)^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (b^2*x*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c + a*d)*(b*e - a*f)*(a + b
*x^2)) + (b*Sqrt[c]*Sqrt[d]*Sqrt[1 - (d*x^2)/c]*Sqrt[e + f*x^2]*EllipticE[A
rcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a*(b*c + a*d)*(b*e - a*f)*S
qrt[c - d*x^2]*Sqrt[1 + (f*x^2)/e]) - (Sqrt[c]*Sqrt[d]*Sqrt[1 - (d*x^2)/c]*
Sqrt[1 + (f*x^2)/e]*EllipticF[ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])
/(2*a*(b*c + a*d)*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]) + (Sqrt[c]*(b^2*c*e - 3*
a^2*d*f + a*b*(2*d*e - 2*c*f))*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*Elli
pticPi[-((b*c)/(a*d)), ArcSin[(Sqrt[d]*x)/Sqrt[c]], -((c*f)/(d*e))])/(2*a^2
*Sqrt[d]*(b*c + a*d)*(b*e - a*f)*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 432

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
/c)*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
```

```
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 538

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-b/a, -d/c]))))))
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 563

```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Dist[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f)), Int[1/((a + b*x^2)*Sqrt[c + d*
x^2]*Sqrt[e + f*x^2]), x], x] - Dist[d*(f/(2*a*(b*c - a*d)*(b*e - a*f))), I
```

```
nt[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c
, d, e, f}, x]
```

Rubi steps

$$\text{integral} = \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)\,(a+bx^2)} + \frac{(df)\int \frac{a+bx^2}{\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx}{2a(bc+ad)(be-af)} + \frac{(b^2ce-3a^2df-2ab(-de+cf))\int \frac{1}{(a+bx^2)\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx}{2a(bc+ad)(be-af)}$$

$$= \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)\,(a+bx^2)} - \frac{d\int \frac{1}{\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx}{2a(bc+ad)} + \frac{(bd)\int \frac{\sqrt{e+fx^2}}{\sqrt{c-dx^2}}\,dx}{2a(bc+ad)(be-af)} + \frac{\left((b^2ce-3a^2df-2ab(-de+cf))\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{1}{(a+bx^2)\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}}\,dx}{2a(bc+ad)(be-af)\sqrt{c-dx^2}}$$

$$= \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)\,(a+bx^2)} + \frac{\left(bd\sqrt{1-\frac{dx^2}{c}}\right)\int \frac{\sqrt{e+fx^2}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{2a(bc+ad)(be-af)\sqrt{c-dx^2}} - \frac{\left(d\sqrt{1+\frac{fx^2}{e}}\right)\int \frac{1}{\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}}\,dx}{2a(bc+ad)\sqrt{e+fx^2}} + \frac{\left((b^2ce-3a^2df-2ab(-de+cf))\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\right)\int \frac{1}{(a+bx^2)\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}}\,dx}{2a(bc+ad)(be-af)\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

$$= \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)\,(a+bx^2)} + \frac{\sqrt{c}(b^2ce-3a^2df+ab(2de-2cf))\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\Pi\left(-\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a^2\sqrt{d}(bc+ad)(be-af)\sqrt{c-dx^2}\sqrt{e+fx^2}} + \frac{\left(bd\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}\right)\int \frac{\sqrt{1+\frac{fx^2}{e}}}{\sqrt{1-\frac{dx^2}{c}}}\,dx}{2a(bc+ad)(be-af)\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}} - \frac{\left(d\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\right)\int \frac{1}{\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}}\,dx}{2a(bc+ad)\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

$$= \frac{b^2x\sqrt{c-dx^2}\sqrt{e+fx^2}}{2a(bc+ad)(be-af)\,(a+bx^2)} + \frac{b\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{e+fx^2}E\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a(bc+ad)(be-af)\sqrt{c-dx^2}\sqrt{1+\frac{fx^2}{e}}}$$
$$- \frac{\sqrt{c}\sqrt{d}\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a(bc+ad)\sqrt{c-dx^2}\sqrt{e+fx^2}}$$
$$+ \frac{\sqrt{c}(b^2ce-3a^2df+ab(2de-2cf))\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(-\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|-\frac{cf}{de}\right)}{2a^2\sqrt{d}(bc+ad)(be-af)\sqrt{c-dx^2}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.21 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.45

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$
$$= \frac{-\frac{b^2cex}{a+bx^2}+\frac{b^2dex^3}{a+bx^2}-\frac{b^2cfx^3}{a+bx^2}+\frac{b^2dfx^5}{a+bx^2}-ibc\sqrt{-\frac{d}{c}}e\sqrt{1-\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\operatorname{arcsinh}\left(\sqrt{-\frac{d}{c}}x\right)|-\frac{cf}{de}\right)+ic\sqrt{-\frac{d}{c}}(b}{}$$

```
[In] Integrate[1/((a + b*x^2)^2*Sqrt[c - d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (-((b^2*c*e*x)/(a + b*x^2)) + (b^2*d*e*x^3)/(a + b*x^2) - (b^2*c*f*x^3)/(a
+ b*x^2) + (b^2*d*f*x^5)/(a + b*x^2) - I*b*c*Sqrt[-(d/c)]*e*Sqrt[1 - (d*x^2
)/c]*Sqrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e)
)] + I*c*Sqrt[-(d/c)]*(b*e - a*f)*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticF[I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*e))] + (I*b^2*c*e*Sqrt[1 - (
d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d
/c)]*x], -((c*f)/(d*e))])/(a*Sqrt[-(d/c)]) - (2*I)*b*c*Sqrt[-(d/c)]*e*Sqrt[
1 - (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqr
t[-(d/c)]*x], -((c*f)/(d*e))] + ((2*I)*b*d*f*Sqrt[1 - (d*x^2)/c]*Sqrt[1 + (
f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/(d*
e))])/(-(d/c))^(3/2) + (3*I)*a*c*Sqrt[-(d/c)]*f*Sqrt[1 - (d*x^2)/c]*Sqrt[1
+ (f*x^2)/e]*EllipticPi[-((b*c)/(a*d)), I*ArcSinh[Sqrt[-(d/c)]*x], -((c*f)/
(d*e))])/(2*a*(b*c + a*d)*(-(b*e) + a*f)*Sqrt[c - d*x^2]*Sqrt[e + f*x^2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 988 vs. $2(369) = 738$.

Time = 6.06 (sec) , antiderivative size = 989, normalized size of antiderivative = 2.32

method	result
elliptic	$\frac{\sqrt{(-d\,x^2+c)(f\,x^2+e)}\left(-\frac{b^2 x\sqrt{-df\,x^4+cf\,x^2-de\,x^2+ce}}{2\left(a^2df+acfb-abde-b^2ce\right)a\left(b\,x^2+a\right)}-\frac{df\sqrt{1-\frac{d\,x^2}{c}}\sqrt{1+\frac{f\,x^2}{e}}\,F\left(x\sqrt{\frac{d}{c}},\sqrt{-1-\frac{cf-de}{ed}}\right)}{2\left(a^2df+acfb-abde-b^2ce\right)\sqrt{\frac{d}{c}}\sqrt{-df\,x^4+cf\,x^2-de\,x^2+ce}}+\frac{bde\sqrt{1-\frac{d\,x^2}{c}}}{2\left(a^2df+acfb-ab\right.}\right.}{}$
default	Expression too large to display

```
[In] int(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] ((-d*x^2+c)*(f*x^2+e))^(1/2)/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(-1/2*b^2/(a^
2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/a*x*(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)/(b*
x^2+a)-1/2*d*f/(a^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/(d/c)^(1/2)*(1-d*x^2/c)^(1
/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticF(x*(d/c
)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))+1/2*b*d/(a^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)
/a*e/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*
x^2+c*e)^(1/2)*EllipticF(x*(d/c)^(1/2),(-1-(c*f-d*e)/e/d)^(1/2))-1/2*b*d/(a
^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/a*e/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/
e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticE(x*(d/c)^(1/2),(-1-(
c*f-d*e)/e/d)^(1/2))+3/2/(a^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/(d/c)^(1/2)*(1-d
*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*Ellipt
icPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*d*f+1/(a^2*d*f+a*b*c*
f-a*b*d*e-b^2*c*e)/a*b/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*
f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(
1/2)/(d/c)^(1/2))*c*f-1/(a^2*d*f+a*b*c*f-a*b*d*e-b^2*c*e)/a*b/(d/c)^(1/2)*(
1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*Ell
ipticPi(x*(d/c)^(1/2),-b*c/a/d,(-f/e)^(1/2)/(d/c)^(1/2))*d*e-1/2/(a^2*d*f+a
*b*c*f-a*b*d*e-b^2*c*e)/a^2*b^2/(d/c)^(1/2)*(1-d*x^2/c)^(1/2)*(1+f*x^2/e)^(
1/2)/(-d*f*x^4+c*f*x^2-d*e*x^2+c*e)^(1/2)*EllipticPi(x*(d/c)^(1/2),-b*c/a/d
,(-f/e)^(1/2)/(d/c)^(1/2))*c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fric
as")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**2/(-d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
[Out] Integral(1/((a + b*x**2)**2*sqrt(c - d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^2\sqrt{-dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxi
ma")
[Out] integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^2\sqrt{-dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^2/(-d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac
")
[Out] integrate(1/((b*x^2 + a)^2*sqrt(-d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c-dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^2\,\sqrt{c-d\,x^2}\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int(1/((a + b*x^2)^2*(c - d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^2*(c - d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.102 $\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 32, antiderivative size = 485

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = -\frac{bfx\sqrt{c+dx^2}}{2a(bc-ad)(be-af)\sqrt{e+fx^2}} + \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)} + \frac{b\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2a(bc-ad)(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{2c(bc-ad)(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{\sqrt{-c}(b^2ce+3a^2df-2ab(de+cf))\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\,\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{2a^2\sqrt{d}(bc-ad)(be-af)\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[Out] -1/2*b*f*x*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(f*x^2+e)^(1/2)+1/2*b*(1
/(1+f*x^2/e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticE(x*f^(1/2)/e^(1/2)/(1+f*x^2/
e)^(1/2),(1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/a/(-a*d+b*c)/(-
a*f+b*e)/(e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)-1/2*d*(1/(1+f*x^2/
e))^(1/2)*(1+f*x^2/e)^(1/2)*EllipticF(x*f^(1/2)/e^(1/2)/(1+f*x^2/e)^(1/2),(
1-d*e/c/f)^(1/2))*e^(1/2)*f^(1/2)*(d*x^2+c)^(1/2)/c/(-a*d+b*c)/(-a*f+b*e)/(
e*(d*x^2+c)/c/(f*x^2+e))^(1/2)/(f*x^2+e)^(1/2)+1/2*b^2*x*(d*x^2+c)^(1/2)*(f
*x^2+e)^(1/2)/a/(-a*d+b*c)/(-a*f+b*e)/(b*x^2+a)+1/2*(b^2*c*e+3*a^2*d*f-2*a*
b*(c*f+d*e))*EllipticPi(x*d^(1/2)/(-c)^(1/2),b*c/a/d,(c*f/d/e)^(1/2))*(-c)^
(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/a^2/(-a*d+b*c)/(-a*f+b*e)/d^(1/2)
/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {563, 552, 551, 545, 429, 506, 422}

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$
$$= \frac{\sqrt{-c}\sqrt{\frac{dx^2}{c}+1}\sqrt{\frac{fx^2}{e}+1}(3a^2df-2ab(cf+de)+b^2ce)\,\text{EllipticPi}\left(\frac{bc}{ad},\arcsin\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right),\frac{cf}{de}\right)}{2a^2\sqrt{d}\sqrt{c+dx^2}\sqrt{e+fx^2}(bc-ad)(be-af)}$$
$$-\frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right),1-\frac{de}{cf}\right)}{2c\sqrt{e+fx^2}(bc-ad)(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{b\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\arctan\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2a\sqrt{e+fx^2}(bc-ad)(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+\frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(a+bx^2)(bc-ad)(be-af)}-\frac{bfx\sqrt{c+dx^2}}{2a\sqrt{e+fx^2}(bc-ad)(be-af)}$$

```
[In] Int[1/((a + b*x^2)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] -1/2*(b*f*x*Sqrt[c + d*x^2])/(a*(b*c - a*d)*(b*e - a*f)*Sqrt[e + f*x^2]) +
(b^2*x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*a*(b*c - a*d)*(b*e - a*f)*(a + b
*x^2)) + (b*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*EllipticE[ArcTan[(Sqrt[f]*x)/Sq
rt[e]], 1 - (d*e)/(c*f)])/(2*a*(b*c - a*d)*(b*e - a*f)*Sqrt[(e*(c + d*x^2))
/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) - (d*Sqrt[e]*Sqrt[f]*Sqrt[c + d*x^2]*Ell
ipticF[ArcTan[(Sqrt[f]*x)/Sqrt[e]], 1 - (d*e)/(c*f)])/(2*c*(b*c - a*d)*(b*e
 - a*f)*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*Sqrt[e + f*x^2]) + (Sqrt[-c]*
(b^2*c*e + 3*a^2*d*f - 2*a*b*(d*e + c*f))*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x
^2)/e]*EllipticPi[(b*c)/(a*d), ArcSin[(Sqrt[d]*x)/Sqrt[-c]], (c*f)/(d*e)])/
(2*a^2*Sqrt[d]*(b*c - a*d)*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
```

```
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 545

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 552

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Dist[Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[1/((a +
b*x^2)*Sqrt[1 + (d/c)*x^2]*Sqrt[e + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e
, f}, x] &&  !GtQ[c, 0]
```

Rule 563

```
Int[1/(((a_) + (b_.)*(x_)^2)^2*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Simp[b^2*x*Sqrt[c + d*x^2]*(Sqrt[e + f*x^2]/(2*a*(b*
c - a*d)*(b*e - a*f)*(a + b*x^2))), x] + (Dist[(b^2*c*e + 3*a^2*d*f - 2*a*b
*(d*e + c*f))/(2*a*(b*c - a*d)*(b*e - a*f)), Int[1/((a + b*x^2)*Sqrt[c + d*
x^2]*Sqrt[e + f*x^2]), x], x] - Dist[d*(f/(2*a*(b*c - a*d)*(b*e - a*f))), I
nt[(a + b*x^2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x], x]) /; FreeQ[{a, b, c
, d, e, f}, x]
```

Rubi steps

$$\text{integral} = \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)} - \frac{(df)\int\frac{a+bx^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2a(bc-ad)(be-af)} + \frac{\left(b^2ce+3a^2df-2ab(de+cf)\right)\int\frac{1}{(a+bx^2)\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2a(bc-ad)(be-af)}$$

$$= \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)} - \frac{(df)\int\frac{1}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2(bc-ad)(be-af)} - \frac{(bdf)\int\frac{x^2}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2a(bc-ad)(be-af)} + \frac{\left(\left(b^2ce+3a^2df-2ab(de+cf)\right)\sqrt{1+\frac{dx^2}{c}}\right)\int\frac{1}{(a+bx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{e+fx^2}}\,dx}{2a(bc-ad)(be-af)\sqrt{c+dx^2}}$$

$$= -\frac{bfx\sqrt{c+dx^2}}{2a(bc-ad)(be-af)\sqrt{e+fx^2}} + \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)} - \frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2c(bc-ad)(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{(bef)\int\frac{\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx}{2a(bc-ad)(be-af)} + \frac{\left(\left(b^2ce+3a^2df-2ab(de+cf)\right)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\right)\int\frac{1}{(a+bx^2)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}}\,dx}{2a(bc-ad)(be-af)\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

$$= -\frac{bfx\sqrt{c+dx^2}}{2a(bc-ad)(be-af)\sqrt{e+fx^2}} + \frac{b^2x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2a(bc-ad)(be-af)\left(a+bx^2\right)} + \frac{b\sqrt{e}\sqrt{f}\sqrt{c+dx^2}E\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2a(bc-ad)(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} - \frac{d\sqrt{e}\sqrt{f}\sqrt{c+dx^2}F\left(\tan^{-1}\left(\frac{\sqrt{f}x}{\sqrt{e}}\right)|1-\frac{de}{cf}\right)}{2c(bc-ad)(be-af)\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}\sqrt{e+fx^2}} + \frac{\sqrt{-c}\left(b^2ce+3a^2df-2ab(de+cf)\right)\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}\Pi\left(\frac{bc}{ad};\sin^{-1}\left(\frac{\sqrt{d}x}{\sqrt{-c}}\right)|\frac{cf}{de}\right)}{2a^2\sqrt{d}(bc-ad)(be-af)\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.31 (sec) , antiderivative size = 587, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

$$= \frac{\frac{b^2cex}{a+bx^2}+\frac{b^2dex^3}{a+bx^2}+\frac{b^2cfx^3}{a+bx^2}+\frac{b^2dfx^5}{a+bx^2}+ibc\sqrt{\frac{d}{c}}e\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{d}{c}}x\right)|\frac{cf}{de}\right)-ic\sqrt{\frac{d}{c}}(be-af)\sqrt{1}}{}$$

[In] Integrate[1/((a + b*x^2)^2*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]

[Out] ((b^2*c*e*x)/(a + b*x^2) + (b^2*d*e*x^3)/(a + b*x^2) + (b^2*c*f*x^3)/(a + b
*x^2) + (b^2*d*f*x^5)/(a + b*x^2) + I*b*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]*S
qrt[1 + (f*x^2)/e]*EllipticE[I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] - I*c*Sqr
t[d/c]*(b*e - a*f)*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticF[I*ArcS
inh[Sqrt[d/c]*x], (c*f)/(d*e)] - (I*b^2*c*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f
*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)])/(a*S
qrt[d/c]) + (2*I)*b*c*Sqrt[d/c]*e*Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*E
llipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (c*f)/(d*e)] + ((2*I)*b*c*f*
Sqrt[1 + (d*x^2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[S
qrt[d/c]*x], (c*f)/(d*e)])/Sqrt[d/c] - (3*I)*a*c*Sqrt[d/c]*f*Sqrt[1 + (d*x^
2)/c]*Sqrt[1 + (f*x^2)/e]*EllipticPi[(b*c)/(a*d), I*ArcSinh[Sqrt[d/c]*x], (
c*f)/(d*e)])/(2*a*(-(b*c) + a*d)*(-(b*e) + a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*
x^2])

Maple [A] (verified)

Time = 4.89 (sec) , antiderivative size = 973, normalized size of antiderivative = 2.01

method	result
elliptic	$\frac{\sqrt{(dx^2+c)(fx^2+e)}\left(\frac{b^2x\sqrt{dfx^4+cfx^2+dex^2+ce}}{2(a^2df-acfb-abde+b^2ce)a(bx^2+a)}-\frac{df\sqrt{1+\frac{dx^2}{c}}\sqrt{1+\frac{fx^2}{e}}F\left(x\sqrt{-\frac{d}{c}},\sqrt{-1+\frac{cf+de}{ed}}\right)}{2(a^2df-acfb-abde+b^2ce)\sqrt{-\frac{d}{c}}\sqrt{dfx^4+cfx^2+dex^2+ce}}+\frac{bde\sqrt{1+\frac{dx^2}{c}}\sqrt{1}}{2(a^2df-acfb-abde+}\right.}{}$
default	Expression too large to display

[In] int(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x,method=_RETURNVERBOSE)

[Out] ((d*x^2+c)*(f*x^2+e))^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)*(1/2*b^2/(a^2*d
*f-a*b*c*f-a*b*d*e+b^2*c*e)/a*x*(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)/(b*x^2+
a)-1/2/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)*d*f/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)
*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticF(x*(-d/c)^(

```
1/2),(-1+(c*f+d*e)/e/d)^(1/2))+1/2*b*d/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/a*
e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2
+c*e)^(1/2)*EllipticF(x*(-d/c)^(1/2),(-1+(c*f+d*e)/e/d)^(1/2))-1/2*b*d/(a^2
*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/a*e/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e
)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticE(x*(-d/c)^(1/2),(-1+(c
*f+d*e)/e/d)^(1/2))+3/2/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/(-d/c)^(1/2)*(1+d
*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*Ellipti
cPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*d*f-1/(a^2*d*f-a*b*c*
f-a*b*d*e+b^2*c*e)/a*b/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*
f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(
1/2)/(-d/c)^(1/2))*c*f-1/(a^2*d*f-a*b*c*f-a*b*d*e+b^2*c*e)/a*b/(-d/c)^(1/2)
*(1+d*x^2/c)^(1/2)*(1+f*x^2/e)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*El
lipticPi(x*(-d/c)^(1/2),b*c/a/d,(-f/e)^(1/2)/(-d/c)^(1/2))*d*e+1/2/(a^2*d*f
-a*b*c*f-a*b*d*e+b^2*c*e)/a^2*b^2/(-d/c)^(1/2)*(1+d*x^2/c)^(1/2)*(1+f*x^2/e
)^(1/2)/(d*f*x^4+c*f*x^2+d*e*x^2+c*e)^(1/2)*EllipticPi(x*(-d/c)^(1/2),b*c/a
/d,(-f/e)^(1/2)/(-d/c)^(1/2))*c*e)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="frica
s")
[Out] Timed out
```

Sympy [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**2/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
[Out] Integral(1/((a + b*x**2)**2*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxim
a")
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(bx^2+a)^2\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^2/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="giac"
)
```

```
[Out] integrate(1/((b*x^2 + a)^2*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx^2)^2\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{(b\,x^2+a)^2\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)^2*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.103 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \text{Int}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}},x\right)$$

```
[Out] Unintegrable((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

```
[In] Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
[Out] Defer[Int][((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 18.21 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

```
[In] Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
[Out] Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/Sqrt[e + f*x^2], x]
```

Maple [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}dx$$

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
[Out] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\text{Timed out}$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
[Out] Timed out
```

Sympy [N/A]

Not integrable

Time = 12.86 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx=\int \frac{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Mupad [N/A]

Not integrable

Time = 5.90 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\left(b\,x^2+a\right)^{3/2}\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

```
[In] int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
[Out] int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

3.104 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 545

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$
$$= \frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}} - \frac{\sqrt{e}\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2f\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$+ \frac{b\sqrt{e}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{c\sqrt{e}(bde-bcf-adf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\operatorname{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

```
[Out] 1/2*d*x*(b*x^2+a)^(1/2)*(f*x^2+e)^(1/2)/f/(d*x^2+c)^(1/2)+1/2*b*(-c*f+d*e)*
EllipticF(x*(-a*f+b*e)^(1/2)/e^(1/2)/(b*x^2+a)^(1/2),((-a*d+b*c)*e/c/(-a*f+
b*e))^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)/d/f/(-
a*f+b*e)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)-1/2*c*(-a*d*
f-b*c*f+b*d*e)*EllipticPi(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-
c*f+d*e),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*e^(1/2)*(b*x^2+a)^(1/2)*(c*(f*
x^2+e)/e/(d*x^2+c))^(1/2)/a/d/f/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^
(1/2)/(f*x^2+e)^(1/2)-1/2*EllipticE(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1
/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*e^(1/2)*(-c*f+d*e)^(1/2)*(b*x^2+a)^
(1/2)*(c*(f*x^2+e)/e/(d*x^2+c))^(1/2)/f/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*
x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {569, 568, 435, 567, 551, 566, 430}

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$
$$= \frac{b\sqrt{e}\sqrt{c+dx^2}(de-cf)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$
$$-\frac{\sqrt{e}\sqrt{a+bx^2}\sqrt{de-cf}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2f\sqrt{e+fx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$-\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}(-adf-bcf+bde)\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$+\frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}}$$

```
[In] Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] (d*x*Sqrt[a + b*x^2]*Sqrt[e + f*x^2])/(2*f*Sqrt[c + d*x^2]) - (Sqrt[e]*Sqrt
[d*e - c*f]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticE
[ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*e)/(
a*(d*e - c*f)))])/(2*f*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[e + f*x^2
]) + (b*Sqrt[e]*(d*e - c*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*
x^2))]*EllipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b
*c - a*d)*e)/(c*(b*e - a*f))])/(2*d*f*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/
(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (c*Sqrt[e]*(b*d*e - b*c*f - a*d*f)*Sqrt
[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi[(d*e)/(d*e - c
*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*
e)/(a*(d*e - c*f)))])/(2*a*d*f*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[e + f*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 566

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x], x
, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 567

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2
/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 568

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 569

```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[d*x*Sqrt[a + b*x^2]*(Sqrt[e + f*x^2]/(2*f*Sqrt[
c + d*x^2])), x] + (-Dist[c*((d*e - c*f)/(2*f)), Int[Sqrt[a + b*x^2]/((c +
d*x^2)^(3/2)*Sqrt[e + f*x^2]), x], x] - Dist[(b*d*e - b*c*f - a*d*f)/(2*d*f
), Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]), x], x] + Dist[b*c
*((d*e - c*f)/(2*d*f)), Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x
^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[(d*e - c*f)/c]
```

Rubi steps

$$\text{integral} = \frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}} - \frac{(c(de-cf))\int \frac{\sqrt{a+bx^2}}{(c+dx^2)^{3/2}\sqrt{e+fx^2}}\,dx}{2f} + \frac{(bc(de-cf))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2df} - \frac{(bde-bcf-adf)\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx}{2df}$$

$$= \frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}} + \frac{\left(b(de-cf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c}}\sqrt{1-\frac{(be-af)x^2}{e}}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2df\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} - \frac{\left((de-cf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{(-bc+ad)x^2}{a}}}{\sqrt{1-\frac{(de-cf)x^2}{e}}}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{2f\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} - \frac{\left(c(bde-bcf-adf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\right)\text{Subst}\left(\int \frac{1}{(1-dx^2)\sqrt{1-\frac{(-bc+ad)x^2}{a}}\sqrt{1-\frac{(de-cf)x^2}{e}}}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right.}{2adf\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$= \frac{dx\sqrt{a+bx^2}\sqrt{e+fx^2}}{2f\sqrt{c+dx^2}} - \frac{\sqrt{e}\sqrt{de-cf}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}E\left(\sin^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2f\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}} + \frac{b\sqrt{e}(de-cf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}F\left(\sin^{-1}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{2df\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} - \frac{c\sqrt{e}(bde-bcf-adf)\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{2adf\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

Mathematica [A] (verified)

Time = 3.28 (sec) , antiderivative size = 503, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$
$$= \frac{\frac{x\sqrt{a+bx^2}(c+dx^2)}{\sqrt{e+fx^2}} - \frac{\sqrt{c}\sqrt{-de+cf}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-de+cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|\frac{bce-acf}{ade-acf}\right)}{f\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}} + \frac{(be-2af)(de-cf)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,\text{EllipticF}}{\sqrt{e}f^2\sqrt{be-af}\sqrt{\frac{a(e+}{e(a-}}}}{2\sqrt{c+dx^2}}$$

```
[In] Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/Sqrt[e + f*x^2],x]
```

```
[Out] ((x*Sqrt[a + b*x^2]*(c + d*x^2))/Sqrt[e + f*x^2] - (Sqrt[c]*Sqrt[-(d*e) + c
*f]*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(e + f*x^2))]*EllipticE[ArcSin[
(Sqrt[-(d*e) + c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], (b*c*e - a*c*f)/(a*d*e -
 a*c*f)])/(f*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]) + ((b*e - 2*a*f)*(d*e -
 c*f)*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*EllipticF[ArcSi
n[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*e - a*d*e)/(b*c*e -
a*c*f)])/(Sqrt[e]*f^2*Sqrt[b*e - a*f]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]
) + (e*(-(b*d*e) + b*c*f + a*d*f)*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x^2))/(c*(
e + f*x^2))]*EllipticPi[(a*f)/(-(b*e) + a*f), ArcSin[(Sqrt[-(b*e) + a*f]*x)
/(Sqrt[a]*Sqrt[e + f*x^2])], (a*d*e - a*c*f)/(b*c*e - a*c*f)])/(Sqrt[a]*f^2
*Sqrt[-(b*e) + a*f]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]))/(2*Sqrt[c + d*x
^2])
```

Maple [F]

$$\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}dx$$

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
[Out] Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/sqrt(e + f*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/sqrt(f*x^2 + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{\sqrt{f\,x^2+e}}\,dx$$

```
[In] int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2),x)
[Out] int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(1/2), x)
```

3.105 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 163

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{a\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

```
[Out] c*EllipticPi(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(-
a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*e^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x
^2+c))^(1/2)/a/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)^(
1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {567, 551}

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{a\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

```
[In] Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (c*Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi
[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])],
-(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 567

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2
/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(c\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\right)\text{Subst}\left(\int\frac{1}{(1-dx^2)\sqrt{1-\frac{(-bc+ad)x^2}{a}}\sqrt{1-\frac{(de-cf)x^2}{e}}}\,dx,x,\frac{x}{\sqrt{c+dx^2}}\right)}{a\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}\\ &= \frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{a\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.65 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.99

$$\begin{aligned}&\int\frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx\\ &=\frac{c\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cfx}}{\sqrt{e}\sqrt{c+dx^2}}\right),\frac{(-bc+ad)e}{a(de-cf)}\right)}{a\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

```
[In] Integrate[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (c*Sqrt[e]*Sqrt[a + b*x^2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi
[(d*e)/(d*e - c*f), ArcSin[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])],
((-(b*c) + a*d)*e)/(a*(d*e - c*f))])/(a*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2)
)/(a*(c + d*x^2))]*Sqrt[e + f*x^2])
```

Maple [F]

$$\int \frac{\sqrt{d\,x^{2}+c}}{\sqrt{b\,x^{2}+a}\,\sqrt{f\,x^{2}+e}}dx$$

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x)
[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2)/(f*x**2+e)**(1/2),x)
[Out] Integral(sqrt(c + d*x**2)/(sqrt(a + b*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
[Out] int((c + d*x^2)^(1/2)/((a + b*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.106 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 148

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

```
[Out] EllipticE(x*(-a*f+b*e)^(1/2)/e^(1/2)/(b*x^2+a)^(1/2),((-a*d+b*c)*e/c/(-a*f+
b*e))^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)/a/(-a*
f+b*e)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {568, 435}

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

```
[In] Int[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticE[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(b*
e - a*f))])/(a*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e
 + f*x^2])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 568

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{(bc-ad)x^2}{c}}}{\sqrt{1-\frac{(be-af)x^2}{e}}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{a\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&= \frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\sin^{-1}\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 4.65 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{be-af}\,x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{a\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

```
[In] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*Sqrt[e + f*x^2]),x]
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticE[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(b*
e - a*f))])/(a*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e
 + f*x^2])
```

Maple [F]

$$\int \frac{\sqrt{d\,x^{2}+c}}{\left(b\,x^{2}+a\right)^{\frac{3}{2}}\sqrt{f\,x^{2}+e}}dx$$

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x)
[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*f*x^6 + (b^2*
e + 2*a*b*f)*x^4 + a^2*e + (2*a*b*e + a^2*f)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{3}{2}}\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2)/(f*x**2+e)**(1/2),x)
[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)**(3/2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)^{3/2}\sqrt{f\,x^2+e}}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2)),x)
[Out] int((c + d*x^2)^(1/2)/((a + b*x^2)^(3/2)*(e + f*x^2)^(1/2)), x)
```

3.107 $\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \text{Int}\left(\frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}},x\right)$$

```
[Out] Unintegrable((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

```
[In] Int[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
[Out] Defer[Int][((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 20.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

```
[In] Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
[Out] Integrate[((a + b*x^2)^(3/2)*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Maple [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{3}{2}}}dx$$

```
[In] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
[Out] int((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 54.45 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fri
cas")
[Out] integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^4 + 2*e*f
*x^2 + e^2), x)
```

Sympy [N/A]

Not integrable

Time = 17.59 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)**(3/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
[Out] Integral((a + b*x**2)**(3/2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="max
ima")
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gia
c")
[Out] integrate((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 6.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{(b\,x^2+a)^{3/2}\sqrt{d\,x^2+c}}{(f\,x^2+e)^{3/2}}\,dx$$

```
[In] int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
[Out] int(((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

3.108 $\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 484

$$\begin{aligned}\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = &-\frac{(de-cf)x\sqrt{a+bx^2}}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}}\\ &+\frac{\sqrt{c}\sqrt{de-cf}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ef\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\\ &-\frac{c^{3/2}(be-af)\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{aef\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\\ &+\frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

```
[Out] -c^(3/2)*(-a*f+b*e)*(1/(1+x^2*(-c*f+d*e)/c/(f*x^2+e)))^(1/2)*(1+x^2*(-c*f+d
*e)/c/(f*x^2+e))^(1/2)*EllipticF(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2)
/(1+x^2*(-c*f+d*e)/c/(f*x^2+e))^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*(
b*x^2+a)^(1/2)/a/e/f/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^
2+c)^(1/2)+(1/(1+x^2*(-c*f+d*e)/c/(f*x^2+e)))^(1/2)*(1+x^2*(-c*f+d*e)/c/(f*
x^2+e))^(1/2)*EllipticE(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2)/(1+x^2*(
-c*f+d*e)/c/(f*x^2+e))^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*c^(1/2)*(-
c*f+d*e)^(1/2)*(b*x^2+a)^(1/2)/e/f/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c
)^(1/2)-(-c*f+d*e)*x*(b*x^2+a)^(1/2)/e/f/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2)+b*
c*EllipticPi(x*(-c*f+d*e)^(1/2)/e^(1/2)/(d*x^2+c)^(1/2),d*e/(-c*f+d*e),(-(-
a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*e^(1/2)*(b*x^2+a)^(1/2)*(c*(f*x^2+e)/e/(d*x
^2+c))^(1/2)/a/f/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(f*x^2+e)
^(1/2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {571, 567, 551, 568, 433, 429, 506, 422}

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\,\text{EllipticPi}\left(\frac{de}{de-cf},\arcsin\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{dx^2+c}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{e+fx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$-\frac{c^{3/2}\sqrt{a+bx^2}(be-af)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{aef\sqrt{c+dx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$
$$+\frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{de-cf}E\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ef\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\frac{x\sqrt{a+bx^2}(de-cf)}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}}$$

```
[In] Int[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
```

```
[Out] -(((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*f*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])) +
(Sqrt[c]*Sqrt[d*e - c*f]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d*e - c*f]*
x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))]])/(e*f*Sq
rt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) - (c^(3/2)*(b*e - a*f)
*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e + f*x
^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))]])/(a*e*f*Sqrt[d*e - c*f]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (b*c*Sqrt[e]*Sqrt[a + b*x^
2]*Sqrt[(c*(e + f*x^2))/(e*(c + d*x^2))]*EllipticPi[(d*e)/(d*e - c*f), ArcS
in[(Sqrt[d*e - c*f]*x)/(Sqrt[e]*Sqrt[c + d*x^2])], -(((b*c - a*d)*e)/(a*(d*
e - c*f)))]])/(a*f*Sqrt[d*e - c*f]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqr
t[e + f*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 567

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2
/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 568

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 571

```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/((e_) + (f_.)*(x_
)^2)^(3/2), x_Symbol] :> Dist[b/f, Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*Sqr
t[e + f*x^2]), x], x] - Dist[(b*e - a*f)/f, Int[Sqrt[c + d*x^2]/(Sqrt[a + b
*x^2]*(e + f*x^2)^(3/2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\text{integral} = \frac{b\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\sqrt{e+fx^2}}\,dx}{f} - \frac{(be-af)\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}(e+fx^2)^{3/2}}\,dx}{f}$$

$$= -\frac{\left((be-af)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{(-de+cf)x^2}{c}}}{\sqrt{1-\frac{(-be+af)x^2}{a}}}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+ \frac{\left(bc\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\right)\text{Subst}\left(\int \frac{1}{(1-dx^2)\sqrt{1-\frac{(-bc+ad)x^2}{a}}\sqrt{1-\frac{(de-cf)x^2}{e}}}\,dx, x, \frac{x}{\sqrt{c+dx^2}}\right)}{af\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

$$= \frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{\left((be-af)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{(-be+af)x^2}{a}}\sqrt{1-\frac{(-de+cf)x^2}{c}}}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{ef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$
$$+ \frac{\left((be-af)(-de+cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{(-be+af)x^2}{a}}\sqrt{1-\frac{(-de+cf)x^2}{c}}}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$$= -\frac{(de-cf)x\sqrt{a+bx^2}}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{c^{3/2}(be-af)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{aef\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+ \frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$
$$- \frac{\left(a(-de+cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{(-be+af)x^2}{a}}}{\left(1-\frac{(-de+cf)x^2}{c}\right)^{3/2}}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{cef\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

$$= -\frac{(de-cf)x\sqrt{a+bx^2}}{ef\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{\sqrt{c}\sqrt{de-cf}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ef\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$- \frac{c^{3/2}(be-af)\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{aef\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$
$$+ \frac{bc\sqrt{e}\sqrt{a+bx^2}\sqrt{\frac{c(e+fx^2)}{e(c+dx^2)}}\Pi\left(\frac{de}{de-cf};\sin^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{e}\sqrt{c+dx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{af\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{e+fx^2}}$$

Mathematica [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx$$

```
[In] Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2),x]
[Out] Integrate[(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(e + f*x^2)^(3/2), x]
```

Maple [F]

$$\int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{\frac{3}{2}}}dx$$

```
[In] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
[Out] int((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fri
cas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(f^2*x^4 + 2*e*f*x
^2 + e^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x**2+a)**(1/2)*(d*x**2+c)**(1/2)/(f*x**2+e)**(3/2),x)
[Out] Integral(sqrt(a + b*x**2)*sqrt(c + d*x**2)/(e + f*x**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="max
ima")
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{bx^2+a}\sqrt{dx^2+c}}{(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)*(d*x^2+c)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gia
c")
[Out] integrate(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)/(f*x^2 + e)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}}{(f\,x^2+e)^{3/2}}\,dx$$

```
[In] int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2),x)
[Out] int(((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2))/(e + f*x^2)^(3/2), x)
```

3.109 $\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\left(e+fx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 319

$$\begin{aligned}\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx &= \frac{(de-cf)x\sqrt{a+bx^2}}{e(be-af)\sqrt{c+dx^2}\sqrt{e+fx^2}} \\ &\quad - \frac{\sqrt{c}\sqrt{de-cf}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{e(be-af)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\ &\quad + \frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{ae\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}\end{aligned}$$

```
[Out] c^(3/2)*(1/(1+x^2*(-c*f+d*e)/c/(f*x^2+e)))^(1/2)*(1+x^2*(-c*f+d*e)/c/(f*x^2
+e))^(1/2)*EllipticF(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2)/(1+x^2*(-c*
f+d*e)/c/(f*x^2+e))^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*(b*x^2+a)^(1/
2)/a/e/(-c*f+d*e)^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/2)-(1/
(1+x^2*(-c*f+d*e)/c/(f*x^2+e)))^(1/2)*(1+x^2*(-c*f+d*e)/c/(f*x^2+e))^(1/2)*
EllipticE(x*(-c*f+d*e)^(1/2)/c^(1/2)/(f*x^2+e)^(1/2)/(1+x^2*(-c*f+d*e)/c/(f
*x^2+e))^(1/2),(-(-a*d+b*c)*e/a/(-c*f+d*e))^(1/2))*c^(1/2)*(-c*f+d*e)^(1/2)
*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)^(1/
2)+(-c*f+d*e)*x*(b*x^2+a)^(1/2)/e/(-a*f+b*e)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2
)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.147, Rules used = {568, 433, 429, 506, 422}

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right),-\frac{(bc-ad)e}{a(de-cf)}\right)}{ae\sqrt{c+dx^2}\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{\sqrt{c}\sqrt{a+bx^2}\sqrt{de-cf}E\left(\arctan\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{fx^2+e}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{e\sqrt{c+dx^2}(be-af)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{x\sqrt{a+bx^2}(de-cf)}{e\sqrt{c+dx^2}\sqrt{e+fx^2}(be-af)}$$

```
[In] Int[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] ((d*e - c*f)*x*Sqrt[a + b*x^2])/(e*(b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[e + f*x
^2]) - (Sqrt[c]*Sqrt[d*e - c*f]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d*e
- c*f]*x)/(Sqrt[c]*Sqrt[e + f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/
(e*(b*e - a*f)*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]) + (c^
(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d*e - c*f]*x)/(Sqrt[c]*Sqrt[e
+ f*x^2])], -(((b*c - a*d)*e)/(a*(d*e - c*f)))])/(a*e*Sqrt[d*e - c*f]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 429

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 433

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 506

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 568

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{(-de+cf)x^2}{c}}}{\sqrt{1-\frac{(-be+af)x^2}{a}}}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} \\
&= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{(-be+af)x^2}{a}}\sqrt{1-\frac{(-de+cf)x^2}{c}}}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{e\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} \\
&\quad - \frac{\left((-de+cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{(-be+af)x^2}{a}}\sqrt{1-\frac{(-de+cf)x^2}{c}}}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{ce\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}} \\
&= \frac{(de-cf)x\sqrt{a+bx^2}}{e(be-af)\sqrt{c+dx^2}\sqrt{e+fx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}\,F\left(\tan^{-1}\left(\frac{\sqrt{de-cf}\,x}{\sqrt{c}\sqrt{e+fx^2}}\right)\middle|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ae\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} \\
&\quad + \frac{\left(a(-de+cf)\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{(-be+af)x^2}{a}}}{\left(1-\frac{(-de+cf)x^2}{c}\right)^{3/2}}\,dx, x, \frac{x}{\sqrt{e+fx^2}}\right)}{ce(be-af)\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}
\end{aligned}
$$

$$= \frac{(de-cf)x\sqrt{a+bx^2}}{e(be-af)\sqrt{c+dx^2}\sqrt{e+fx^2}} - \frac{\sqrt{c}\sqrt{de-cf}\sqrt{a+bx^2}E\left(\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{e(be-af)\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}} + \frac{c^{3/2}\sqrt{a+bx^2}F\left(\tan^{-1}\left(\frac{\sqrt{de-cf}x}{\sqrt{c}\sqrt{e+fx^2}}\right)|-\frac{(bc-ad)e}{a(de-cf)}\right)}{ae\sqrt{de-cf}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\sqrt{c+dx^2}}$$

Mathematica [A] (verified)

Time = 5.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{a(e+fx^2)}}E\left(\arcsin\left(\frac{\sqrt{-be+af}x}{\sqrt{a}\sqrt{e+fx^2}}\right)|\frac{a(-de+cf)}{c(-be+af)}\right)}{e\sqrt{-be+af}\sqrt{a+bx^2}\sqrt{\frac{e(c+dx^2)}{c(e+fx^2)}}}$$

```
[In] Integrate[Sqrt[c + d*x^2]/(Sqrt[a + b*x^2]*(e + f*x^2)^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[c + d*x^2]*Sqrt[(e*(a + b*x^2))/(a*(e + f*x^2))]*EllipticE[Ar
cSin[(Sqrt[-(b*e) + a*f]*x)/(Sqrt[a]*Sqrt[e + f*x^2])], (a*(-(d*e) + c*f))/
(c*(-(b*e) + a*f))])/(e*Sqrt[-(b*e) + a*f]*Sqrt[a + b*x^2]*Sqrt[(e*(c + d*x
^2))/(c*(e + f*x^2))])
```

Maple [F]

$$\int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}\,\left(f\,x^2+e\right)^{\frac{3}{2}}}dx$$

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x)
```

Fricas [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="fri
cas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*f^2*x^6 + (2*b*
e*f + a*f^2)*x^4 + a*e^2 + (b*e^2 + 2*a*e*f)*x^2), x)
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(1/2)/(f*x**2+e)**(3/2),x)
[Out] Integral(sqrt(c + d*x**2)/(sqrt(a + b*x**2)*(e + f*x**2)**(3/2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="max
ima")
[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(1/2)/(f*x^2+e)^(3/2),x, algorithm="gia
c")
[Out] integrate(sqrt(d*x^2 + c)/(sqrt(b*x^2 + a)*(f*x^2 + e)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}}{\sqrt{a+bx^2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{\sqrt{b\,x^2+a}\,{\left(f\,x^2+e\right)}^{3/2}}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2)),x)
[Out] int((c + d*x^2)^(1/2)/((a + b*x^2)^(1/2)*(e + f*x^2)^(3/2)), x)
```

3.110 $\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}} \, dx = \text{Int}\left(\frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}, x\right)$$

```
[Out] Unintegrable((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}} \, dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}} \, dx$$

```
[In] Int[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
```

```
[Out] Defer[Int][Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}} \, dx$$

Mathematica [N/A]

Not integrable

Time = 21.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}\,dx$$

```
[In] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),x]
[Out] Integrate[Sqrt[c + d*x^2]/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)), x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)^{\frac{3}{2}}\,(f\,x^2+e)^{\frac{3}{2}}}dx$$

```
[In] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x)
[Out] int((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 3.26

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="fri
cas")
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*f^2*x^8 + 2*(
b^2*e*f + a*b*f^2)*x^6 + (b^2*e^2 + 4*a*b*e*f + a^2*f^2)*x^4 + a^2*e^2 + 2*
(a*b*e^2 + a^2*e*f)*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 11.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{\frac{3}{2}}\,(e+fx^2)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)/(b*x**2+a)**(3/2)/(f*x**2+e)**(3/2),x)
[Out] Integral(sqrt(c + d*x**2)/((a + b*x**2)**(3/2)*(e + f*x**2)**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="max
ima")
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}\,(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{dx^2+c}}{(bx^2+a)^{\frac{3}{2}}(fx^2+e)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)/(b*x^2+a)^(3/2)/(f*x^2+e)^(3/2),x, algorithm="gia
c")
[Out] integrate(sqrt(d*x^2 + c)/((b*x^2 + a)^(3/2)*(f*x^2 + e)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 6.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{c+dx^2}}{(a+bx^2)^{3/2}(e+fx^2)^{3/2}}\,dx = \int \frac{\sqrt{d\,x^2+c}}{(b\,x^2+a)^{3/2}\,(f\,x^2+e)^{3/2}}\,dx$$

```
[In] int((c + d*x^2)^(1/2)/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)),x)
[Out] int((c + d*x^2)^(1/2)/((a + b*x^2)^(3/2)*(e + f*x^2)^(3/2)), x)
```

3.111 $\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 541

$$\begin{aligned}
&\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx \\
&= \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}} - \frac{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}} \\
&\quad + \frac{(bc-ad)\sqrt{e}(2be-af)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} \\
&\quad - \frac{a(adf-b(de+cf))\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^2\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}
\end{aligned}$$

```
[Out] 1/2*x*(d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2)-1/2*EllipticE(x*(-a*d
+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2),(c*(-a*f+b*e)/(-a*d+b*c)/e)^(1/2))*c^(1
/2)*(-a*d+b*c)^(1/2)*(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)*(f*x^2+e)^(1/2)/b/(d*x
^2+c)^(1/2)/(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)-1/2*a*(a*d*f-b*(c*f+d*e))*Ellip
ticPi(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(-a*f+b*
e)/(-a*d+b*c)/e)^(1/2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)/b^2
/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)+1
/2*(-a*d+b*c)*(-a*f+2*b*e)*EllipticF(x*(-a*f+b*e)^(1/2)/e^(1/2)/(b*x^2+a)^(
1/2),((-a*d+b*c)*e/c/(-a*f+b*e))^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)*(a*(f*x^2+e
)/e/(b*x^2+a))^(1/2)/b^2/c/(-a*f+b*e)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.206, Rules used = {570, 568, 435, 567, 551, 566, 430}

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{e}\sqrt{c+dx^2}(bc-ad)(2be-af)\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}(adf-b(cf+de))\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{2b^2\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}E\left(\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}} + \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}}$$

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/Sqrt[a + b*x^2],x]
```

```
[Out] (x*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/(2*Sqrt[a + b*x^2]) - (Sqrt[c]*Sqrt[b*c
 - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]*EllipticE[Arc
Sin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e - a*f))/((b*c -
 a*d)*e)])/(2*b*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]) + ((
b*c - a*d)*Sqrt[e]*(2*b*e - a*f)*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a
 + b*x^2))]*EllipticF[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])]
, ((b*c - a*d)*e)/(c*(b*e - a*f))])/(2*b^2*c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d
*x^2))/(c*(a + b*x^2))]*Sqrt[e + f*x^2]) - (a*(a*d*f - b*(d*e + c*f))*Sqrt[
c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(b*c - a*
d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e - a*f))/
((b*c - a*d)*e)])/(2*b^2*Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*(a
 + b*x^2))]*Sqrt[e + f*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 566

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x], x
, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 567

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2
/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 568

```
Int[Sqrt[(c_) + (d_.)*(x_)^2]/(((a_) + (b_.)*(x_)^2)^(3/2)*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(a*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[Sqrt[1 - (b*c - a*d)*(x^2/c)]/Sqrt[1 - (b*e - a*f)*(x^2/e)], x], x, x/
Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rule 570

```
Int[(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2])/Sqrt[(e_) + (f_.)
*(x_)^2], x_Symbol] :> Simp[x*Sqrt[a + b*x^2]*(Sqrt[c + d*x^2]/(2*Sqrt[e +
f*x^2])), x] + (Dist[e*((b*e - a*f)/(2*f)), Int[Sqrt[c + d*x^2]/(Sqrt[a + b
*x^2]*(e + f*x^2)^(3/2)), x], x] - Dist[(b*d*e - b*c*f - a*d*f)/(2*f^2), In
t[Sqrt[e + f*x^2]/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[(b*e - a
*f)*((d*e - 2*c*f)/(2*f^2)), Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e
+ f*x^2]), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NegQ[(d*e - c*f)/c]
```

Rubi steps

$$\text{integral} = \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}} - \frac{(a(bc-ad))\int \frac{\sqrt{e+fx^2}}{(a+bx^2)^{3/2}\sqrt{c+dx^2}}\,dx}{2b} + \frac{((bc-ad)(2be-af))\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2b^2} + \frac{(bde+bcf-adf)\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx}{2b^2}$$

$$= \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}} - \frac{\left((bc-ad)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\right)\text{Subst}\left(\int \frac{\sqrt{1-\frac{(be-af)x^2}{e}}}{\sqrt{1-\frac{(bc-ad)x^2}{c}}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}} + \frac{\left((bc-ad)(2be-af)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c}}\sqrt{1-\frac{(be-af)x^2}{e}}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b^2c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} + \frac{\left(a(bde+bcf-adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\right)\text{Subst}\left(\int \frac{1}{(1-bx^2)\sqrt{1-\frac{(bc-ad)x^2}{c}}\sqrt{1-\frac{(be-af)x^2}{e}}}\,dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{2b^2c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

$$= \frac{x\sqrt{c+dx^2}\sqrt{e+fx^2}}{2\sqrt{a+bx^2}} - \frac{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}\,E\left(\sin^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2b\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}} + \frac{(bc-ad)\sqrt{e}(2be-af)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}F\left(\sin^{-1}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{2b^2c\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}} + \frac{a(bde+bcf-adf)\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\Pi\left(\frac{bc}{bc-ad};\sin^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{2b^2\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

Mathematica [A] (verified)

Time = 3.48 (sec) , antiderivative size = 512, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \frac{\sqrt{a+bx^2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\left(b^2c\sqrt{bc-ad}x\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}(e+fx^2)-bc\sqrt{bc-ad}\sqrt{e}\sqrt{be-af}\sqrt{a+bx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}E\left(a\right.\right.}{}$$

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[e + f*x^2])/Sqrt[a + b*x^2],x]
```

```
[Out] (Sqrt[a + b*x^2]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*(b^2*c*Sqrt[b*c - a*
d]*x*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*(e + f*x^2) - b*c*Sqrt[b*c - a*d
]*Sqrt[e]*Sqrt[b*e - a*f]*Sqrt[a + b*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^
2))]*EllipticE[ArcSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*
e - a*d*e)/(b*c*e - a*c*f)] + Sqrt[b*c - a*d]*(2*b*c - a*d)*Sqrt[e]*Sqrt[b*
e - a*f]*Sqrt[a + b*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], (b*c*e - a*d*e)/(b*c*e
 - a*c*f)] - a*Sqrt[c]*(a*d*f - b*(d*e + c*f))*Sqrt[a + b*x^2]*Sqrt[(a*(e +
 f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(b*c - a*d), ArcSin[(Sqrt[b*c -
a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (b*c*e - a*c*f)/(b*c*e - a*d*e)]))/(2*a
*b^2*Sqrt[b*c - a*d]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2])
```

Maple [F]

$$\int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}}dx$$

```
[In] int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

```
[Out] int((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x)
```

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \text{Timed out}$$

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="fri
cas")
```

```
[Out] Timed out
```

Sympy [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx$$

```
[In] integrate((d*x**2+c)**(1/2)*(f*x**2+e)**(1/2)/(b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(c + d*x**2)*sqrt(e + f*x**2)/sqrt(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="max
ima")
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{fx^2+e}}{\sqrt{bx^2+a}}\,dx$$

```
[In] integrate((d*x^2+c)^(1/2)*(f*x^2+e)^(1/2)/(b*x^2+a)^(1/2),x, algorithm="gia
c")
[Out] integrate(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/sqrt(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{c+dx^2}\sqrt{e+fx^2}}{\sqrt{a+bx^2}}\,dx = \int \frac{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}{\sqrt{b\,x^2+a}}\,dx$$

```
[In] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(1/2),x)
[Out] int(((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2))/(a + b*x^2)^(1/2), x)
```

3.112 $\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Int}\left(\frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}},x\right)$$

```
[Out] Unintegrable((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

```
[In] Int[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] Defer[Int][(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 5.07 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

```
[In] Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
[Out] Integrate[(a + b*x^2)^(3/2)/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{(b\,x^2+a)^{\frac{3}{2}}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}dx$$

```
[In] int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
[Out] int((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 60.52 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fri
cas")
[Out] integral((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(d*f*x^4 + (d*e
+ c*f)*x^2 + c*e), x)
```

Sympy [N/A]

Not integrable

Time = 9.86 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(a+bx^2)^{\frac{3}{2}}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

```
[In] integrate((b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
[Out] Integral((a + b*x**2)**(3/2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="max
ima")
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{(bx^2+a)^{\frac{3}{2}}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
[Out] integrate((b*x^2 + a)^(3/2)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [N/A]

Not integrable

Time = 7.78 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^2)^{3/2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{{\left(b\,x^2+a\right)}^{3/2}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
[Out] int((a + b*x^2)^(3/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.113 $\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 159

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

```
[Out] a*EllipticPi(x*(-a*d+b*c)^(1/2)/c^(1/2)/(b*x^2+a)^(1/2),b*c/(-a*d+b*c),(c*(
-a*f+b*e)/(-a*d+b*c)/e)^(1/2))*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1
/2)/c^(1/2)/(-a*d+b*c)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2
)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {567, 551}

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{bx^2+a}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{e+fx^2}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

```
[In] Int[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(
b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e
- a*f))/((b*c - a*d)*e)])/(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*
(a + b*x^2))]*Sqrt[e + f*x^2])
```

Rule 551

```
Int[1/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*(x
_)^2]), x_Symbol] :> Simp[(1/(a*Sqrt[c]*Sqrt[e]*Rt[-d/c, 2]))*EllipticPi[b*
(c/(a*d)), ArcSin[Rt[-d/c, 2]*x], c*(f/(d*e))], x] /; FreeQ[{a, b, c, d, e,
 f}, x] &&  !GtQ[d/c, 0] && GtQ[c, 0] && GtQ[e, 0] &&  !( !GtQ[f/e, 0] && S
implerSqrtQ[-f/e, -d/c])
```

Rule 567

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/(Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.)*
(x_)^2]), x_Symbol] :> Dist[a*Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/((1 - b*x^2)*Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2
/e)]), x], x, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\text{integral}&=\frac{\left(a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\right)\text{Subst}\left(\int\frac{1}{(1-bx^2)\sqrt{1-\frac{(bc-ad)x^2}{c}}\sqrt{1-\frac{(be-af)x^2}{e}}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}\\&=\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\Pi\left(\frac{bc}{bc-ad};\sin^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right)|\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.60 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int\frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx\\&=\frac{a\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticPi}\left(\frac{bc}{bc-ad},\arcsin\left(\frac{\sqrt{bc-ad}x}{\sqrt{c}\sqrt{a+bx^2}}\right),\frac{c(be-af)}{(bc-ad)e}\right)}{\sqrt{c}\sqrt{bc-ad}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

```
[In] Integrate[Sqrt[a + b*x^2]/(Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```



```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticPi[(b*c)/(
b*c - a*d), ArcSin[(Sqrt[b*c - a*d]*x)/(Sqrt[c]*Sqrt[a + b*x^2])], (c*(b*e
- a*f))/((b*c - a*d)*e)])/(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(a*(c + d*x^2))/(c*
(a + b*x^2))]*Sqrt[e + f*x^2])
```

Maple [F]

$$\int \frac{\sqrt{b\,x^{2}+a}}{\sqrt{d\,x^{2}+c}\,\sqrt{f\,x^{2}+e}}dx$$

[In] int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)

[Out] int((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)

Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \text{Timed out}$$

[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

[In] integrate((b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)

[Out] Integral(sqrt(a + b*x**2)/(sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)

Maxima [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)

Giac [F]

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate((b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate(sqrt(b*x^2 + a)/(sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx^2}}{\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{\sqrt{b\,x^2+a}}{\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int((a + b*x^2)^(1/2)/((c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.114 $\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 148

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}$$

```
[Out] EllipticF(x*(-a*f+b*e)^(1/2)/e^(1/2)/(b*x^2+a)^(1/2),((-a*d+b*c)*e/c/(-a*f+
b*e))^(1/2))*e^(1/2)*(d*x^2+c)^(1/2)*(a*(f*x^2+e)/e/(b*x^2+a))^(1/2)/c/(-a*
f+b*e)^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(f*x^2+e)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {566, 430}

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{bx^2+a}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{e+fx^2}\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

```
[In] Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(b*
```

```
e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e
 + f*x^2])
```

Rule 430

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1/(Sqrt[a]*Sqrt[c]*Rt[-d/c, 2]))*EllipticF[ArcSin[Rt[-d/c, 2]*x], b*(c
/(a*d))], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a,
0] &&  !(NegQ[b/a] && SimplerSqrtQ[-b/a, -d/c])
```

Rule 566

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]*Sqrt[(e_) + (f_.
)*(x_)^2]), x_Symbol] :> Dist[Sqrt[c + d*x^2]*(Sqrt[a*((e + f*x^2)/(e*(a +
b*x^2)))]/(c*Sqrt[e + f*x^2]*Sqrt[a*((c + d*x^2)/(c*(a + b*x^2)))])), Subst
[Int[1/(Sqrt[1 - (b*c - a*d)*(x^2/c)]*Sqrt[1 - (b*e - a*f)*(x^2/e)]), x], x
, x/Sqrt[a + b*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\left(\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\right)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{(bc-ad)x^2}{c}}\sqrt{1-\frac{(be-af)x^2}{e}}}\,dx,x,\frac{x}{\sqrt{a+bx^2}}\right)}{c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}\\ &= \frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}F\left(\sin^{-1}\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right)|\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

Mathematica [A] (verified)

Time = 2.94 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int\frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx\\ &=\frac{\sqrt{e}\sqrt{c+dx^2}\sqrt{\frac{a(e+fx^2)}{e(a+bx^2)}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{be-af}x}{\sqrt{e}\sqrt{a+bx^2}}\right),\frac{(bc-ad)e}{c(be-af)}\right)}{c\sqrt{be-af}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{e+fx^2}}\end{aligned}$$

```
[In] Integrate[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] (Sqrt[e]*Sqrt[c + d*x^2]*Sqrt[(a*(e + f*x^2))/(e*(a + b*x^2))]*EllipticF[Ar
cSin[(Sqrt[b*e - a*f]*x)/(Sqrt[e]*Sqrt[a + b*x^2])], ((b*c - a*d)*e)/(c*(b*
e - a*f))])/(c*Sqrt[b*e - a*f]*Sqrt[(a*(c + d*x^2))/(c*(a + b*x^2))]*Sqrt[e
 + f*x^2])
```

Maple [F]

$$\int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}dx$$

```
[In] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b*d*f*x^6 + (b*d*
e + (b*c + a*d)*f)*x^4 + a*c*e + (a*c*f + (b*c + a*d)*e)*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(1/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(a + b*x**2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="m
axima")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(1/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="g
iac")
```

```
[Out] integrate(1/(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx^2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx = \int \frac{1}{\sqrt{b\,x^2+a}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
```

```
[Out] int(1/((a + b*x^2)^(1/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

3.115 $\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\text{Int}\left(\frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}},x\right)$$

```
[Out] Unintegrable(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

```
[In] Int[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
[Out] Defer[Int][1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Rubi steps

$$\text{integral}=\int \frac{1}{\left(a+bx^2\right)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 10.59 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

```
[In] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]),x]
```

```
[Out] Integrate[1/((a + b*x^2)^(3/2)*Sqrt[c + d*x^2]*Sqrt[e + f*x^2]), x]
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{1}{(b\,x^2+a)^{\frac{3}{2}}\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}dx$$

```
[In] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

```
[Out] int(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 3.74

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="f
ricas")
```

```
[Out] integral(sqrt(b*x^2 + a)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)/(b^2*d*f*x^8 + (b^
2*d*e + (b^2*c + 2*a*b*d)*f)*x^6 + ((b^2*c + 2*a*b*d)*e + (2*a*b*c + a^2*d)
*f)*x^4 + a^2*c*e + (a^2*c*f + (2*a*b*c + a^2*d)*e)*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 7.33 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(a+bx^2)^{\frac{3}{2}}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx$$

```
[In] integrate(1/(b*x**2+a)**(3/2)/(d*x**2+c)**(1/2)/(f*x**2+e)**(1/2),x)
[Out] Integral(1/((a + b*x**2)**(3/2)*sqrt(c + d*x**2)*sqrt(e + f*x**2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="m
axima")
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{(bx^2+a)^{\frac{3}{2}}\sqrt{dx^2+c}\sqrt{fx^2+e}}\,dx$$

```
[In] integrate(1/(b*x^2+a)^(3/2)/(d*x^2+c)^(1/2)/(f*x^2+e)^(1/2),x, algorithm="g
iac")
[Out] integrate(1/((b*x^2 + a)^(3/2)*sqrt(d*x^2 + c)*sqrt(f*x^2 + e)), x)
```

Mupad [N/A]

Not integrable

Time = 9.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{1}{(a+bx^2)^{3/2}\sqrt{c+dx^2}\sqrt{e+fx^2}}\,dx=\int \frac{1}{{\left(b\,x^2+a\right)}^{3/2}\,\sqrt{d\,x^2+c}\,\sqrt{f\,x^2+e}}\,dx$$

```
[In] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)),x)
[Out] int(1/((a + b*x^2)^(3/2)*(c + d*x^2)^(1/2)*(e + f*x^2)^(1/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_count
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).___name___ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation ="none"
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```